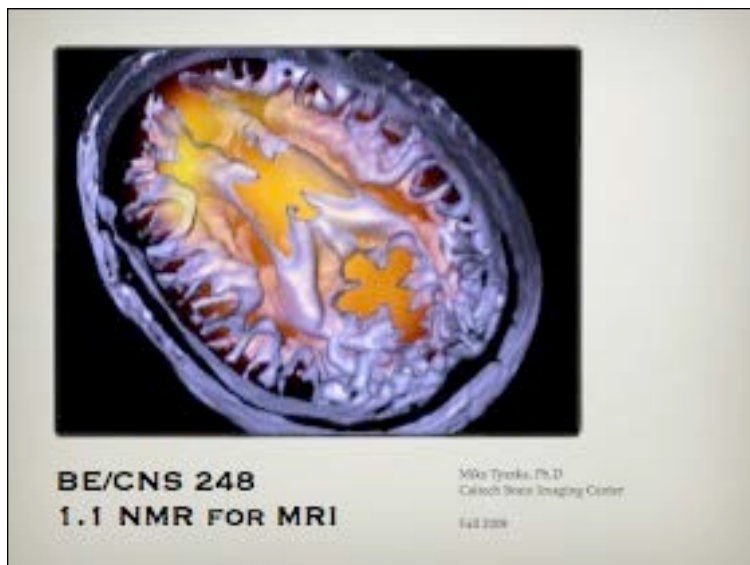
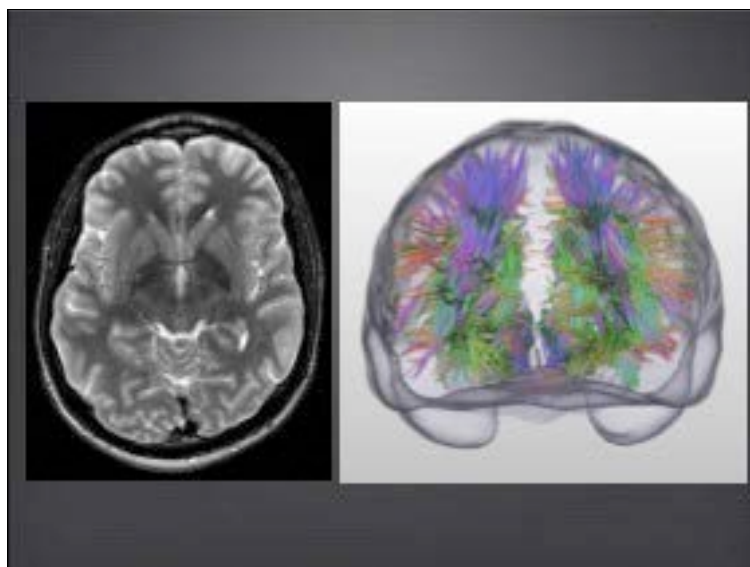
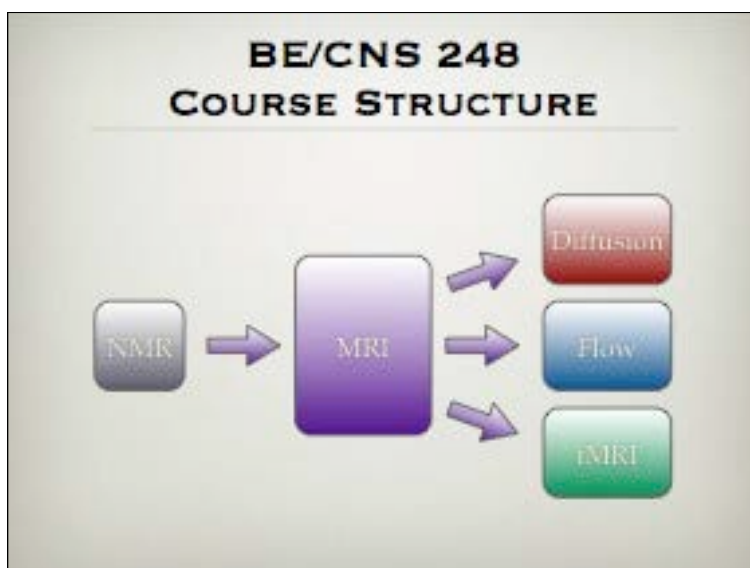




1



2



3

WEB RESOURCES



<http://www.its.caltech.edu/~jmt/BE248>

4

LECTURE SUMMARY

- 1.1 Basic NMR
 - a. A Brief History of NMR and MRI
 - b. Magnetism and Magnetic Materials
 - c. The Zeeman effect
 - d. Nuclear spin polarization
 - e. The Larmor relation
 - f. Nuclear shielding and chemical shift
 - g. Nuclear spin quantum mechanics
 - h. The Semi-classical Vector Model
 - i. The Bloch Equation
 - j. The Rotating Frame

5



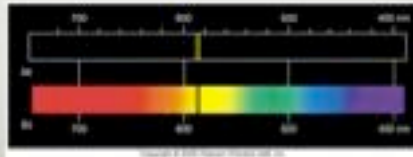
HISTORY OF MRI

6

1891: OPTICAL HYPERFINE SPLITTING



A.A. Michelson observes optical hyperfine splitting as early as 1891. A full explanation of the effect had to wait more than 40 years for Pauli to suggest that the nucleus had a magnetic dipole. The orbital angular momentum of the electron interacts with the nuclear dipole, resulting in hyperfine splitting of absorption and emission lines.

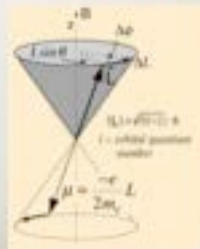


7

1896: LARMOR PRECESSION



1896 Sir Joseph Larmor publishes magnetic field effect on radiation frequency. Relates the precession frequency of the magnetic dipole associated with an orbiting electron to an externally applied magnetic field.



8

1896: ZEEMAN EFFECT



1896 Pieter Zeeman observes sodium D line splitting in an applied magnetic field. The effect arises from the interaction of the total (ie orbital + intrinsic) angular momentum of an electron.



Zeeman P. Nature 1897:55:347

9

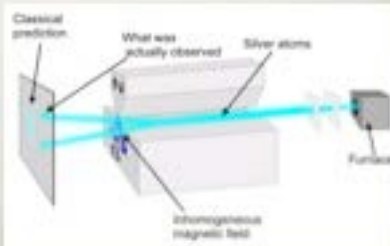
1920 : THE STERN-GERLACH EXPERIMENT



Performed in Frankfurt, Germany in 1920. Nobel Prize in Physics followed in 1943.



1921 Stern and Gerlach observed silver atom beam splitting in a magnetic field gradient due to quantization of total angular momentum of electron.



10

1924-25: ELECTRON SPIN



Ralph Kronig



George Uhlenbeck



Samuel Goudsmit



Wolfgang Pauli

Regarding electron spin: "It is indeed very clever but of course has nothing to do with reality" - Pauli

11

1933: NUCLEAR SPIN



1933: Stern and Gerlach improve their molecular beam experiments, detecting deflection of a hydrogen beam.



Can only be explained in terms of a nuclear magnetic moment.

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1936-37: NUCLEAR SPIN RESONANCE



1936 C.J. Gorter attempts to use nuclear resonance to study nuclear paramagnetism.



1937 I.I. Rabi succeeds with first NMR experiment. LiCl beam passed through a combined static and oscillating magnetic field. RF absorption increased at Larmor frequency.

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1945: BLOCH AND PURCELL



Felix Bloch
Stanford



Edward Purcell
Harvard

1945 within 3 weeks of each other, Bloch at Stanford and Purcell at Harvard observe ^1H NMR in water at 8MHz and Paraffin at 30MHz respectively.



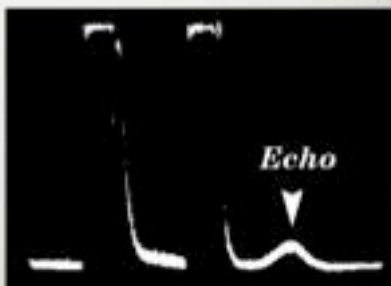
1952 Nobel Prize in Physics

14

1950: THE HAHN ECHO



1950 Erwin Hahn observes the first nuclear spin echo.



15

1973: PAUL LAUTERBUR



Paul Lauterbur

1973 Paul Lauterbur at SUNY publishes Nature article describing a method for generating MR images using gradient frequency encoding and back projection reconstruction.

