Basic Magnetic Resonance Imaging

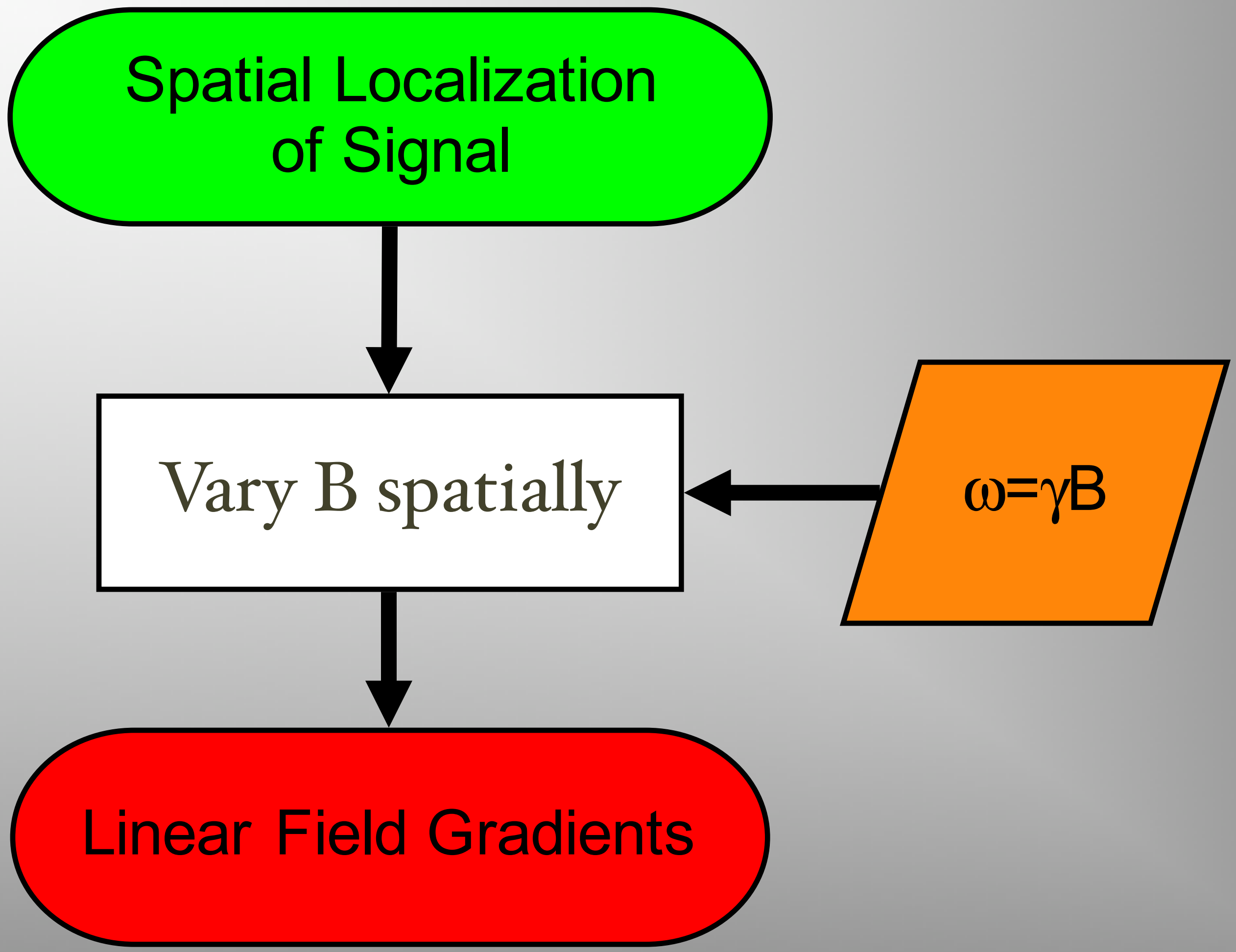
Mike Tyszka, Ph.D.

Magnetic Resonance Theory Course 2004

Caltech Brain Imaging Center

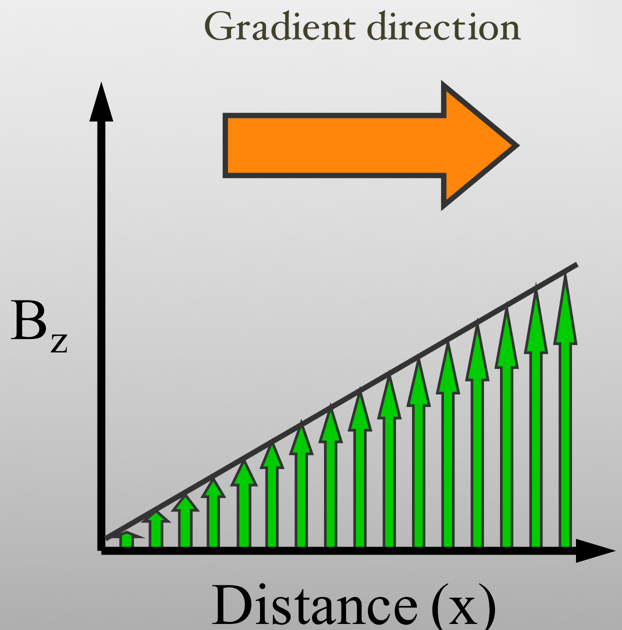
California Institute of Technology

Spatial Encoding



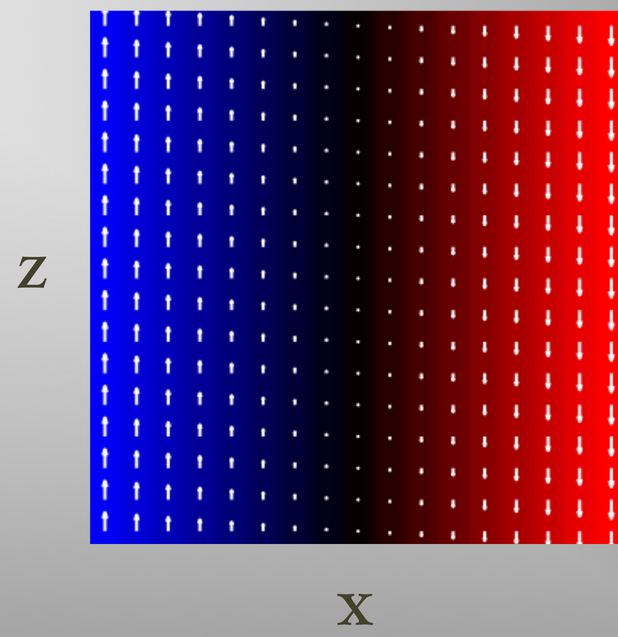
Linear Magnetic Field Gradients

One-Dimensional



Two-Dimensional

$B_z(x,z)$



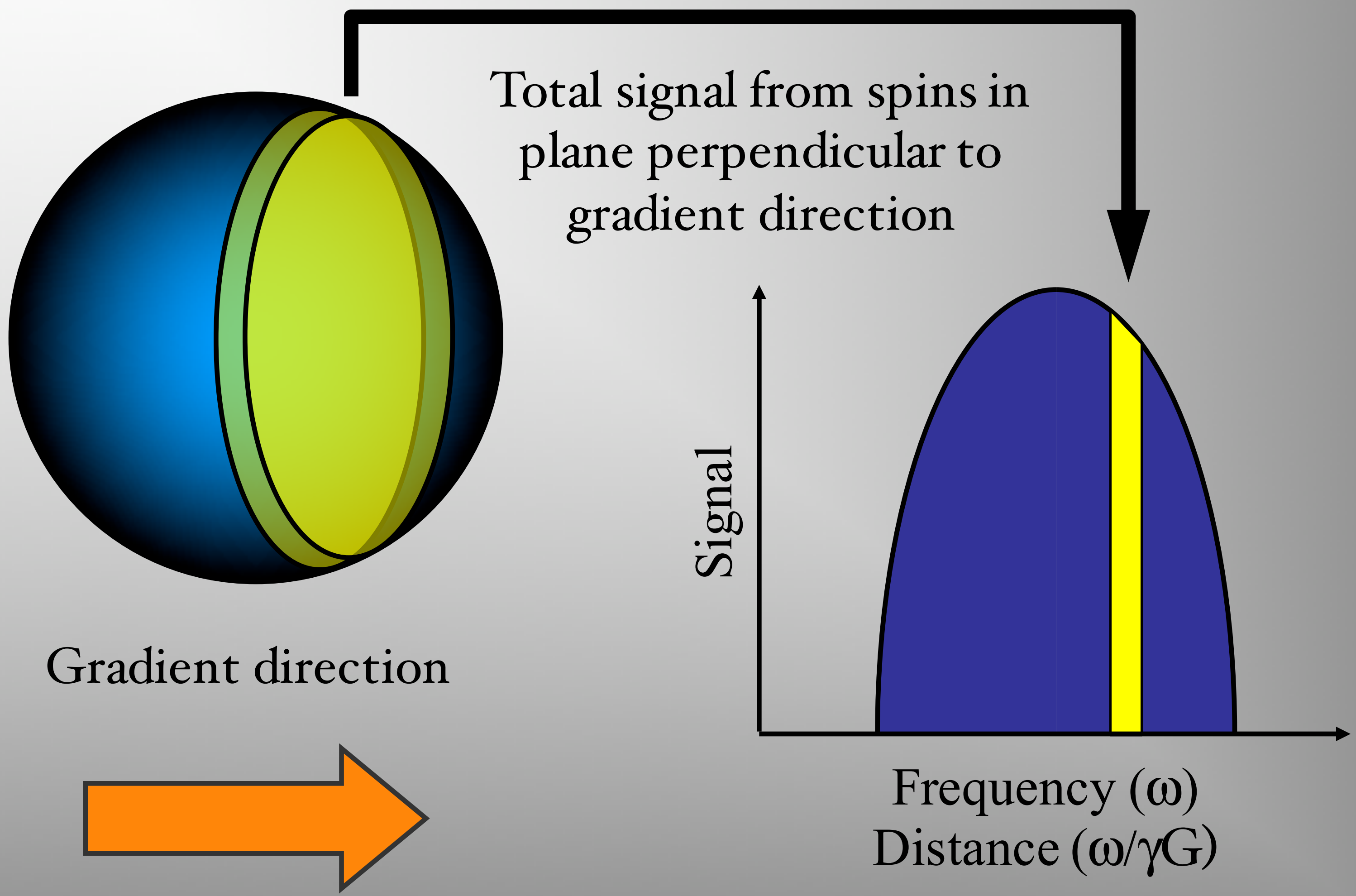
$$\frac{\partial B_z}{\partial x} = G_x$$

$$\frac{\partial B_z}{\partial y} = 0$$

$$\frac{\partial B_z}{\partial z} = 0$$

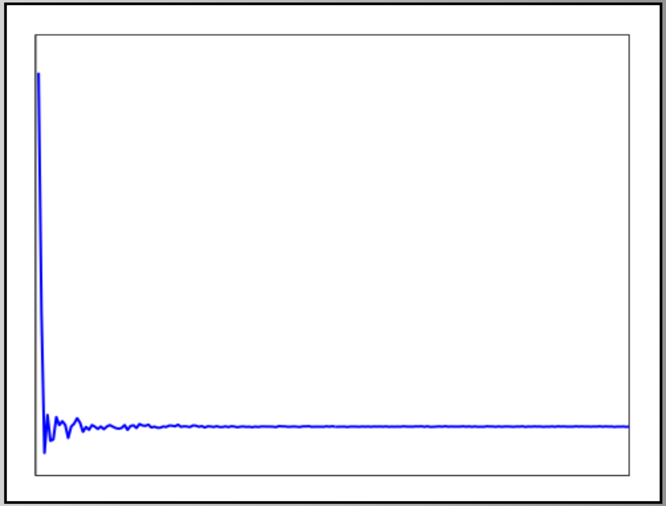
Deviation from B_o typically very small (~0.1%)

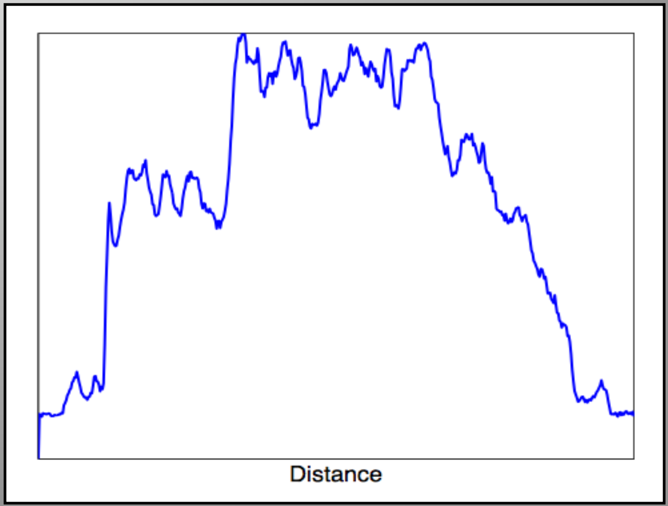
Frequency Encoding



The FID in a Field Gradient

- FID contains total signal from all spins at all frequencies/locations.
- Each frequency component represents the total signal from a plane perpendicular to the gradient direction.
- Frequency analysis of FID reveals "projection" of object.