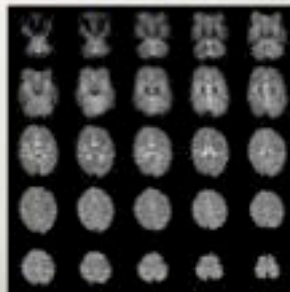


16

1977: ECHO PLANAR IMAGING

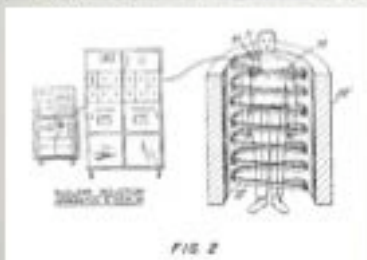


1977 Peter Mansfield publishes echo planar imaging method. EPI later becomes workhorse imaging sequence for fMRI with the advent of high performance gradient hardware.



17

1970s RAYMOND DAMADIAN



Patent awarded 1974

1970 Damadian at SUNY publishes Science article on relaxation time differences between normal and cancerous tissue. His 1974 patent did not describe a methods to generate images. By 1977 constructs human imager ("Indomitable") and acquires first MRI in a human.

See www.fodor.com for Damadian's argument supporting his claim to the 2003 Nobel Prize.

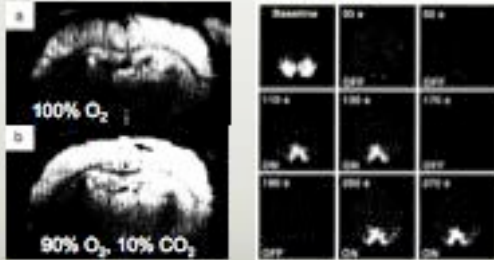
18

1990s BOLD fMRI



Seiji Ogawa
Bell Labs

1990-1992 Seiji Ogawa and Ken Kwong introduce blood oxygenation level dependent (BOLD) contrast MRI. SFN is never the same again.



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MAGNETISM

20

FORMS OF MAGNETISM

- Paramagnetism
 - Electronic Paramagnetism
 - NUCLEAR PARAMAGNETISM
 - Superparamagnetism
- Diamagnetism
 - Most organic compounds
- Ferromagnetism
 - Metallic Fe, Ni, Co

21

MAGNETIC SUSCEPTIBILITY

Susceptibility is dimensionless and relates the magnetization of a material to an applied magnetic field.

$$\mathbf{M} = \chi \mathbf{H}$$

$$\begin{aligned}\mathbf{B} &= \mu_0 (\mathbf{H} + \mathbf{M}) \\ &= \mu_0 \mathbf{H} (1 + \chi)\end{aligned}$$

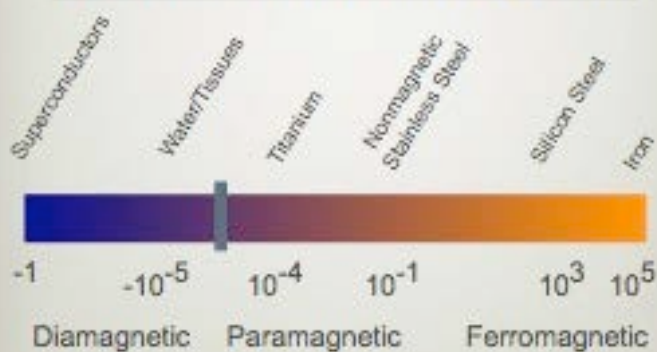
22

"MAGNETIC FIELD"

- H and B are strictly not the same thing.
- H aka the "auxilliary field"
- B aka the "magnetic induction"
- B is the total flux density due to external H field and resulting magnetization.
- B has units of Tesla ($\text{kg s}^{-2} \text{A}^{-1}$)

23

SUSCEPTIBILITY SPECTRUM



24

MEISNER EFFECT

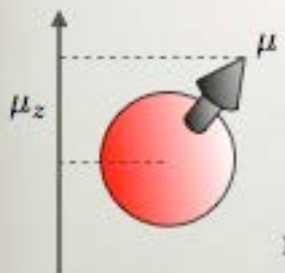


Magnet floating over HTC superconductor in liquid nitrogen

25

SPIN ANGULAR MOMENTUM

For a nucleus with a spin quantum number, I



Magnetic field
parallel to z axis

Dipole moment

$$\mu = \gamma \hbar \sqrt{I(I+1)}$$

z-component of dipole moment

$$\mu_z = \gamma m_I \hbar$$

Magnetic quantum number

$$m_I = -I, -I+1, \dots, I$$

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SPIN 1/2 NUCLEUS

Dipole moment

$$\mu = \gamma \hbar \frac{\sqrt{3}}{2}$$

z-component of dipole moment

$$\mu_z = \pm \gamma \frac{\hbar}{2}$$

Magnetic quantum number

$$m_I = \pm \frac{1}{2}$$



27

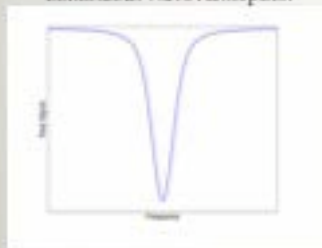
IMPORTANT NMR NUCLEI

Nucleus	Spin	γ (MHz/T)	Nat Abundance	Rel Sensitivity
^1H	$\frac{1}{2}$	42.58	99.99	100%
^2H	1	6.54	0.015	1.0%
^{12}C	0	-	-	-
^{13}C	$\frac{1}{2}$	10.71	1.1	1.6%
^{15}N	$\frac{1}{2}$	4.32	0.37	0.1%
^{19}F	$\frac{1}{2}$	40.01	100.0	83%
^{31}P	$\frac{1}{2}$	17.24	100.0	6.6%
^{23}Na	$\frac{3}{2}$	11.26	100.0	9.3%

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NUCLEAR MAGNETIC RESONANCE

Continuous Wave Absorption



Increasing RF Frequency
Constant Field Strength

Increasing Field Strength
Constant RF Frequency

Pulsed RF Excitation

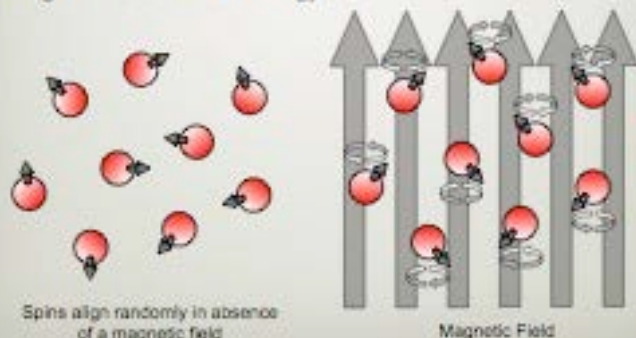


Increasing RF Frequency
Constant Field Strength

29

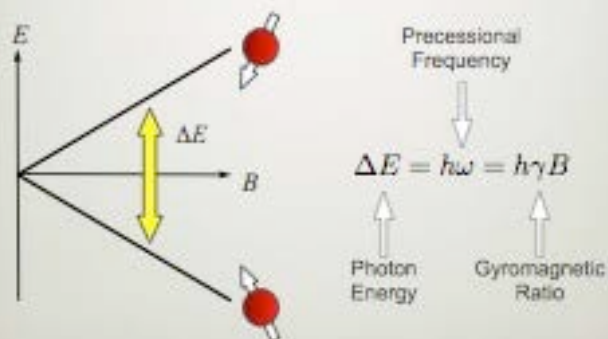
NUCLEAR ZEEMAN EFFECT

A magnetic field forces nuclei with a net magnetic moment to align in two or more energy states.



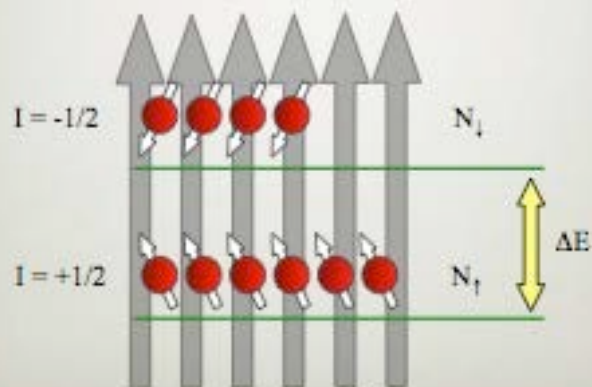
30

LARMOR EQUATION



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POPULATION OF ENERGY LEVELS



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POPULATION OF ENERGY LEVELS

Boltzmann distribution of ^1H energy level populations

$$N_R = \frac{N_{\downarrow}}{N_{\uparrow}} = e^{-\Delta E/KT} = e^{-h\gamma B/KT} < 1$$

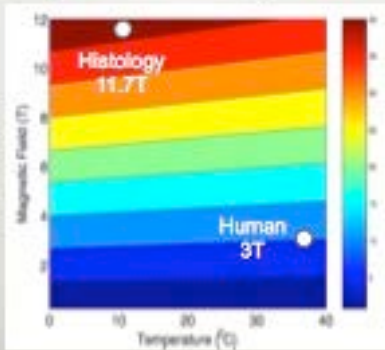
Excess fraction in low energy state (paramagnetic)

$$\Delta N_R = \frac{N_{\uparrow} - N_{\downarrow}}{N_{\uparrow} + N_{\downarrow}} = \frac{1 - N_R}{1 + N_R}$$

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POPULATION OF ENERGY LEVELS

Excess spins in low energy state in ppm



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BULK MAGNETIZATION

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BULK MAGNETIZATION

Spin Ensemble

Bulk Magnetization

$$\int_V \text{Spin Ensemble} = \mathbf{M} \uparrow$$

- Equilibrium Bulk Magnetization does NOT precess
- A typical MRI voxel (1 μ l) contains roughly 10^{12} spins

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FORCES ON A DIPOLE IN A MAGNETIC FIELD

A magnetic dipole or current loop experiences a torque in a magnetic field



$$\mathbf{N} = \boldsymbol{\mu} \times \mathbf{B}$$

37

THE BLOCH EQUATION

Provides a classical description for the evolution of the magnetization with time.