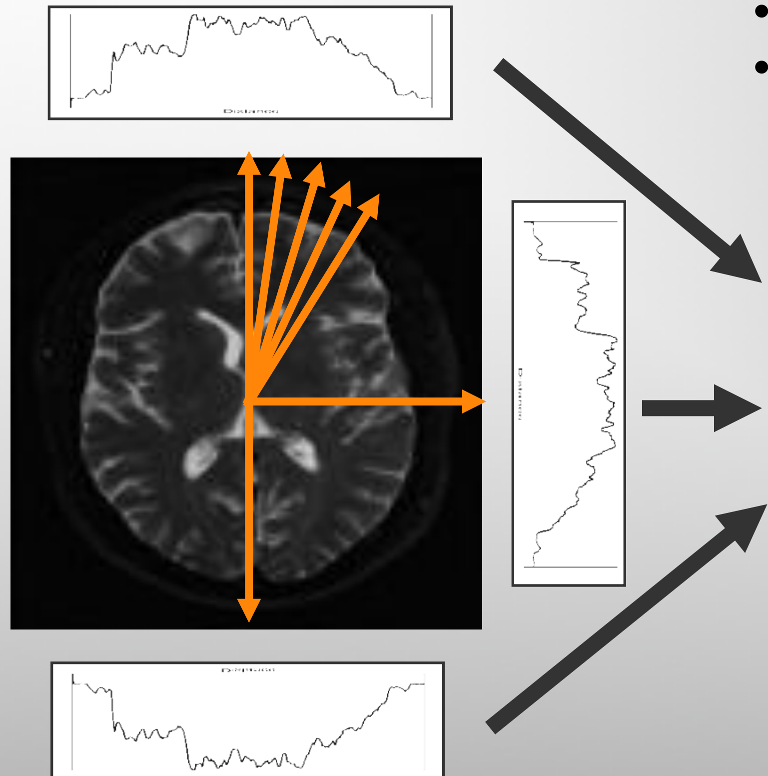


MRI from Projections

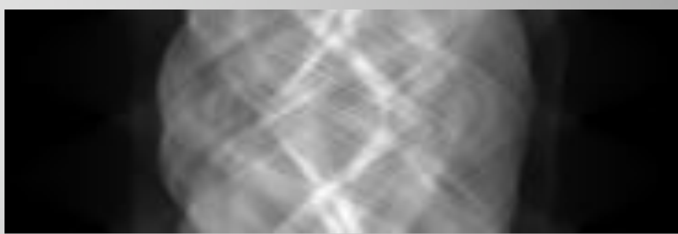
- Can projections be turned into an image?

Back Projection

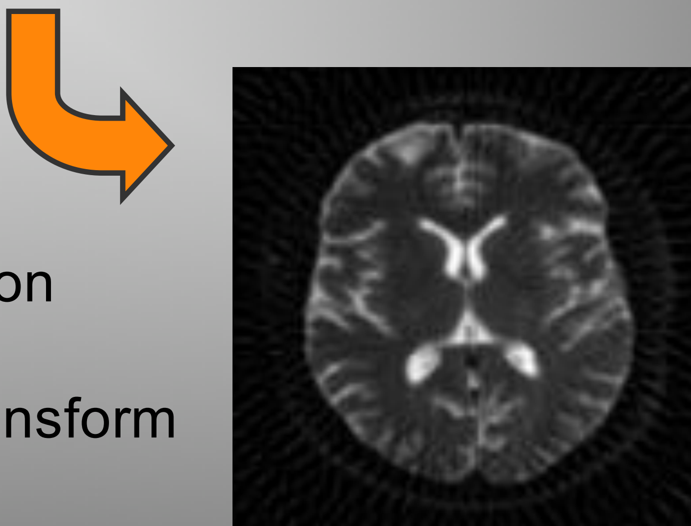
- Sweep gradient direction through 180°
- Acquire projections of object for each direction



Sinogram



Back Projection
or
Inverse Radon Transform

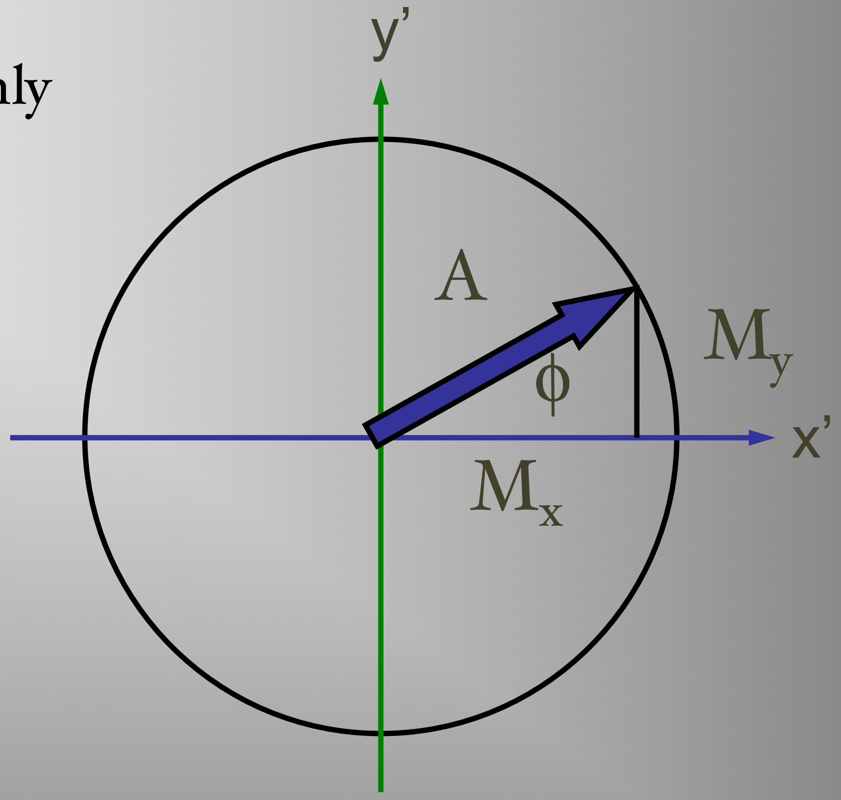


Frequency Analysis for MRI

MR Signal as a Complex Quantity

- Both the magnitude and phase of the MR signal are measured.
- Two signal channels are commonly referred to as "real" and "imaginary".

$$M = Ae^{i\phi}$$
$$= A\cos(\phi) + iA\sin(\phi)$$
$$= M_x + iM_y$$



The Fourier Transform (FT)

The Fourier Transform is an Integral Transform which, for MRI, relates a time varying waveform to the corresponding frequency spectrum.

FORWARD TRANSFORM (time -> frequency)

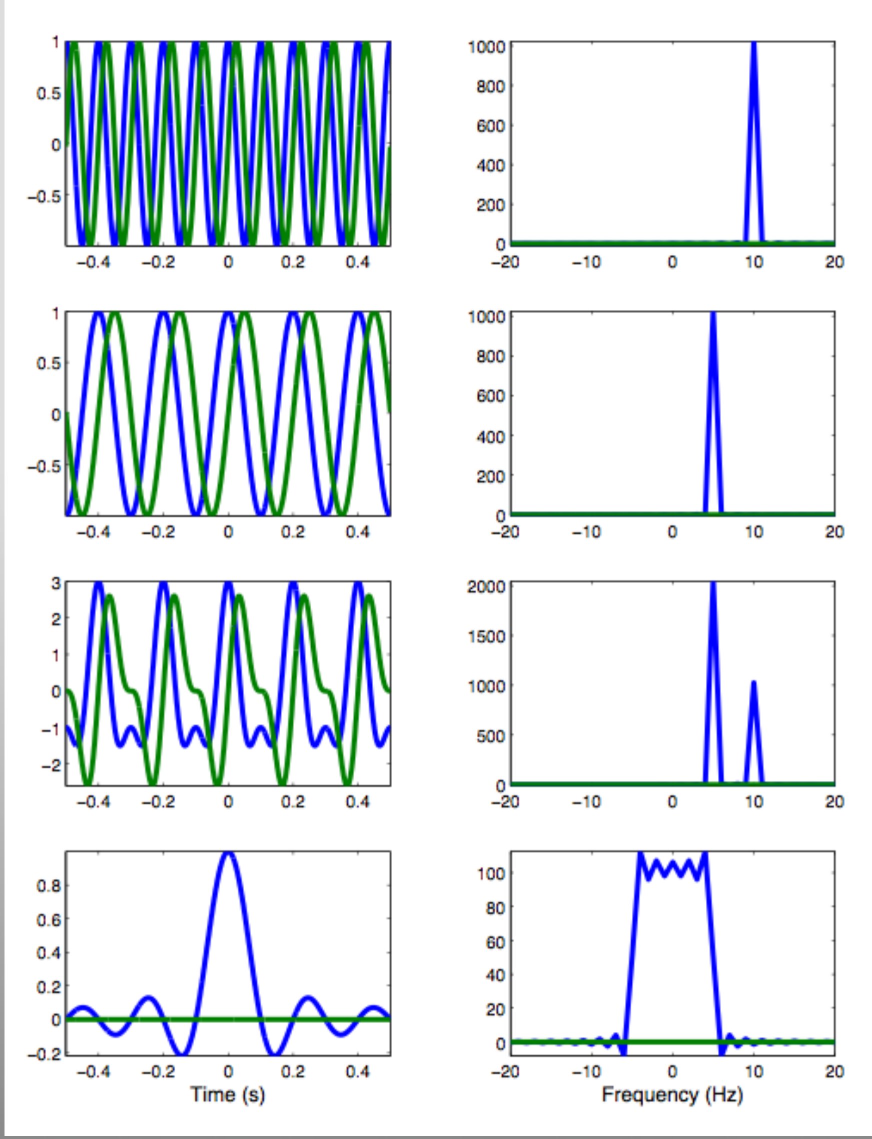
$$\bar{S}(\omega) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, S(t)\, e^{i\omega t}$$

REVERSE TRANSFORM (frequency -> time)

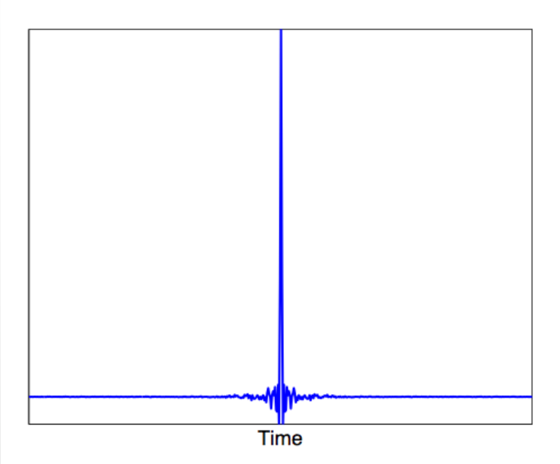
$$S(t) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} dt\, \bar{S}(\omega) e^{-i\omega t}$$

The Fast Fourier Transform (FFT)

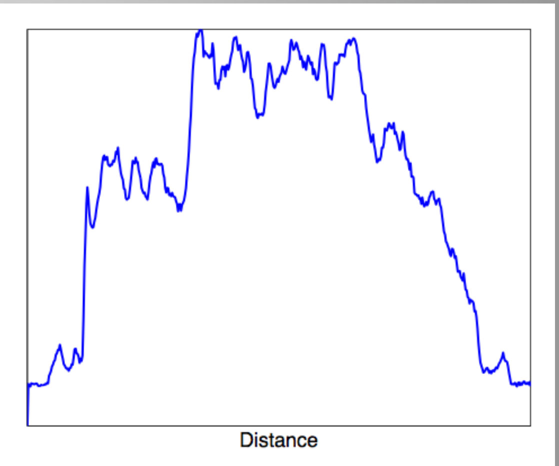
- Numerical algorithm for efficient computation of the discrete Fourier Transform.
- Requires discrete complex valued samples.
- Both forward and reverse transforms can be calculated.

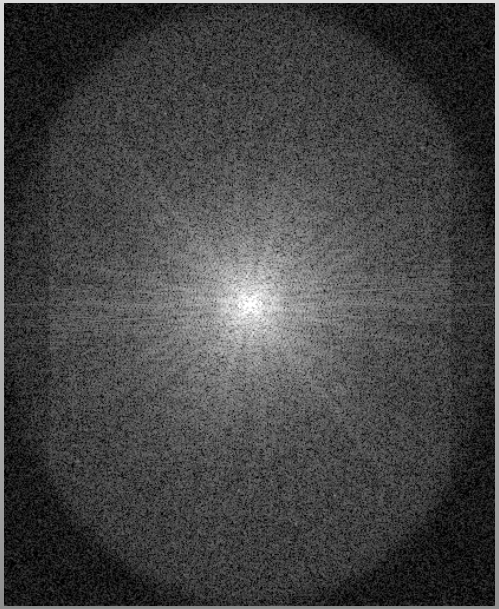


The Two Dimensional Fourier Transform

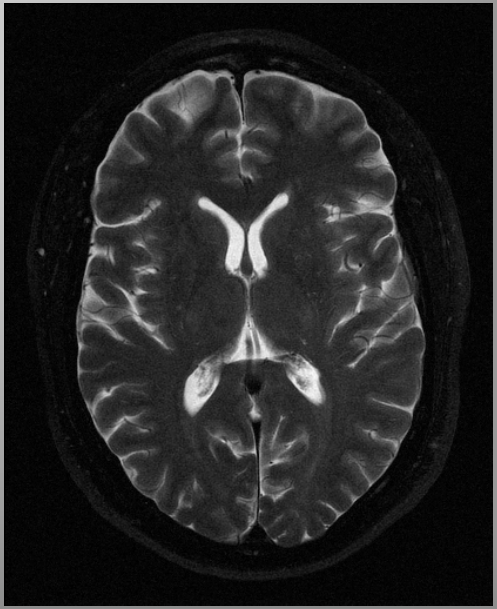


1D FT

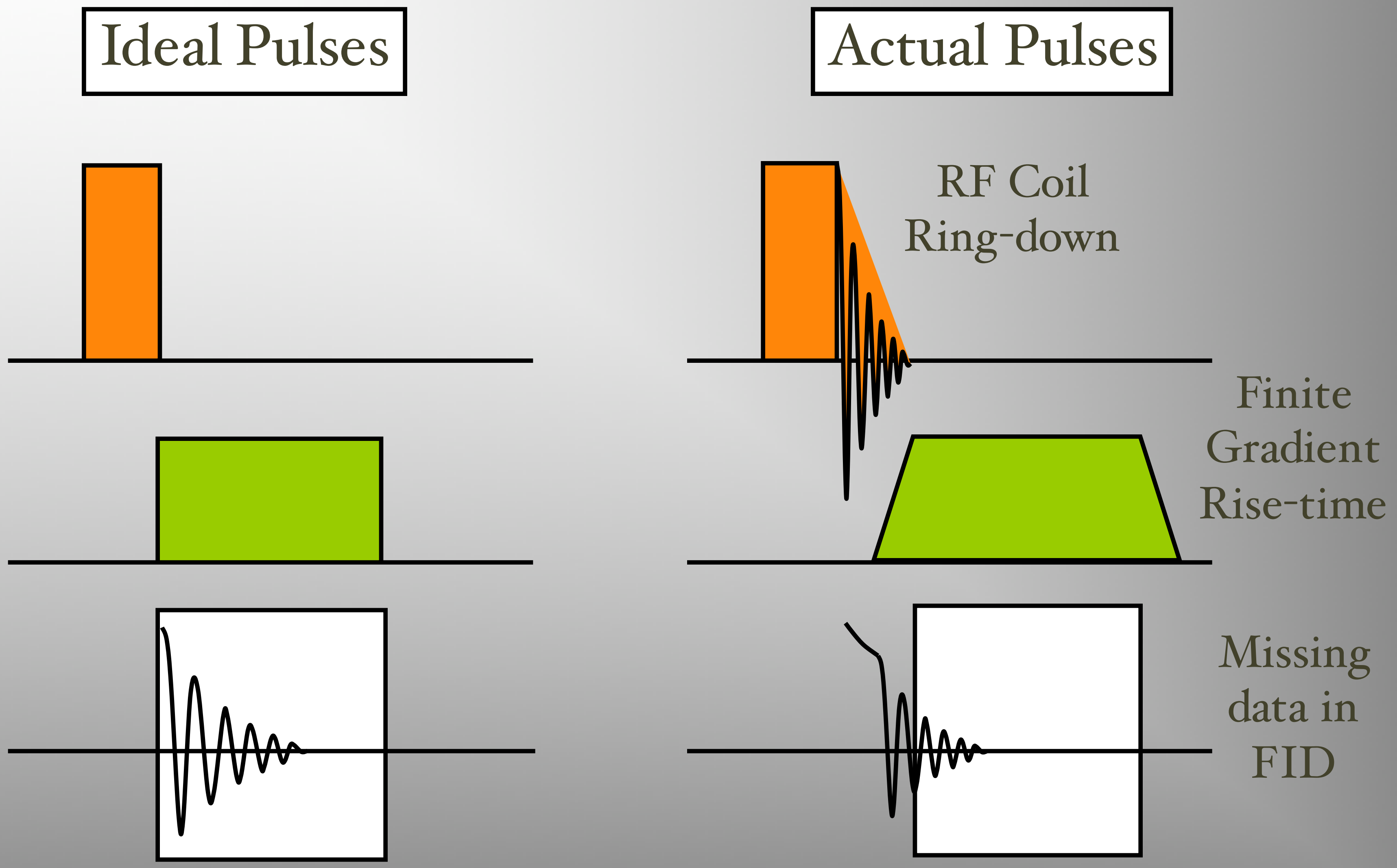




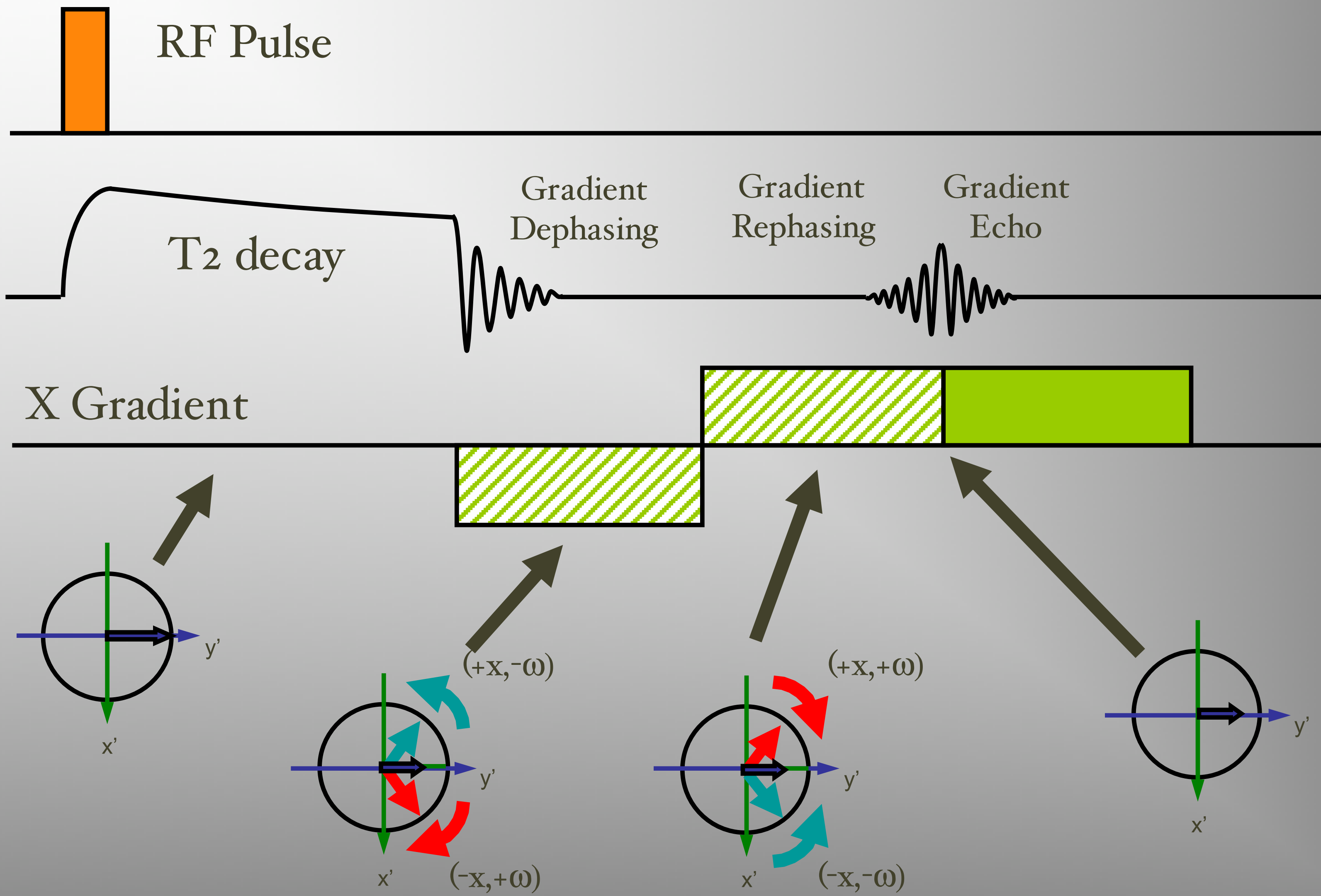
2D FT



Limitations of FID Imaging

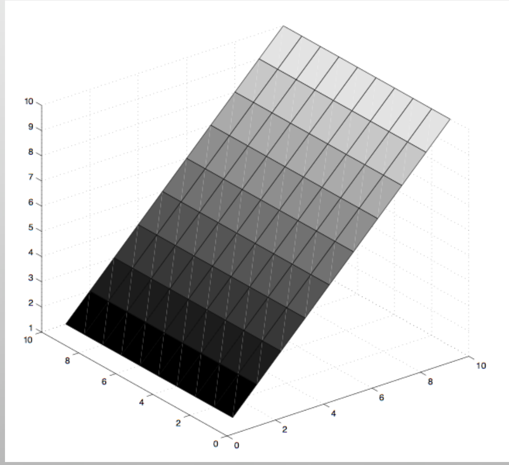


The Gradient Echo

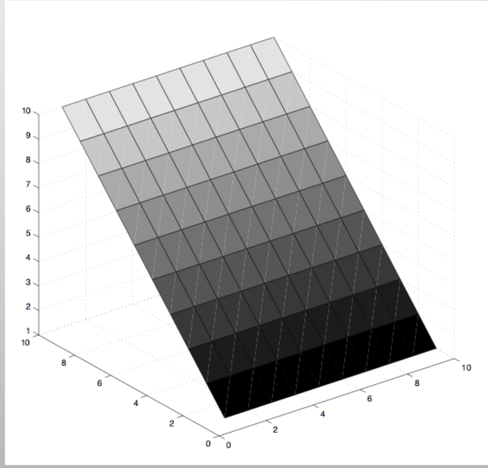


More than One Dimension

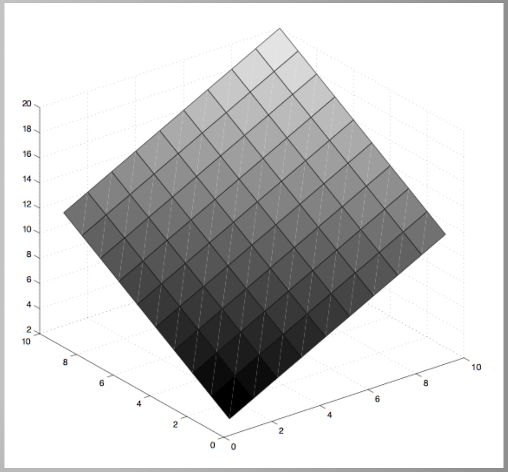
- Sum of two linear gradients is another linear gradient.
- Cannot frequency encode all spatial dimensions simultaneously.