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times \mathbf{B}$$

Rate of change of magnetization vector due to the application of an external field

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FARADAY INDUCTION

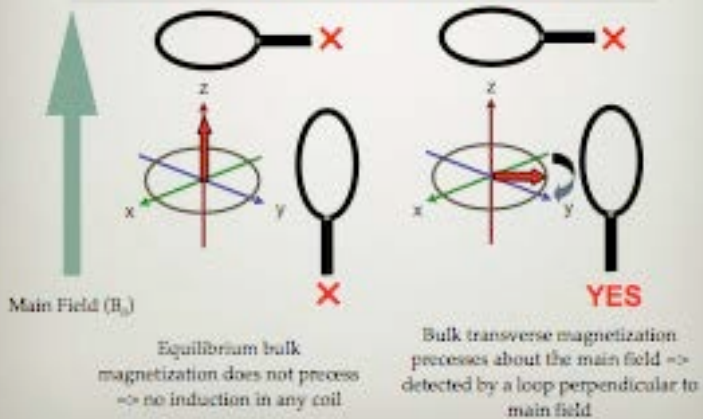
$$emf = \frac{d\Phi}{dt}$$

$$\Phi = \int_A \mathbf{B} \cdot d\mathbf{S}$$

See Haacke 7.1, 7.2, 7.3 for details

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DETECTING MAGNETIZATION



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CREATING TRANSVERSE MAGNETIZATION

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LINEAR POLARIZATION

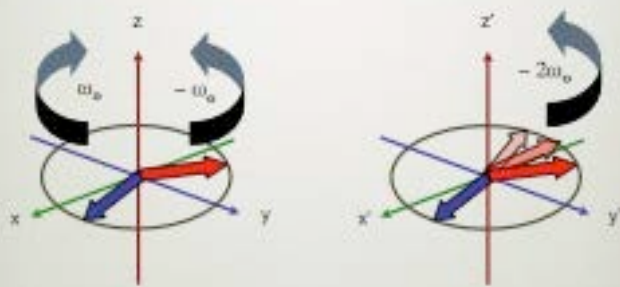
For an oscillating current in a circular loop



$$B_z(t) = \frac{\mu_0 I_0 \cos(\omega t)}{2a} \quad \text{at center of loop}$$

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CIRCULARLY POLARIZED B_1 FIELD

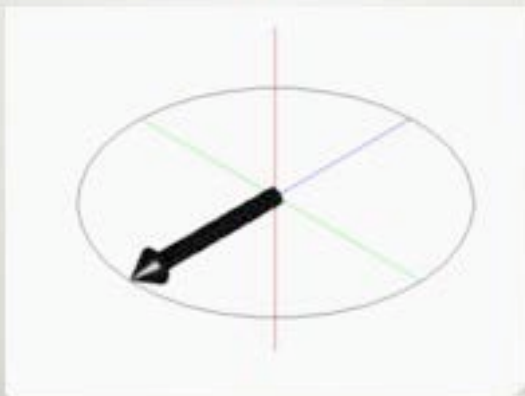


Laboratory Frame

Rotating Frame

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LINEAR TO CIRCULAR POLARIZATION

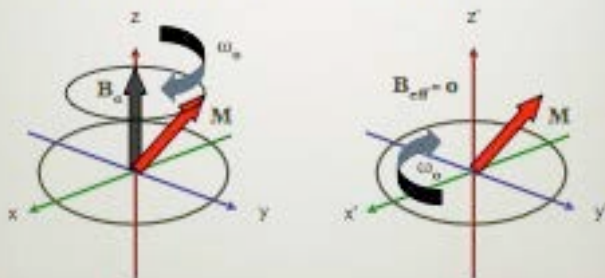


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THE ROTATING FRAME

Laboratory Frame

Rotating Frame



45

BLOCH EQUATION IN THE ROTATING FRAME

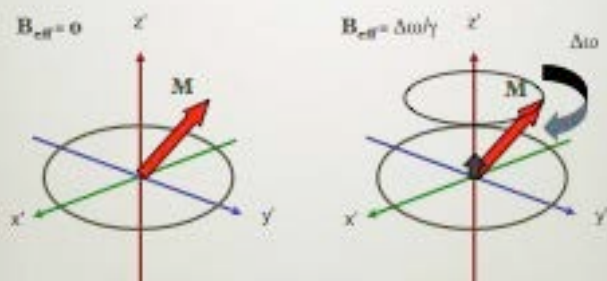
Introduce effective magnetic field seen by magnetization

$$\mathbf{B}_{\text{eff}} = \left(B_0 - \frac{\omega_{\text{rf}}}{\gamma} \right) \hat{\mathbf{z}}$$

$$\frac{\partial \mathbf{M}}{\partial t} = \gamma \mathbf{M} \times \mathbf{B}_{\text{eff}}$$

46

OFF-RESONANCE IN THE ROTATING FRAME



47

ADDING RF $\mathbf{B}_1$ FIELD TO THE BLOCH EQUATION

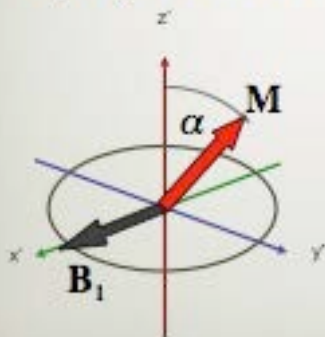
In the rotating frame:

$$\mathbf{B}_{\text{eff}} = \left(B_0 - \frac{\omega_{\text{rf}}}{\gamma} \right) \hat{\mathbf{z}} + B_1 \hat{\mathbf{x}}$$

48

PRECESSION ABOUT B_1

Flip Angle due to a finite duration RF pulse



$$\alpha = \gamma B_1 \tau$$

See Haacke 5.4 for QM description of RF spin tipping

49

RF IN THE ROTATING FRAME

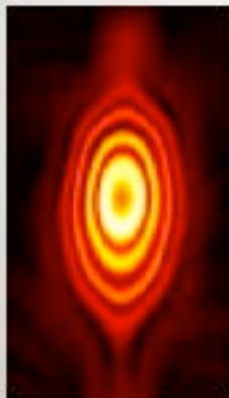
Laboratory Frame

Rotating Frame



50

QUIZ

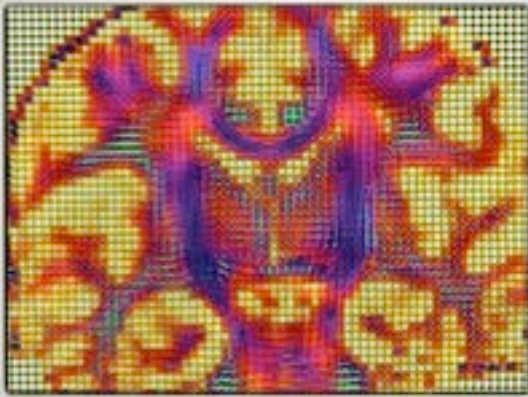


- 2D MRI immediately above a surface coil.
- Image plane is parallel to coil plane.
- Sample is a large volume of water.
- Question: why does the image look like this?