$$B_z(x,y) = G_x x$$



$$B_z(x,y) = G_y y$$



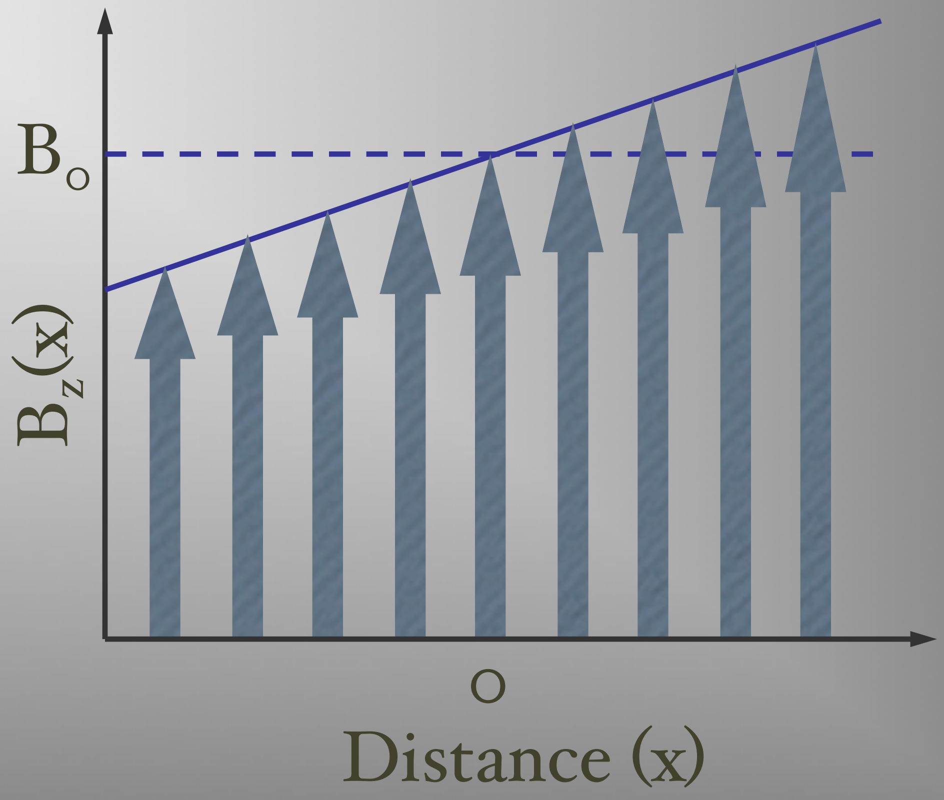
$$B_z(x,y) = G_x x + G_y y$$
$$= \sqrt{G_x^2 + G_y^2}\left[\frac{G_x x + G_y y}{\sqrt{G_x^2 + G_y^2}}\right]$$
$$= G_u u$$

MRI without Back-projection?

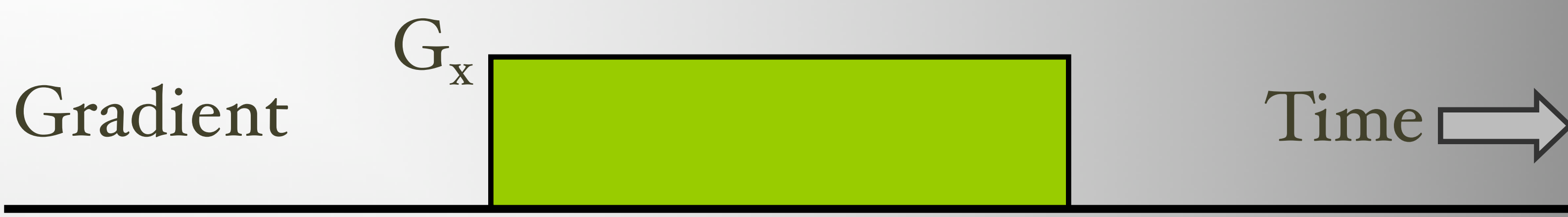
Field Gradients and Magnetization Phase

$$B_z(x) = B_0 + G_x x$$

$$\phi(x,t) = \omega(x)t$$
$$= \gamma B_z(x)t$$
$$= \gamma G_x x t$$

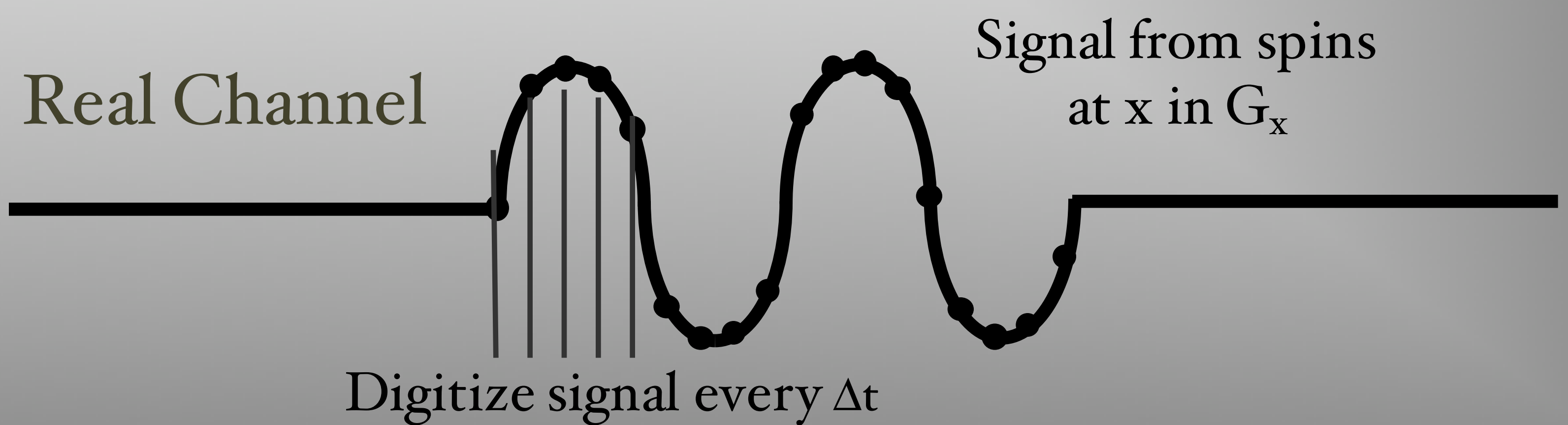


Phase Encoding (Part 1)



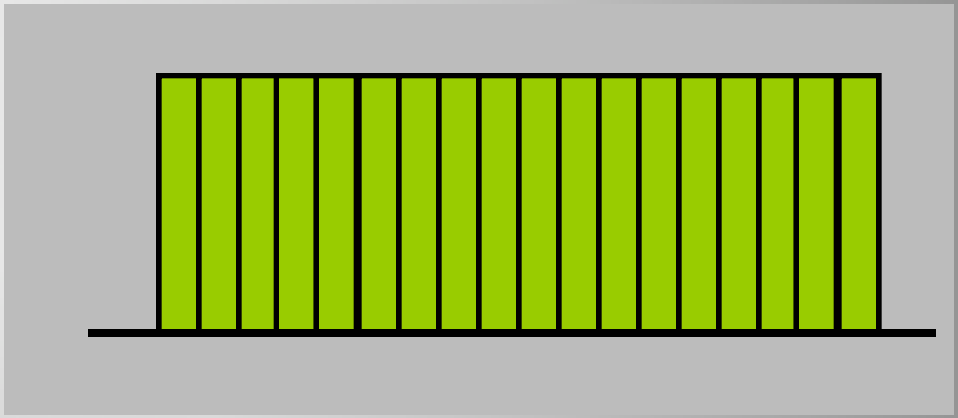
Precession Frequency $\omega = \gamma G_x\, x$

Digitized Signal $\mathrm{Re}[S(x,t)] = \sin(\omega \cdot n\Delta t)$

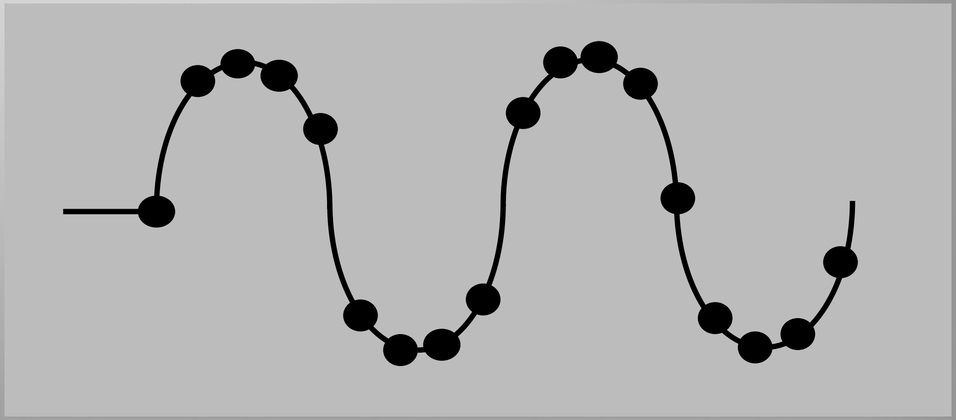


Phase Encoding (Part 2)

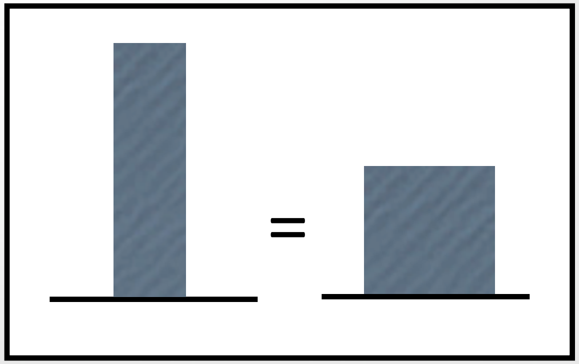
Increase gradient pulse duration by Δt for each repeat