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NEXT LECTURE

- 1.2 NMR for MRI



BE/CNS 248 1.2 NMR FOR MRI

Mike Tycko, Ph.D.
Caltech Stem Imaging Center
Fall Semester 2006-2007

1

LECTURE SUMMARY

- 1.2 Basic NMR
 - The Free Induction Decay
 - Relaxation
 - Spectral density function
 - Spin-lattice interactions and T_1
 - Spin-spin interactions and T_2
 - Field homogeneity and T_2^*
 - Spin-locked relaxation and $T_{1\rho}$
 - Basic Pulse Sequences
 - Pulse-acquire
 - Hahn Spin Echo
- CPMG Sequence
- Inversion Recovery
- Saturation Recovery
- Stimulated Echo
- Steady-State NMR

2



FREE INDUCTION DECAY

3

RECAP: RF IN THE ROTATING FRAME

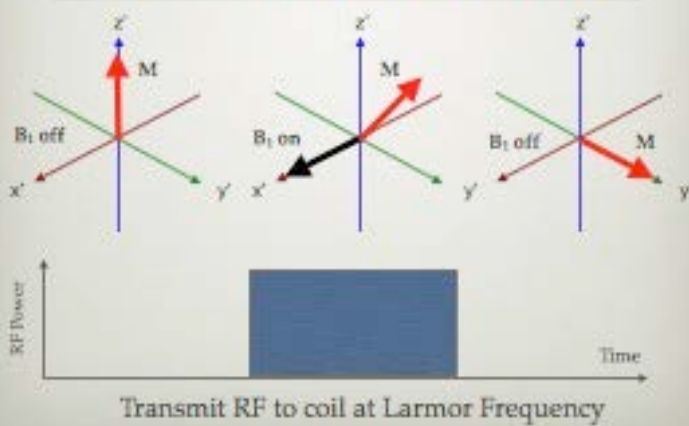
Laboratory Frame

Rotating Frame



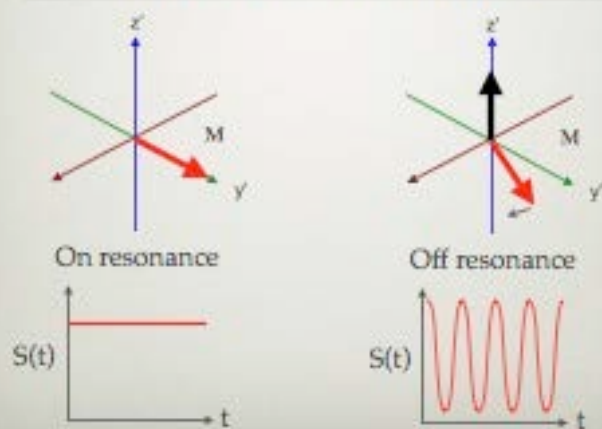
4

PULSED RF EXCITATION



5

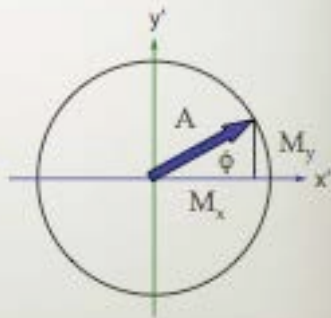
FREE PRECESSION



6

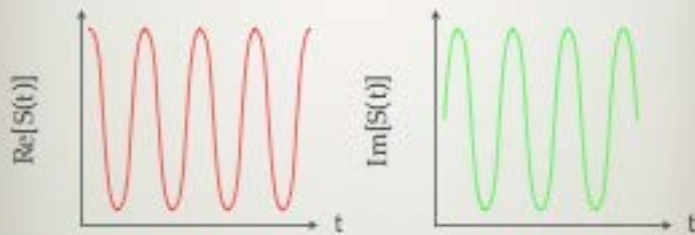
MR SIGNAL AS A COMPLEX QUANTITY

- Signal detection is phase sensitive, so both M_x and M_y components of precessing magnetization can be measured.
- Two signal channels are commonly referred to as "real" and "imaginary".



7

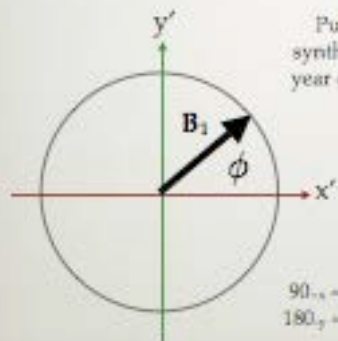
COMPLEX SIGNAL



$$\begin{aligned}
 M &= Ae^{i\phi} \\
 &= A \cos(\phi) + iA \sin(\phi) \\
 &= M_x + iM_y
 \end{aligned}$$

8

RF PULSE PHASING



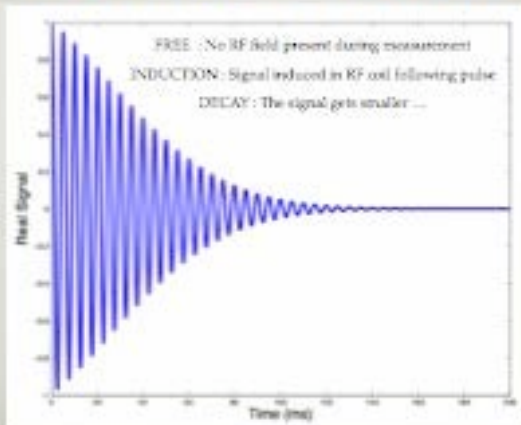
Pulse phase defined in relation to synthesizer (Typical accuracy: 1 ppm/year drift, < 0.5 degree phase precision at 100MHz)

Example Notation:

90_{-x} = 90 degree pulse phased along -x
 180_y = 180 degree pulse phased along -y

9

THE FREE INDUCTION DECAY



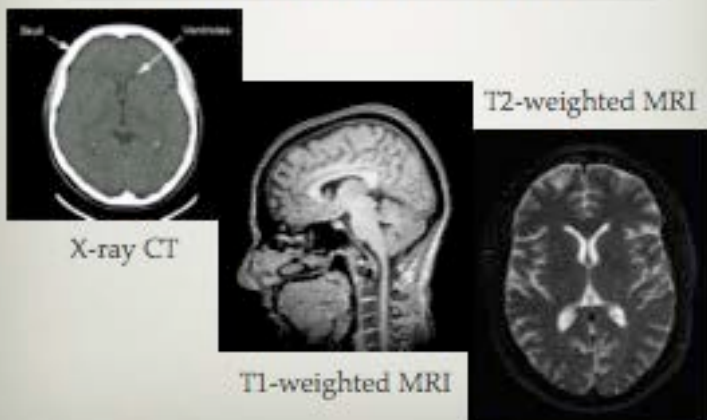
10



RELAXATION

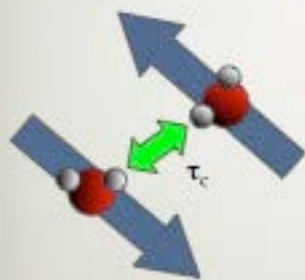
11

THE IMPORTANCE OF TISSUE CONTRAST IN MRI



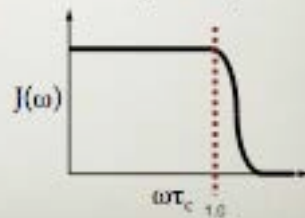
12

SPECTRAL DENSITY FUNCTION



Basis of Bloembergen-Percell-Pound (BPP) relaxation theory

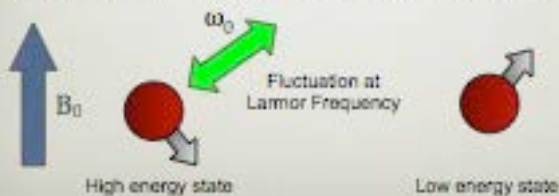
Nuclear spins experience short duration (τ_c) dipole-dipole interactions giving rise to a spectrum of magnetic field fluctuation frequencies (spectral density function)



13

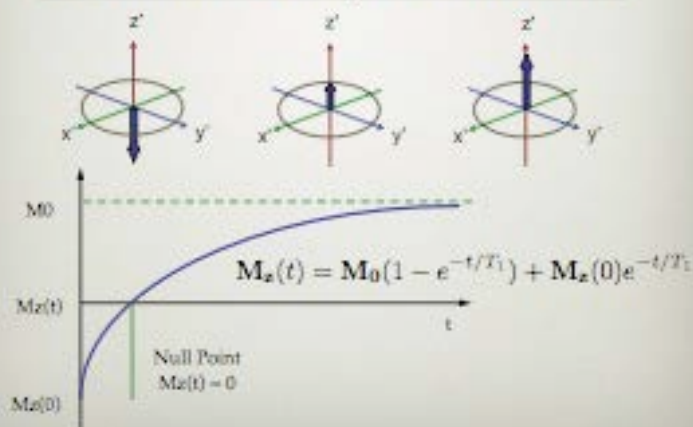
SPIN-LATTICE OR T_1 RELAXATION

- Spins do not spontaneously decay from higher to lower energy states because the resonance frequency is very low (MHz).
- T_1 relaxation is driven by field fluctuations only at the Larmor frequency.
- The longitudinal magnetization decays exponentially to equilibrium.
- Typical T_1 for liquid water in tissues ranges from ms to seconds.



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T_1 (SPIN-LATTICE OR LONGITUDINAL) RELAXATION



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T2 (SPIN-SPIN OR TRANSVERSE) RELAXATION

- T2 relaxation is driven by field fluctuations at both zero frequency and the Larmor frequency.
- Spins are in phase following an RF pulse.
- The transverse magnetization decays exponentially to zero due to cumulative dephasing.
- Typical T2 for liquid water in tissues ranges from ms to seconds.