Digitize one signal sample at end of gradient pulse for each repeat

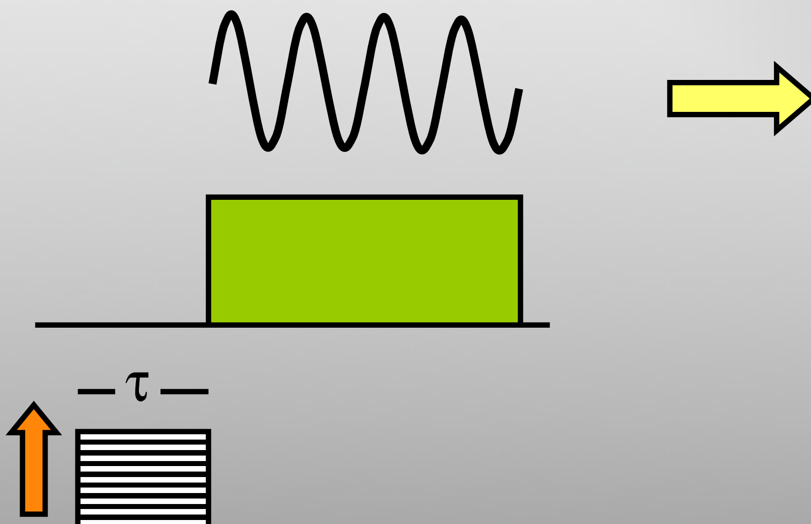


Phase Encoding (Part 3)

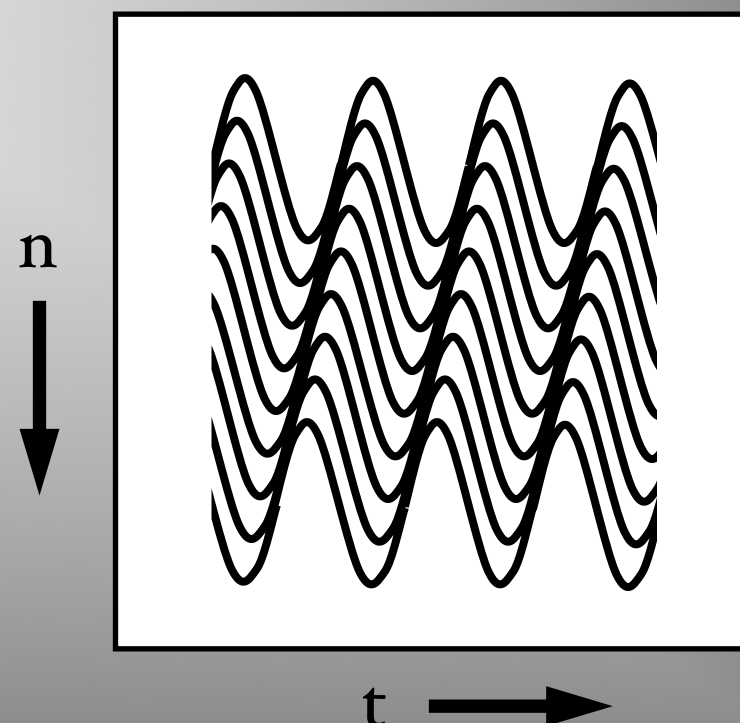


$$\phi_x(x,t) = \gamma G_x x t$$

$$\phi_y(y,n) = \gamma\left(n\Delta G_y\right) y \tau$$



Increment Y "phase encoding" gradient pulse and repeat signal acquisition



k-space

k measures phase accumulated per unit distance over a time interval (radians/m) due to a time varying gradient (pulse, waveform, etc).

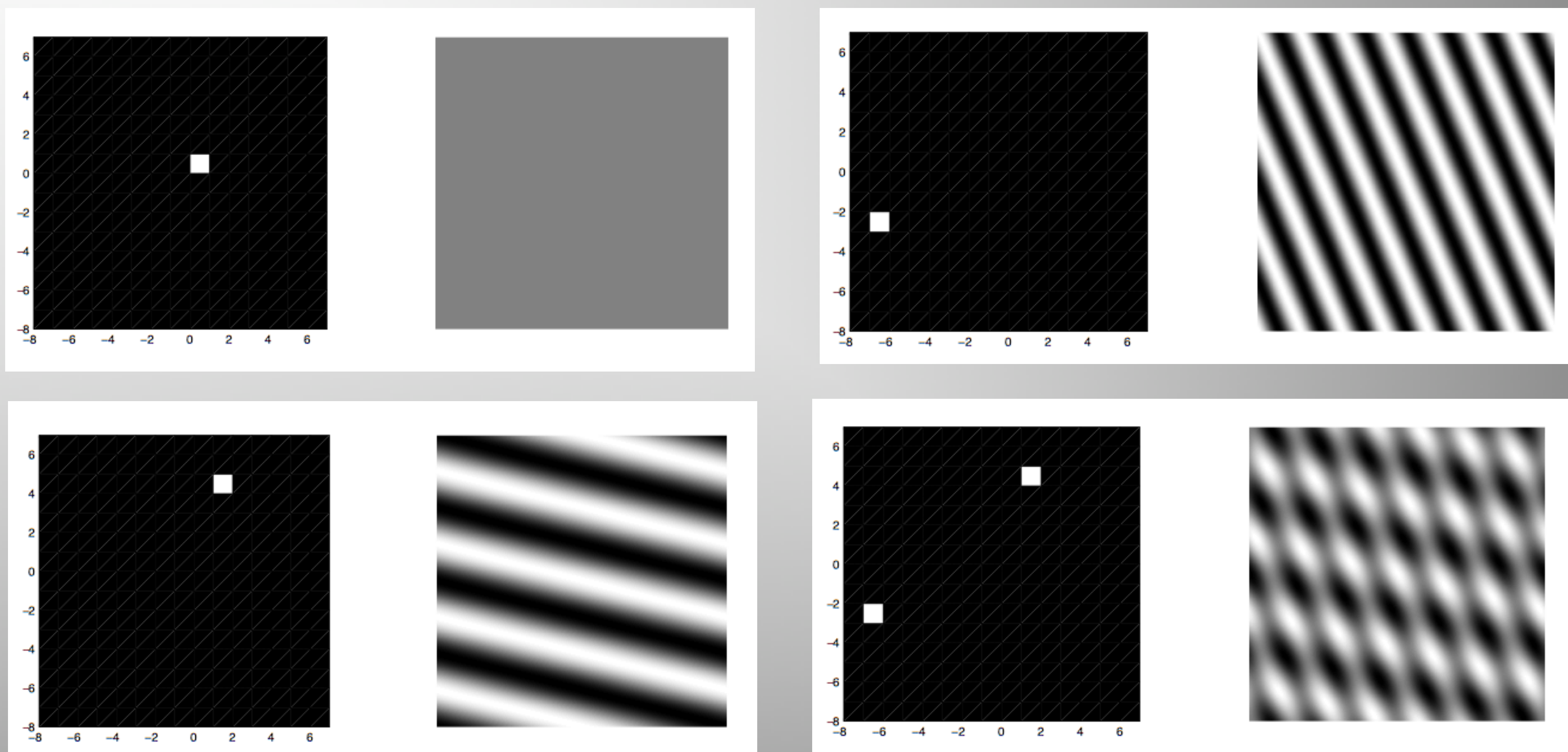
$$\mathbf{k}(t) = \int_0^t dt' \gamma \mathbf{G}(t')$$

k is a vector => k-space can be multidimensional.

The components of **k** (k_x, k_y, etc.) have units of radians/m.

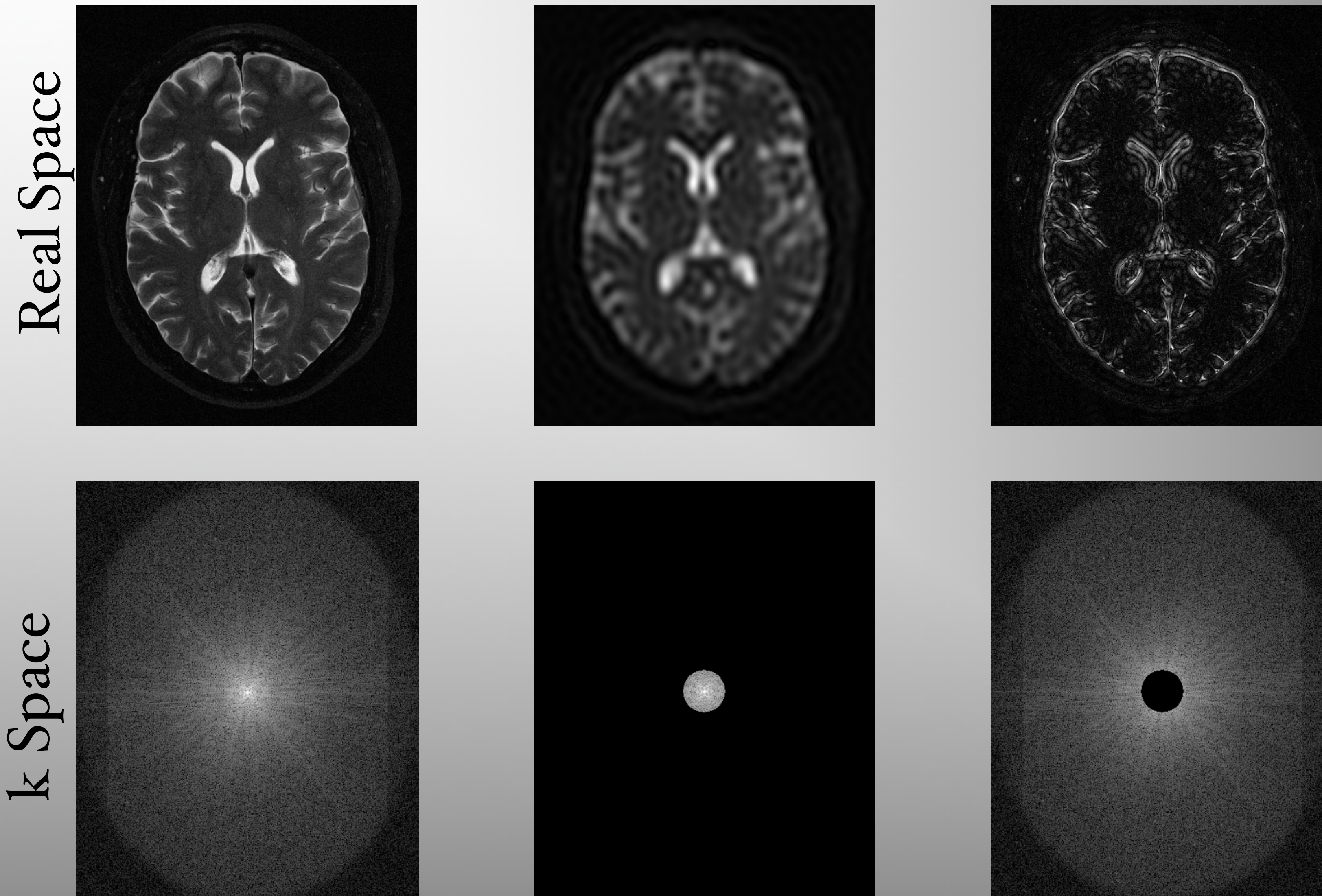
Spatial Frequencies and k-space

k-space Real space



Coordinates in k-space equate to spatial frequency.