16

T2 (SPIN-SPIN OR TRANSVERSE) RELAXATION

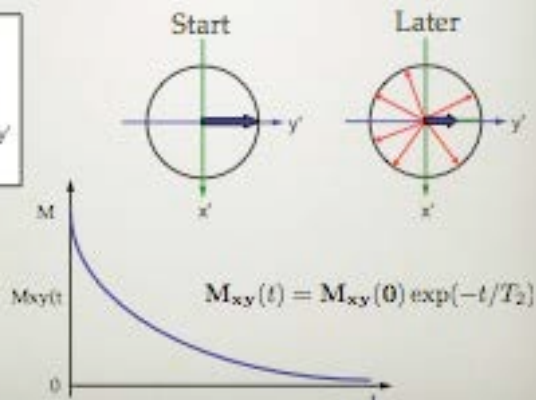
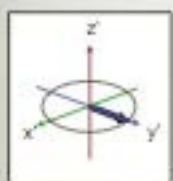


Fluctuation at either zero frequency or the Larmor Frequency causes spin-spin energy exchange.

Total energy preserved. Phase randomized.

17

T2 (SPIN-SPIN) RELAXATION



18

RELAXATION TIMES AT 3T

Table 1

T_1 and T_2 Relaxation Times at 3T and 1.5T Measured at 37°C. Literature data is also shown.

Tissue	$T_1=0.7$ [ms]		$T_1=0.7$ [ms]		$T_2=1.5$ T [ms]		$T_2=1.5$ T [ms]	
	This study	Literature	This study	Literature	This study	Literature	This study	Literature
Liver	42 ± 3		812 ± 54		45 ± 5	54 ± 6 ⁽¹⁾	519 ± 33	~900 ⁽¹⁾
Skeletal muscle	50 ± 4	32 ± 2 ⁽²⁾	1472 ± 13	1425 ± 34 ⁽³⁾	44 ± 5	35 ± 4 ⁽⁴⁾	1008 ± 33	1085 ± 15 ⁽⁵⁾
Heart	47 ± 11		1471 ± 31		42 ± 5	44 ± 5 ⁽⁶⁾	1030 ± 34	
Kidney	58 ± 4		1194 ± 27		55 ± 3	61 ± 1 ⁽⁷⁾	890 ± 30	758 ± 62 ⁽⁸⁾
Cartilage 0°	27 ± 3	27 ± 4 ⁽⁹⁾	1198 ± 18	~1200 ⁽¹⁰⁾	33 ± 4	42 ± 1 ⁽¹¹⁾	1014 ± 72	~1000 ⁽¹²⁾
Cartilage 30°	43 ± 2	45 ± 5 ⁽¹³⁾	1138 ± 15		44 ± 5		1048 ± 87	
White matter	60 ± 3	56 ± 4 ⁽¹⁴⁾	1084 ± 40	1110 ± 40 ⁽¹⁵⁾	72 ± 4	78 ± 8 ⁽¹⁶⁾	884 ± 50	778 ± 64 ⁽¹⁷⁾
Gray matter	89 ± 7	71 ± 12 ⁽¹⁸⁾	1622 ± 119	1475 ± 53 ⁽¹⁹⁾	65 ± 5	~90 ⁽²⁰⁾	1124 ± 52	1088 ± 226 ⁽²¹⁾
Optic nerve	78 ± 5		1283 ± 39		77 ± 5		815 ± 30	
Spinal cord	78 ± 2		865 ± 47		74 ± 5		745 ± 37	
Blood	275 ± 58		1832 ± 85	~1800 ⁽²²⁾	285 ± 31	327 ± 42 ⁽²³⁾	1441 ± 120	~1200 ⁽²⁴⁾

Shenoy GJ, et al. "T1, T2 Relaxation and Magnetization Transfer in Tissue at 3T" Magn Reson Med 2005;54:507-512.

19

RELAXATION AND THE BLOCH EQUATION

Separate equations of motion for each magnetization component. Add relaxation terms.

$$\begin{aligned}\frac{\partial M_x}{\partial t} &= \gamma(M_y B_z - M_z B_y) - \frac{1}{T_2} M_x \\ \frac{\partial M_y}{\partial t} &= \gamma(M_z B_x - M_x B_z) - \frac{1}{T_2} M_y \\ \frac{\partial M_z}{\partial t} &= \gamma(M_x B_y - M_y B_x) - \frac{1}{T_1} (1 - M_z)\end{aligned}$$

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ANALYTICAL SOLUTIONS

Torque on magnetization causes precession:

Rotation about B_{eff}

Relaxation processes cause:

Exponential Decays

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OTHER CONTRAST MECHANISMS

22

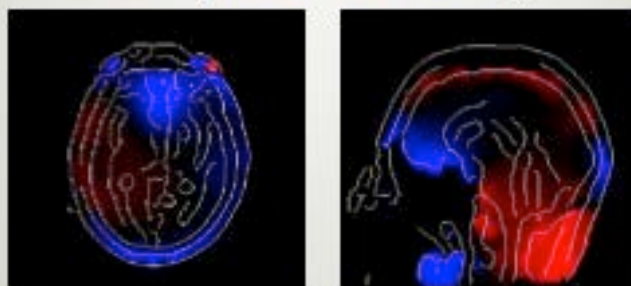
T₂* RELAXATION

- B₀ inhomogeneities within a volume (right) cause additional dephasing of total M_{xy}.
- Decreases apparent T₂ by providing an additional dephasing mechanism.
- T₂* is NOT an exponential process - B₀ distribution (below right) would have to be Lorentzian).
- Basis of BOLD and susceptibility sensitive imaging.
- Addressed in part by magnetic field shimming (homogeneity optimization)

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B₀ HOMOGENEITY IN THE HUMAN HEAD

Following whole brain ideal linear shimming

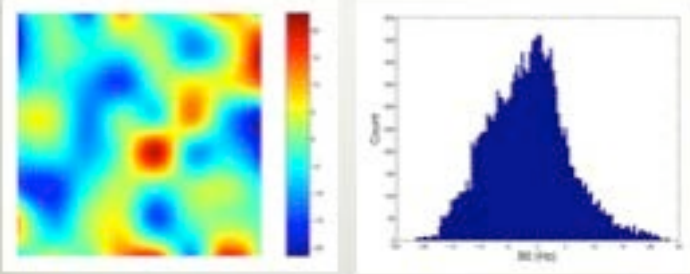


Off-resonance Frequency (Hz) at 1.5T

-50 -40 -30 -20 -10 0 10 20 30 40 50

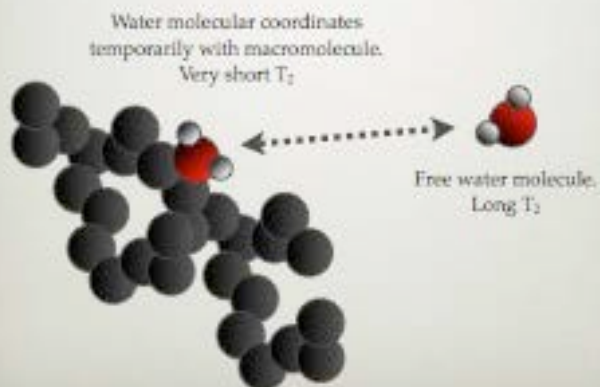
24

T2* RELAXATION



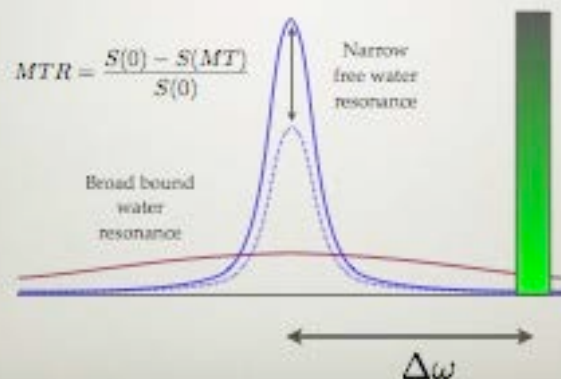
25

WATER INTERACTION WITH MACROMOLECULES



26

MAGNETIZATION TRANSFER



27

MAGNETIZATION TRANSFER RATIOS AT 3T

Table 2
Magnetization Transfer Parameters at 3 T Compared to Literature Data at 1.5 T