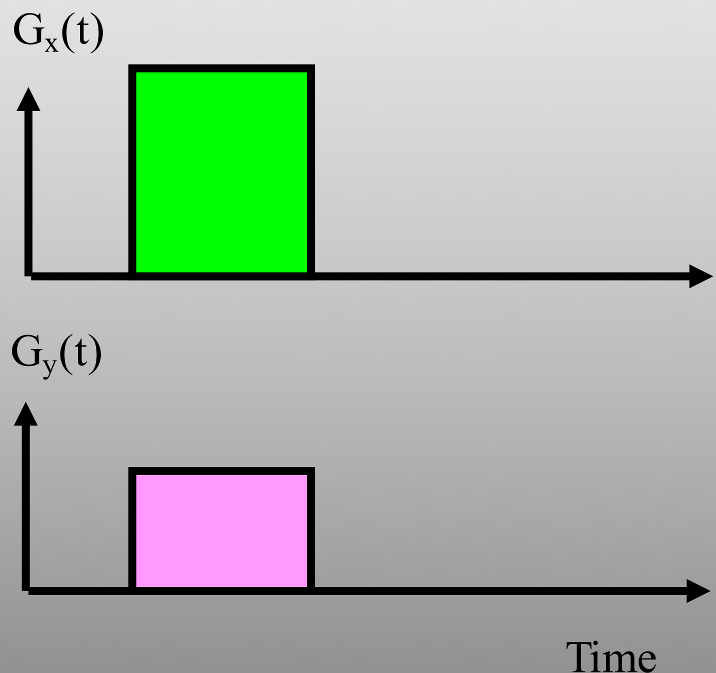
k-space Spatial Frequency Regions

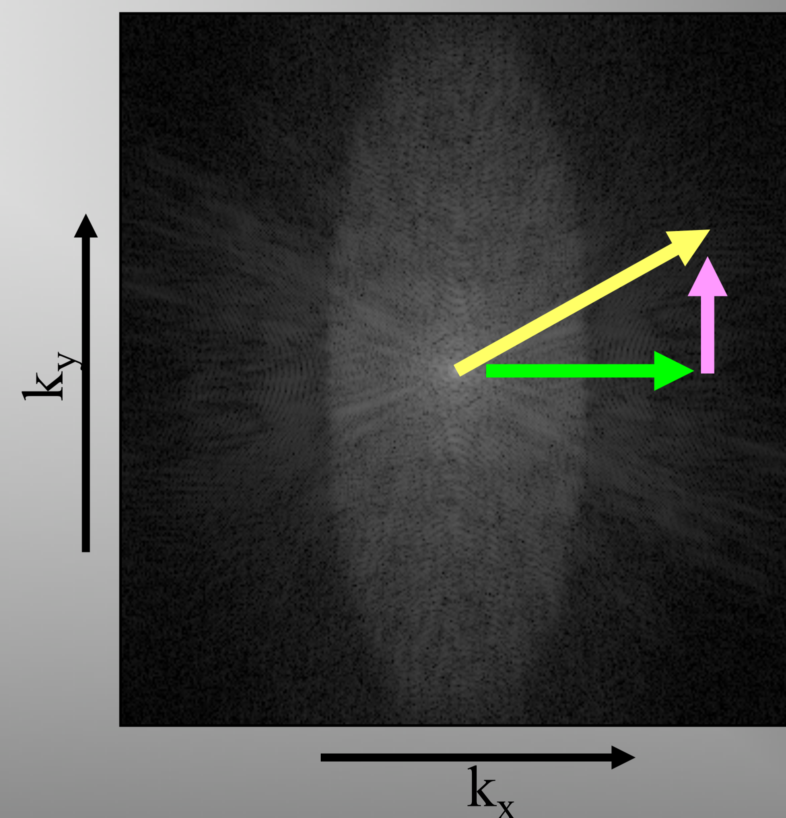


k-space Maneuvers

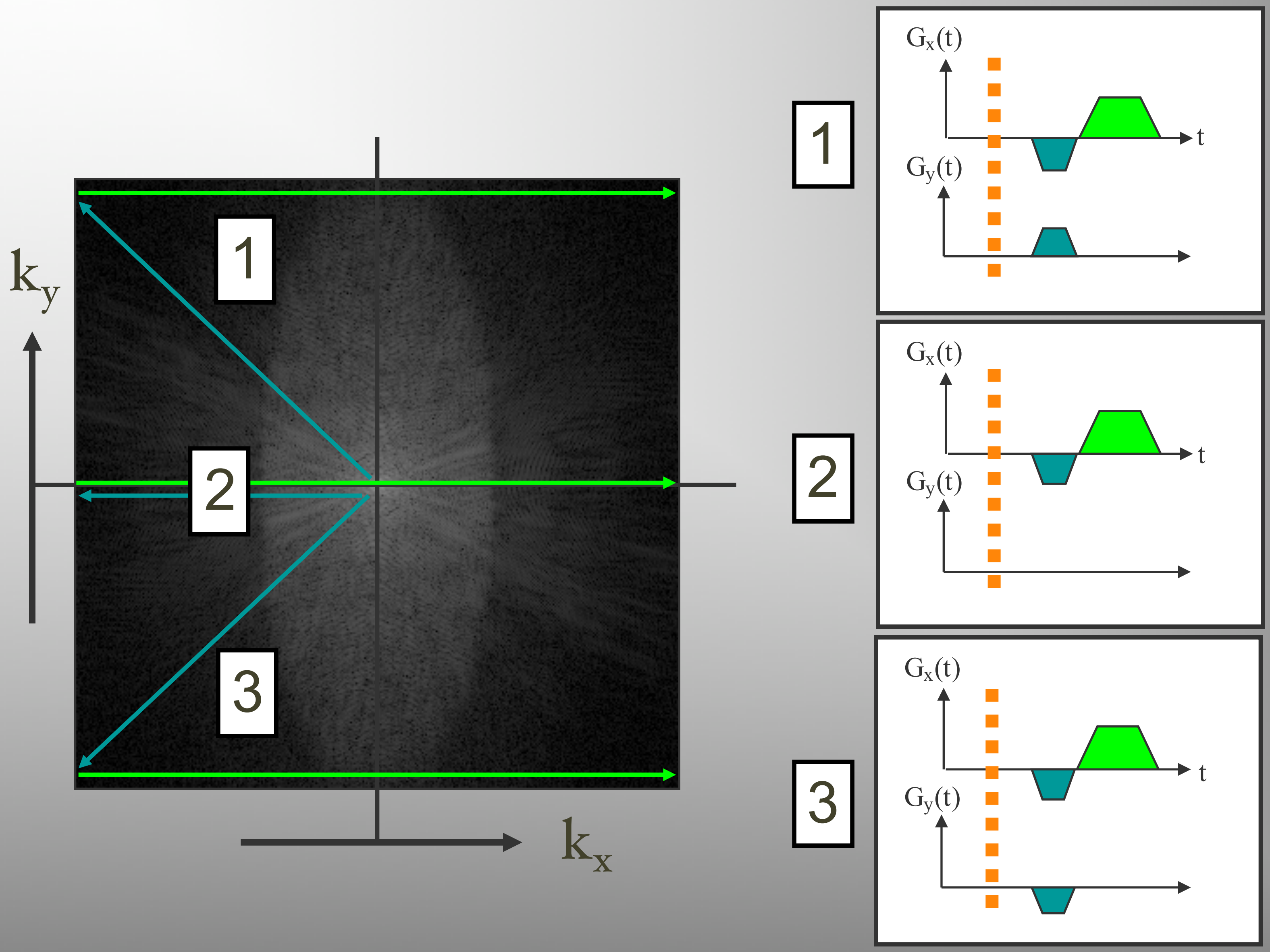
$$\mathbf{k}(t) = \int_0^t dt' \gamma \mathbf{G}(t')$$

Gradient pulses move the spin system in k-space.