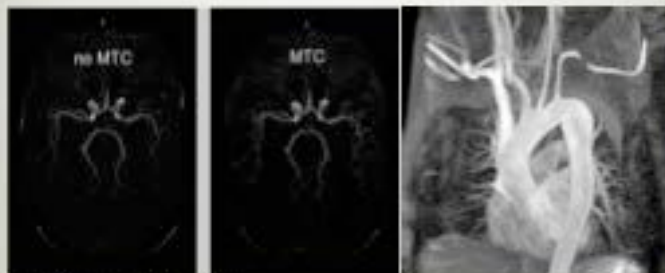
Tissue	This paper measured at 3 T				Literature at 1.5 T		
	M_{0z} (%)	R_2 (s ⁻¹)	$T_{2\rho}$ (s)	MTR (%)	M_{0z} (%)	R_2 (s ⁻¹)	$T_{2\rho}$ (s)
Liver	6.9 ± 0.7	51 ± 10	1.7 ± 0.2	37 ± 6			
Skeletal muscle	7.4 ± 1.5	60 ± 8	0.7 ± 0.1	66 ± 2	9.9 ± 1.8 ¹⁰	53 ± 9 ¹⁰	7.8 ± 0.6 ¹⁰
Heart	9.7 ± 0.2	52 ± 7	0.7 ± 0.1	90 ± 1	7.2 ± 0.9 ¹⁰	47 ± 9 ¹⁰	6.4 ± 0.4 ¹⁰
Kidney	7.1 ± 1.0	46 ± 7	0.7 ± 0.1	92 ± 1			
Cartilage 2°	17.1 ± 2.4	57 ± 5	0.3 ± 0.1	95 ± 1			
Cartilage 3°	16.2 ± 2.4	60 ± 5	0.3 ± 0.1	96 ± 1			
White matter	10.0 ± 2.8	33 ± 4	0.3 ± 0.1	90 ± 1	16.2 ± 3.8 ¹⁰	30 ± 9 ¹⁰	11.3 ± 1.8 ¹⁰
Gray matter	5.0 ± 0.5	40 ± 1	0.1 ± 0.1	94 ± 1	7.2 ± 1.8 ¹⁰	35 ± 9 ¹⁰	11.1 ± 1.1 ¹⁰
Optic nerve	15.0 ± 1.1	33 ± 2	0.3 ± 0.1	90 ± 2			
Spinal cord	12.8 ± 1.9	39 ± 3	0.3 ± 0.1	93 ± 1			
Brain	2.8 ± 0.7	55 ± 1	200 ± 10	~ 0	2.3 ± 0.8 ¹⁰	40 ± 9 ¹⁰	340 ± 40 ¹⁰

MTR is measured at the 90° saturation pulse amplitude, $\omega_{RF} = 40$ Hz, and the offset frequency of the saturation, $\Delta = 5$ kHz (optimum MTR experimental parameters to achieve maximum MTR effect for most of the tissues).

Maritz GJ, et al. T1, T2 Relaxation and Magnetization Transfer in Tissue at 3T Magn Reson Med 2005;54:507-512.

28

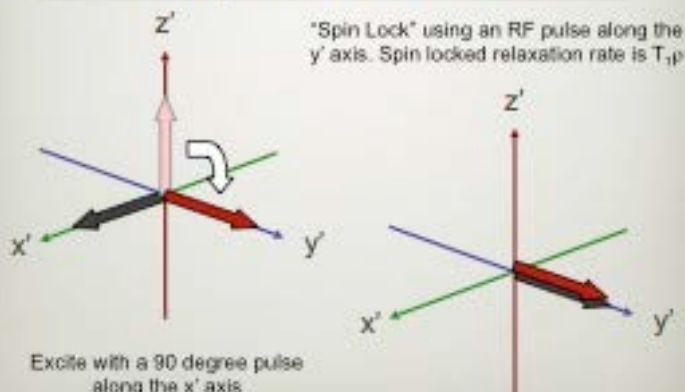
MAGNETIZATION TRANSFER BACKGROUND SUPPRESSION



de Boer, (c) Philips Medical Systems

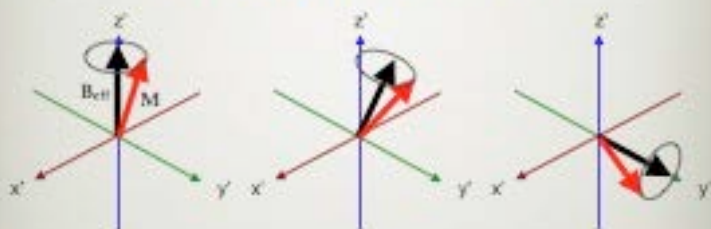
29

T1ρ RELAXATION

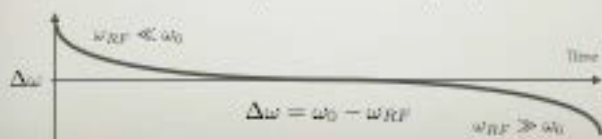


30

ADIABATIC PULSES



Rotating Frame follows RF Frequency



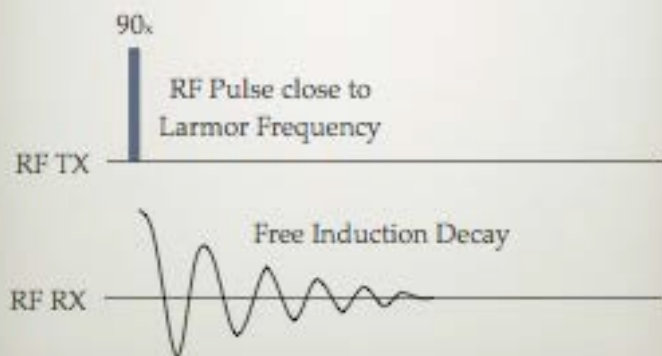
31



PULSE SEQUENCES

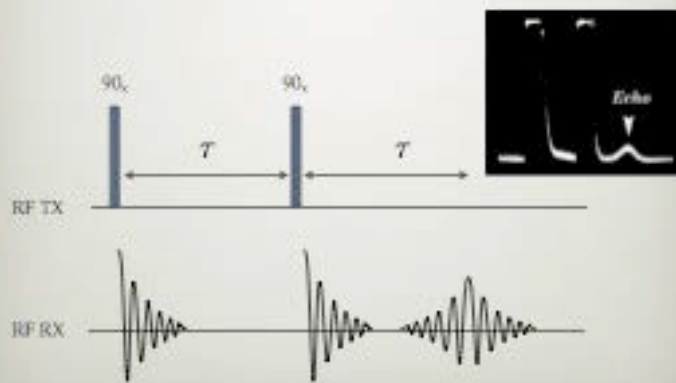
32

PULSE ACQUIRE



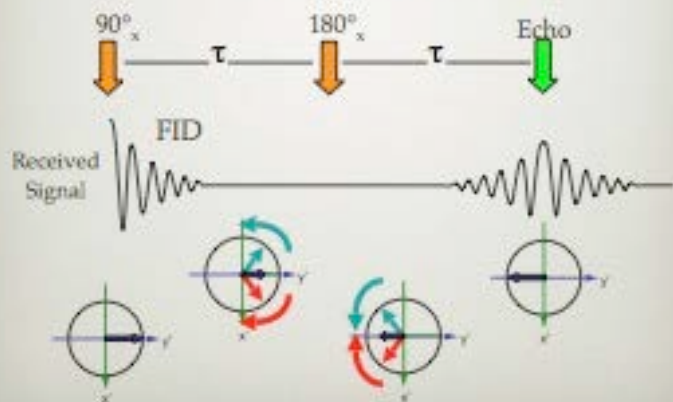
33

TWO RF PULSES



34

THE HAHN SPIN ECHO EXPLAINED



35

GENERALIZED SPIN ECHO CONTRAST

Proton Density $\downarrow$ T_2 Weighting $\downarrow$

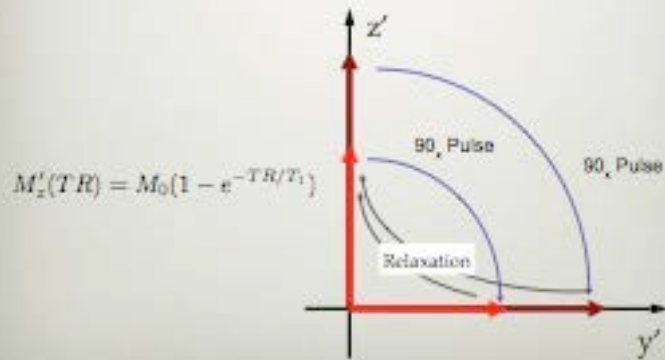
$$S(TR, TE) = S(0)(1 - e^{-TR/T_1})e^{-TE/T_2}$$