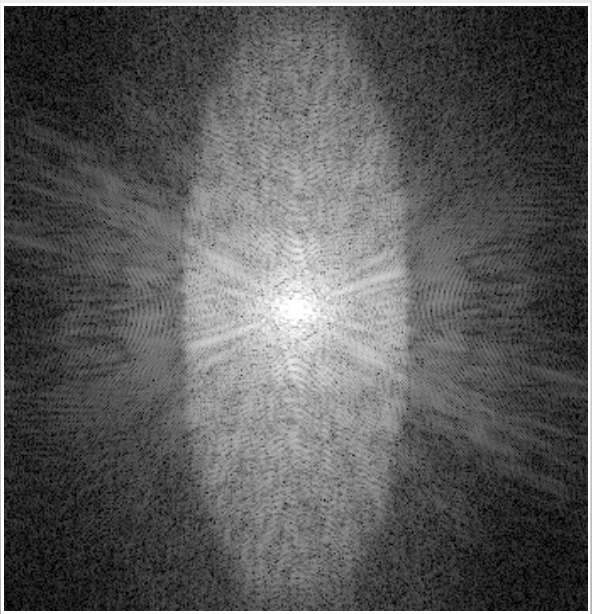


k-space Trajectories



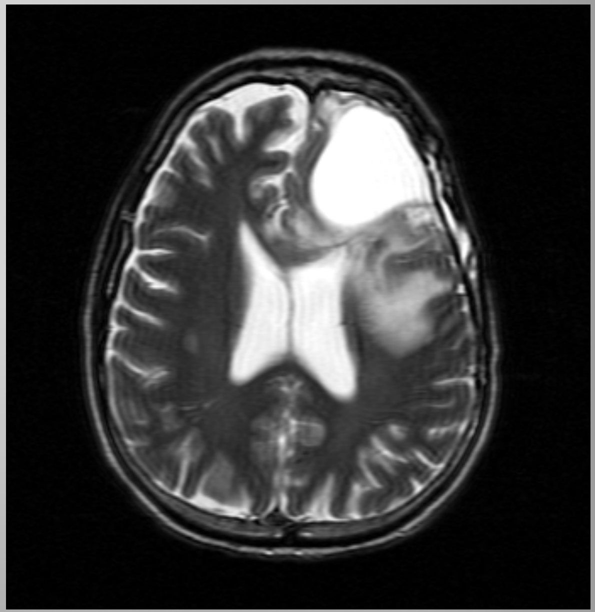
MR Imaging as Diffraction

MRI data is acquired in the inverse space of the final image



k-space

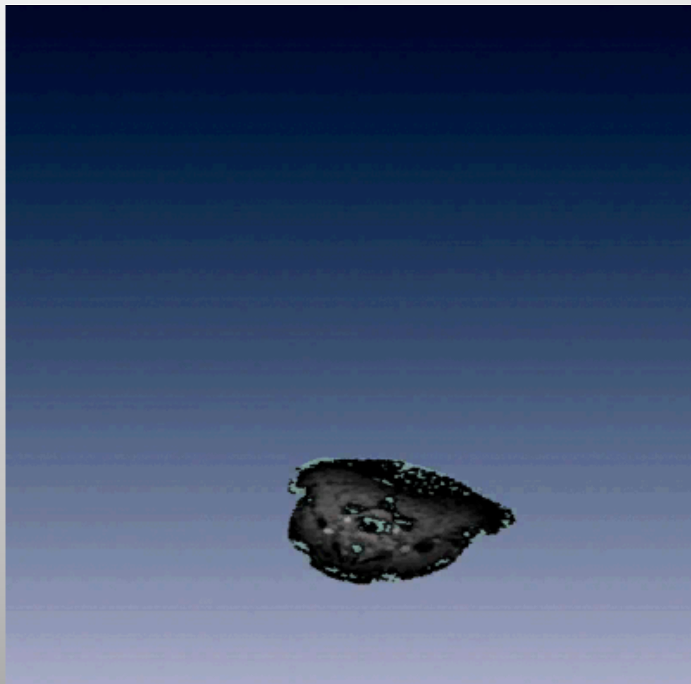
Fourier
Transform

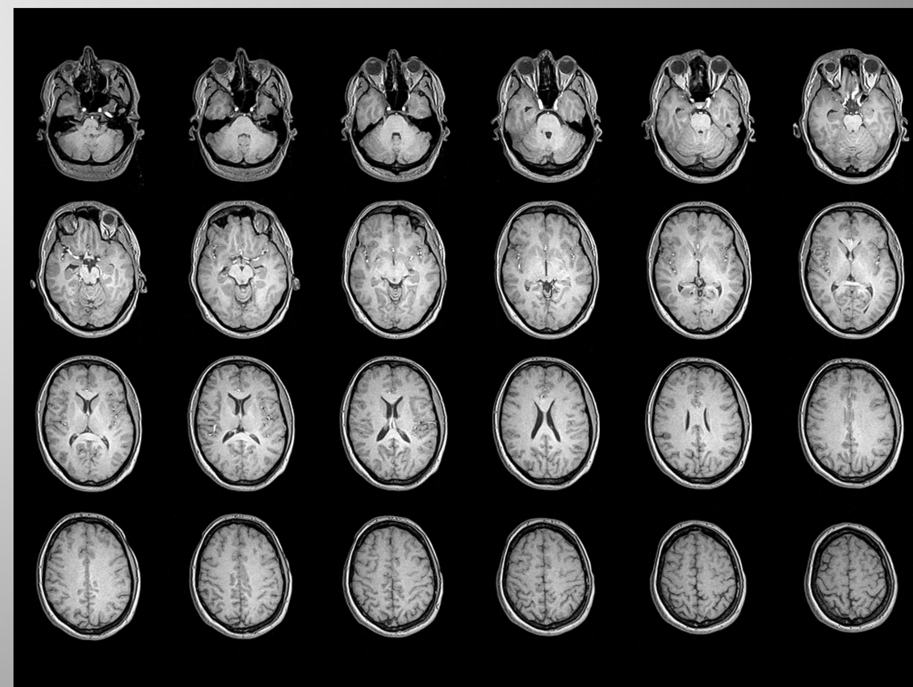


Frequency Encoded
Space

Volume Localization

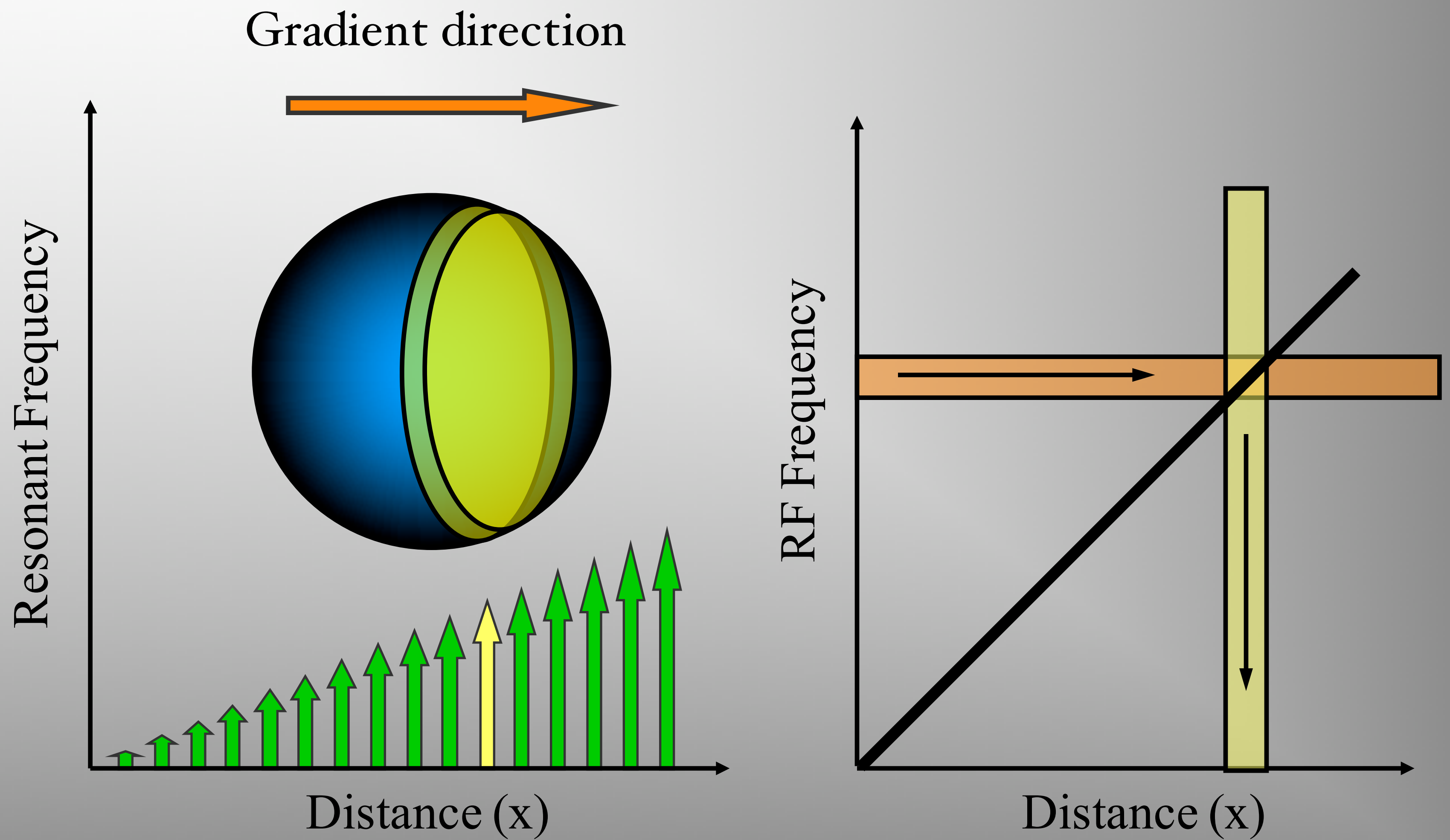
We frequently want to localize MR signal from a subvolume of tissue: slice, slab, pencil, cube ...





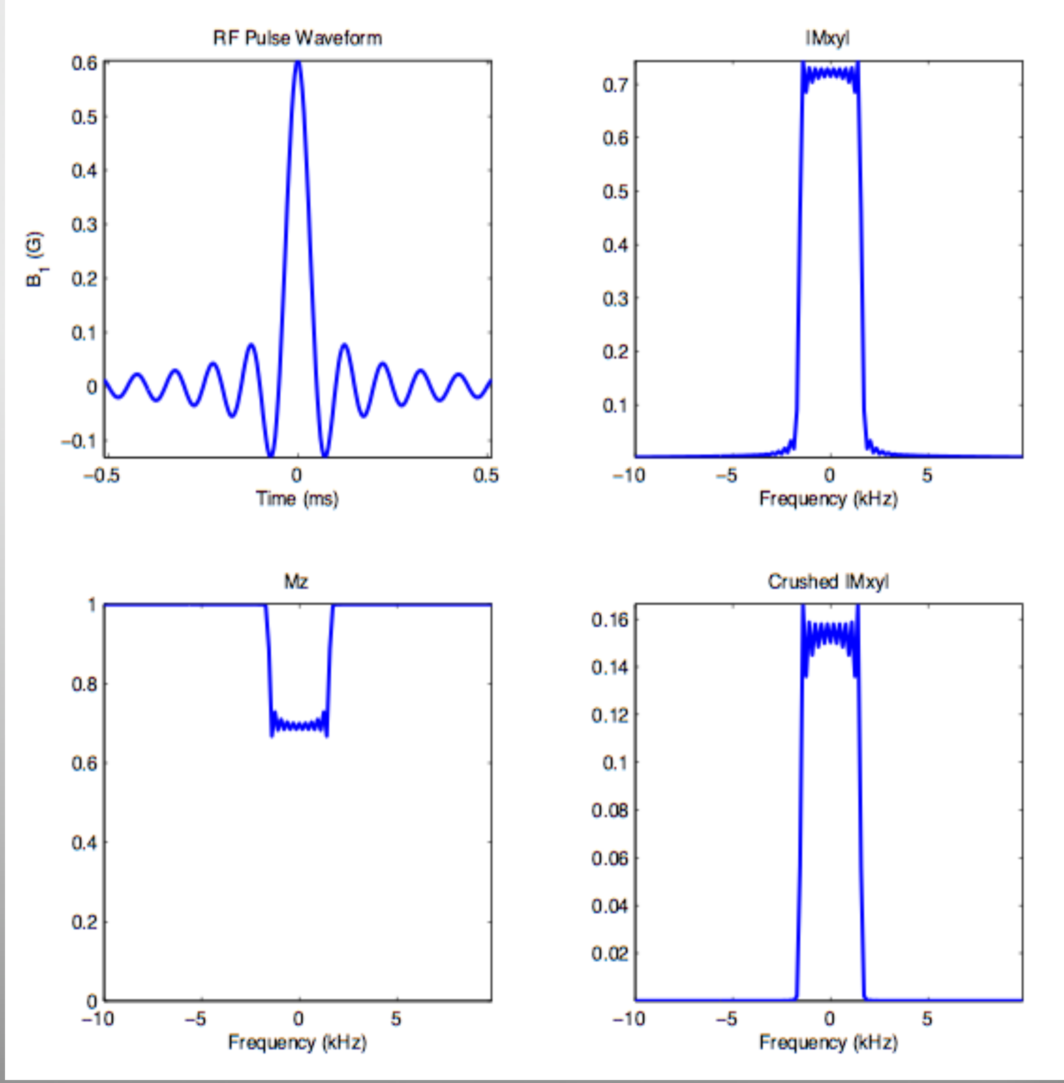
Can we achieve this by frequency encoding space?

Slice Selection with Gradients

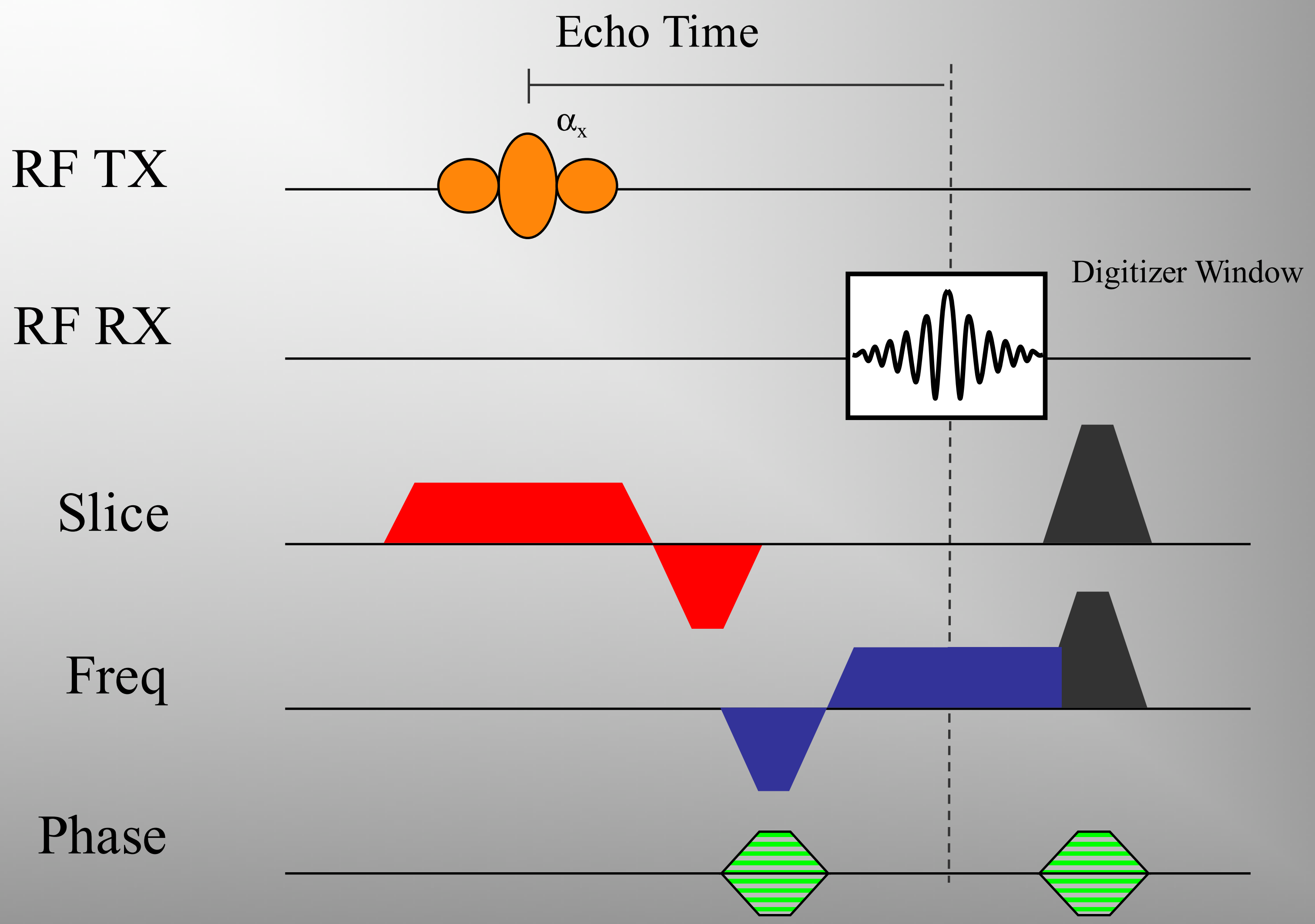


Bandlimited RF Pulses

- Pulse waveform modulates Larmor carrier frequency.
- Affects frequency band centered on Larmor frequency.