$\uparrow$ Signal Intensity $\uparrow$ T_1 Weighting

36

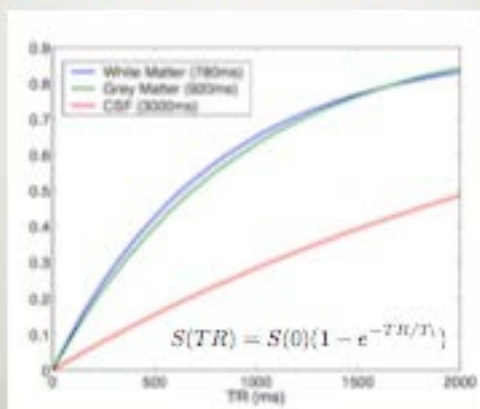
MAGNETIZATION SATURATION

- Saturation is most pronounced when TR is short compared to T_1



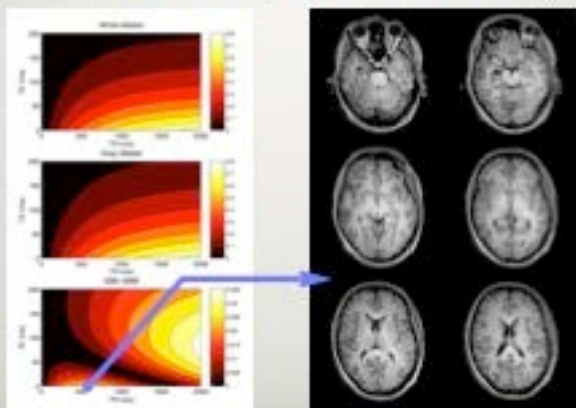
37

SATURATION RECOVERY



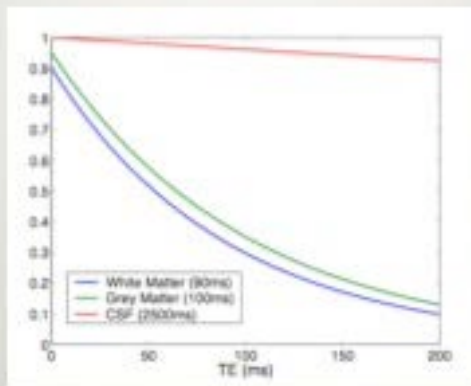
38

SATURATION RECOVERY CONTRAST (T_1 -WEIGHTED)