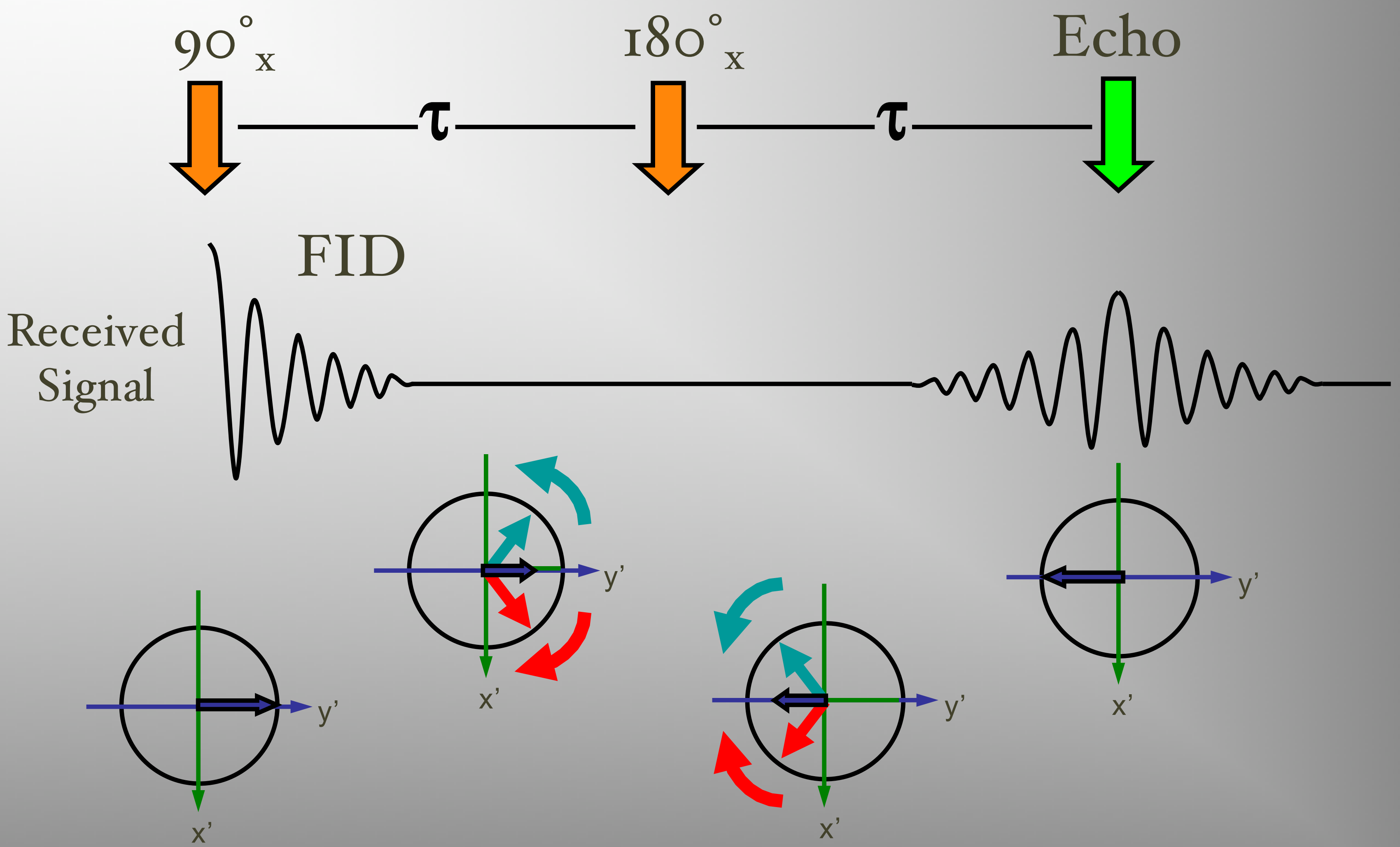
Full Gradient Echo Pulse Sequence



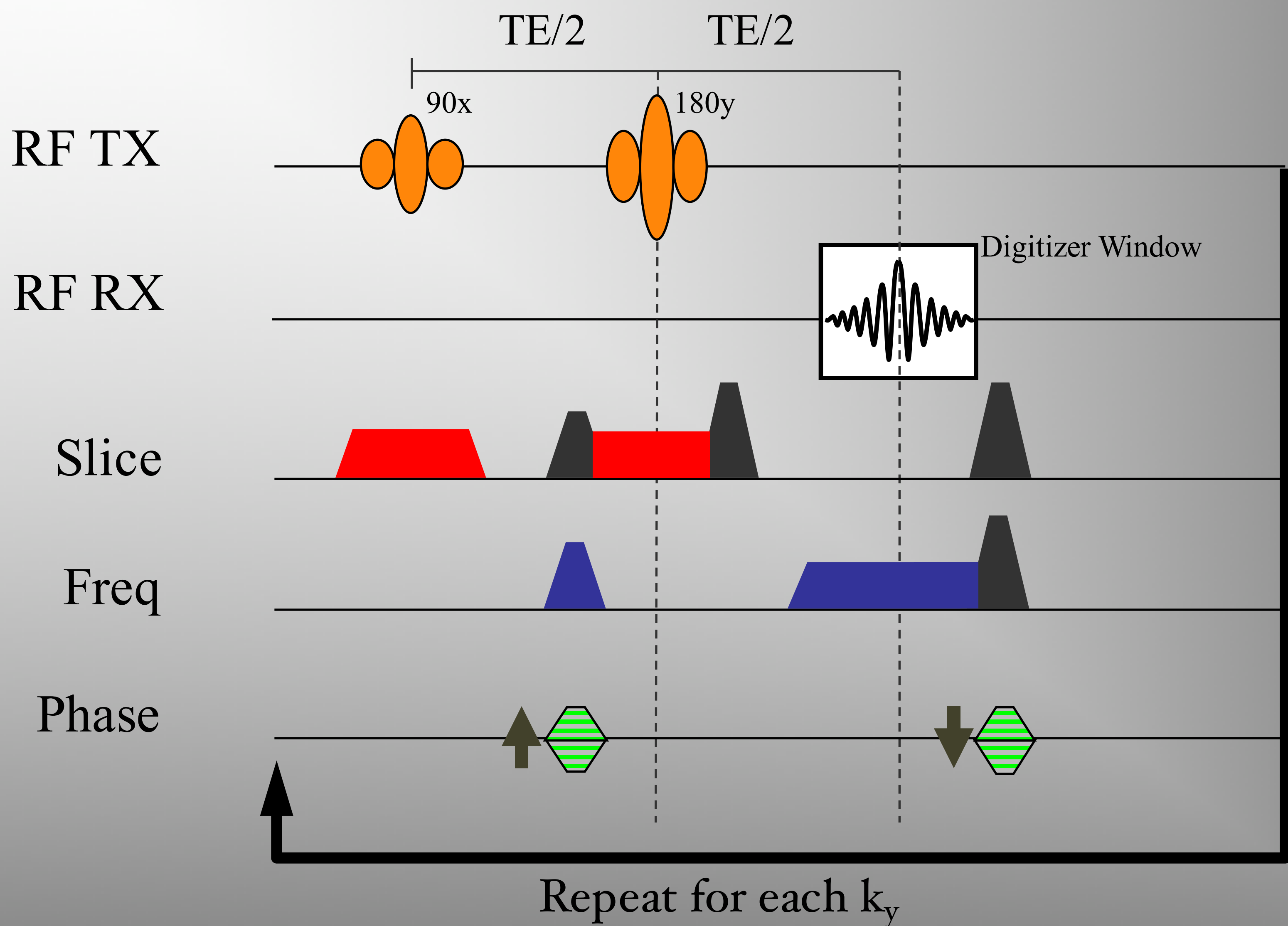
The Hahn Spin Echo

- General case has two RF pulses separated by a short delay, τ, causing a signal "echo" at time 2τ.
- Maximum signal observed when pulse flip angles are 90° followed by 180°.
- Dephasing by static B_0 inhomogeneities is refocused.

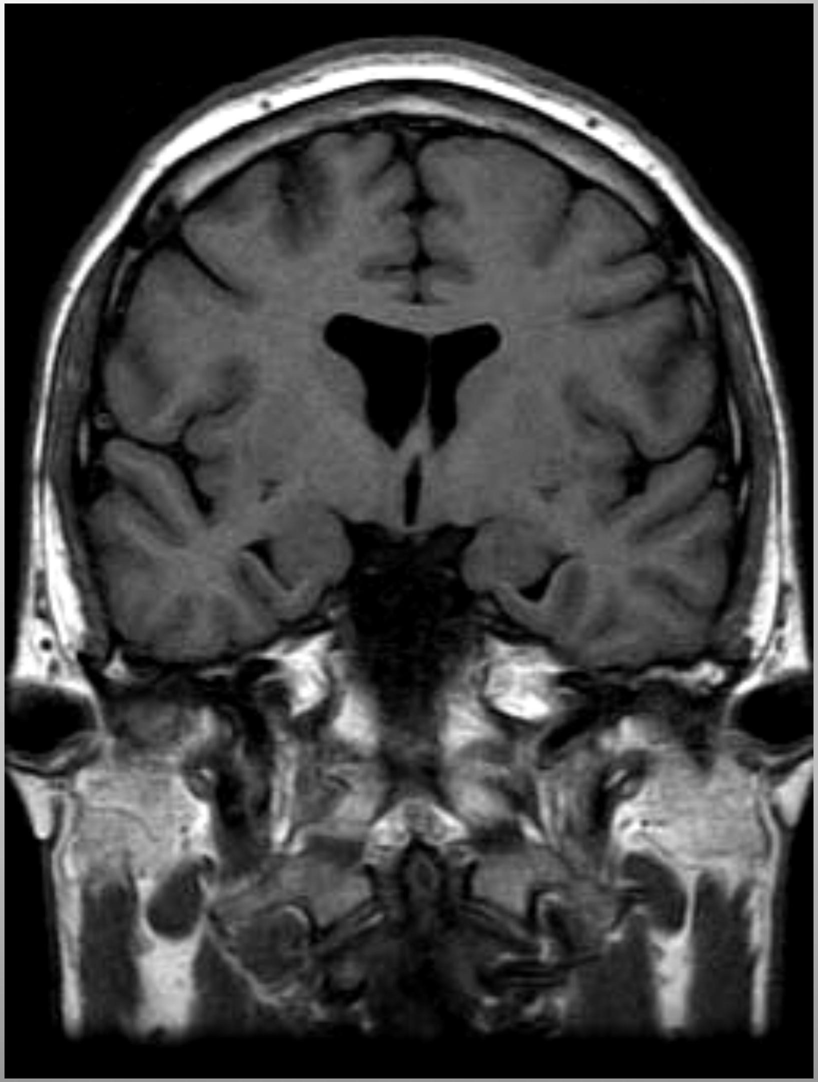
The Hahn Spin Echo Explained



Spin Echo Pulse Sequence



T_1-weighted SE Imaging



http://www.gemedicalsystems.com/rad/mri/images/med/mri/neuro/15_neuro1_01_image3.jpg

Next Lecture

- Contrast Manipulation in MRI