39

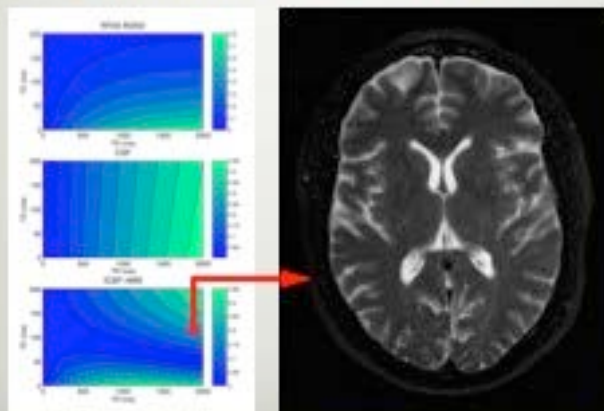
T₂ DECAY AND ECHO TIME



$$S(TE) = S(0)e^{-TE/T_2}$$

40

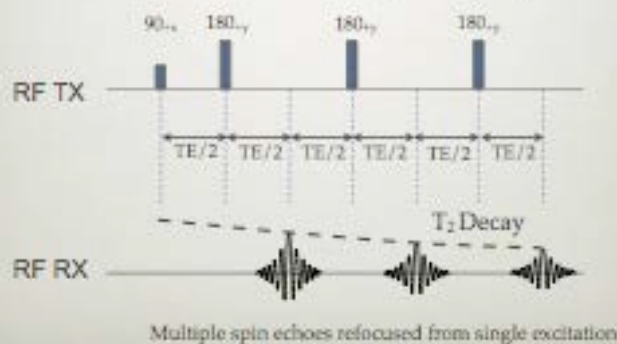
T₂-WEIGHTED CONTRAST



41

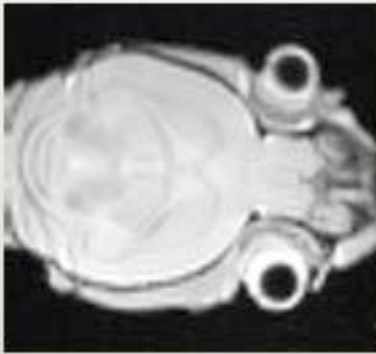
REFOCUSING MORE THAN ONE SPIN ECHO

Carr-Purcell-Meiboom-Gill (CPMG) Sequence



42

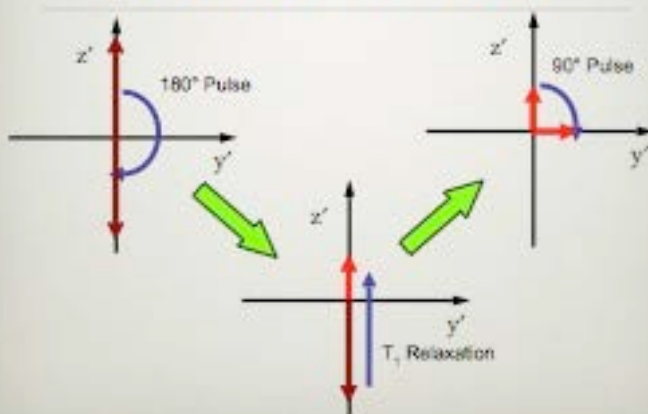
T2 DECAY



CPMG sequence: One image per echo, TE interval 7ms

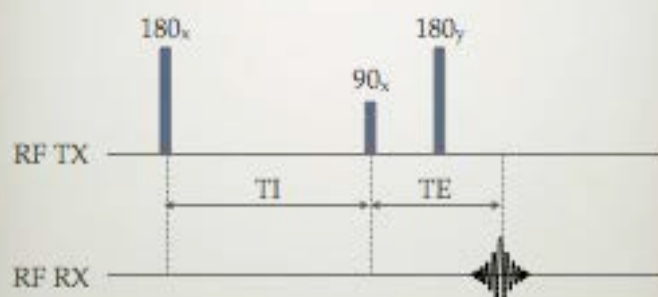
43

INVERSION RECOVERY



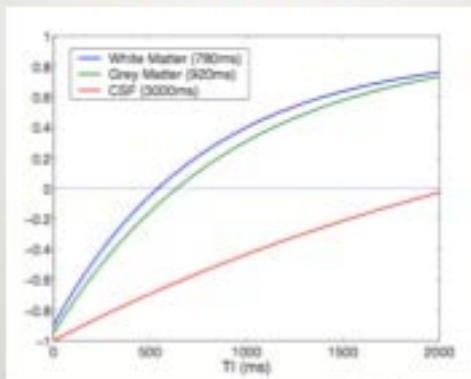
44

INVERSION RECOVERY SPIN ECHO



45

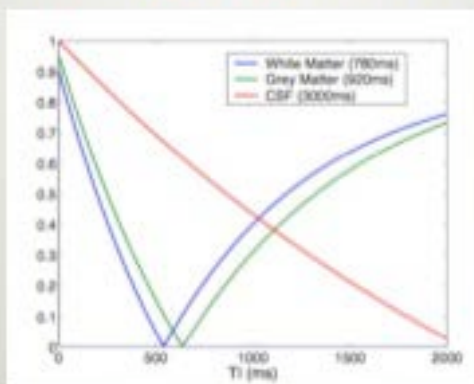
INVERSION RECOVERY WEIGHTING



$$S(TI, TE) = S(0)(1 - 2e^{-TR/T_1})e^{-TE/T_2}$$

46

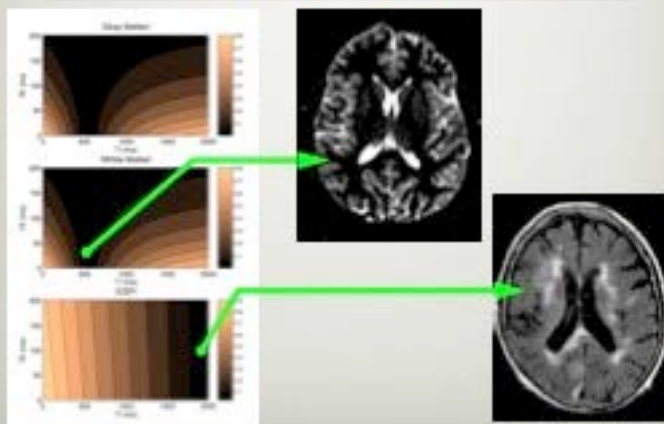
MAGNITUDE INVERSION RECOVERY WEIGHTING



$$S(TI, TE) = |S(0)(1 - 2e^{-TR/T_1})e^{-TE/T_2}|$$

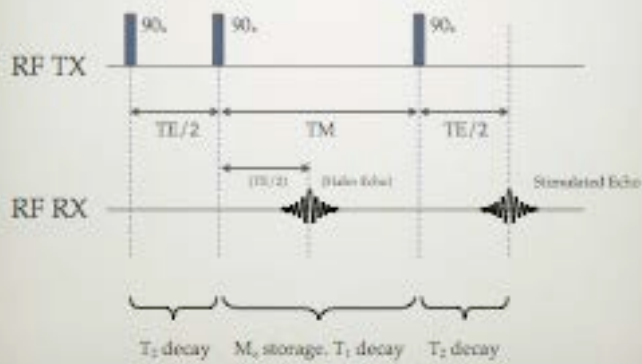
47

INVERSION RECOVERY CONTRAST



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STIMULATED ECHO SEQUENCE



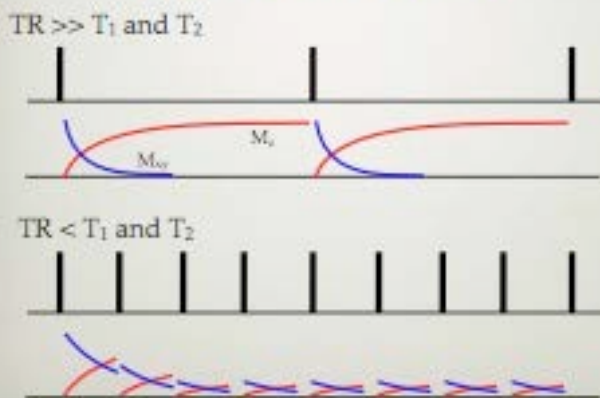
49



THE STEADY STATE

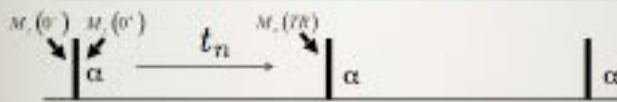
50

RAPID RF PULSING



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MAGNETIZATION AND THE STEADY-STATE



Define Normalized Time within interpulse period

$$t_n = t - nTR$$

Transverse magnetization T_2 decay

$$M_{xy}(t_n) = M_{xy}(0^+)e^{-t_n/T_2}$$

Longitudinal magnetization T_1 recovery

$$M_z(t_n) = M_0(1 - e^{-t_n/T_1}) + M_z(0^-)e^{-t_n/T_1}$$

52

STEADY-STATE INCOHERENT (SSI)

Assume $T_2 \ll TR$ so no memory of M_{xy} across TR intervals

$$E_1 = e^{TR/T_1} \quad E_2 = e^{TR/T_2} = 0$$

If a steady-state exists then the following must be true:

$$M_{nTR^-} = M_{ze} = M_{ze}E_1 \cos \theta + M_0(1 - E_1)$$

and therefore

$$M_{ze} = \frac{M_0(1 - E_1)}{(1 - E_1 \cos \theta)}$$

53

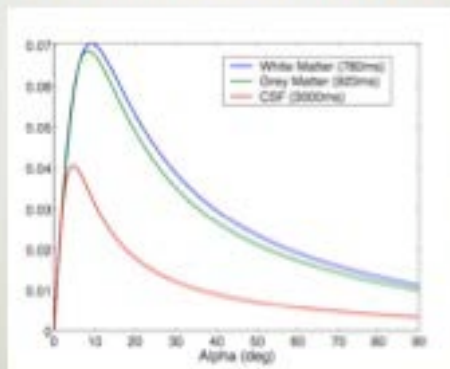
SSI SIGNAL

Applying a flip and T_2 decay gives us the signal

$$M_{xy}(\theta, t_n) = \frac{M_0(1 - E_1)}{(1 - E_1 \cos \theta)} e^{-t_n/T_2} \sin \theta$$

54

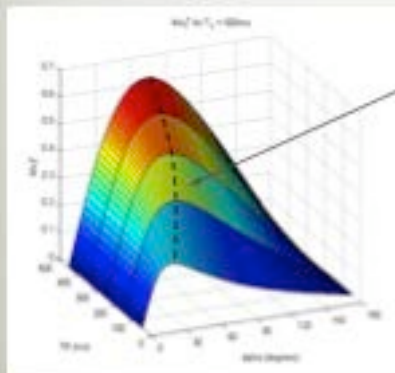
SSI TISSUE CONTRAST



$$S(TR, TE, \alpha) = S(0) \sin(\alpha) \frac{1 - e^{-TR/T_1}}{1 - \cos(\alpha)e^{-TR/T_1}} e^{-TE/T_2}$$

55

THE ERNST ANGLE

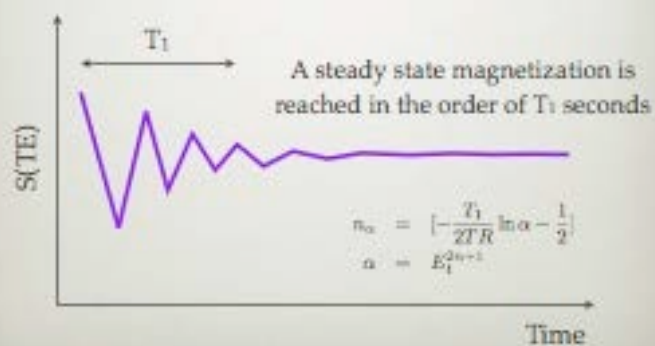


$$\theta_E = \cos^{-1}(e^{-TR/T_1})$$

Maximum signal
obtained at Ernst
angle, **not 90°** when
 $TR < T_1$

56

APPROACH TO SSI



$$n_{\alpha} = \left\lceil \frac{T_1}{2TR} \ln \alpha - \frac{1}{2} \right\rceil$$

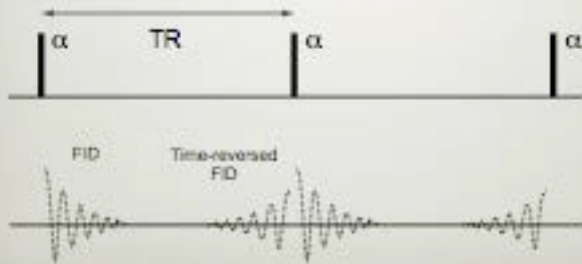
$$\alpha = E_I^{2n_{\alpha} + 1}$$

57

STEADY-STATE COHERENT (SSC)

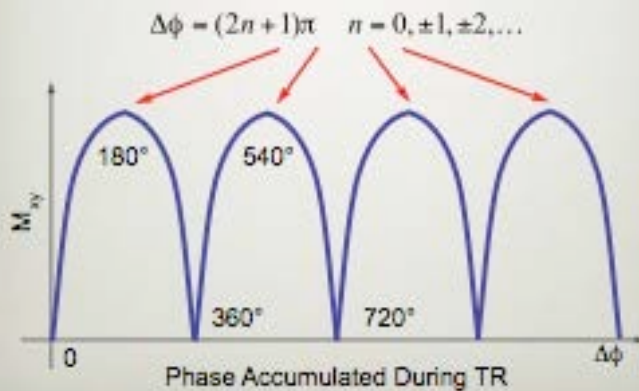
Transverse Magnetization Preserved. $TR < T_2$

Both M_x and M_{xy} attain a steady-state.



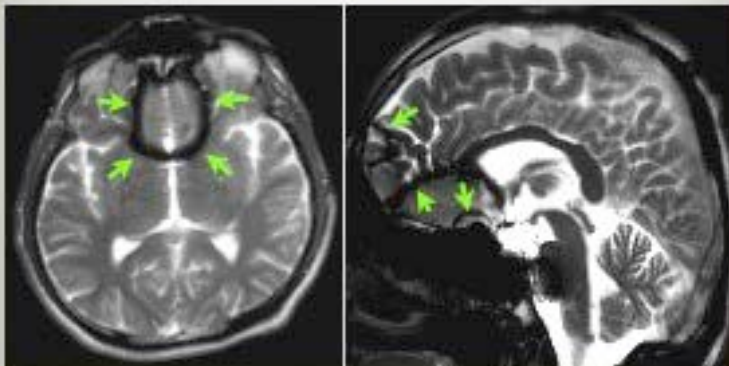
58

OFF-RESONANCE EFFECTS IN SSC



59

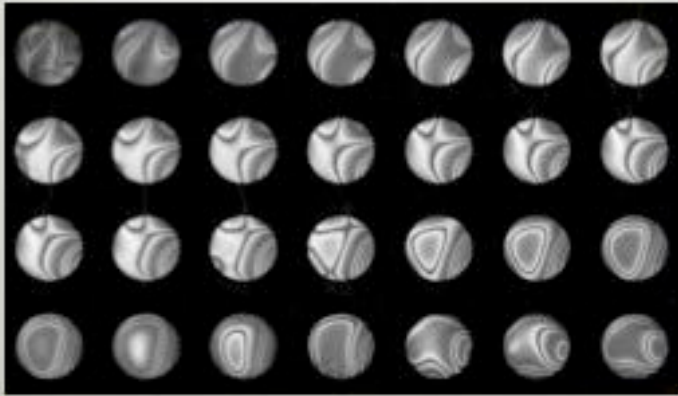
QUIZ



SSFP Imaging Sequence at 3T

60

OFF-RESONANCE ARTIFACTS SSFP AT 12 TESLA

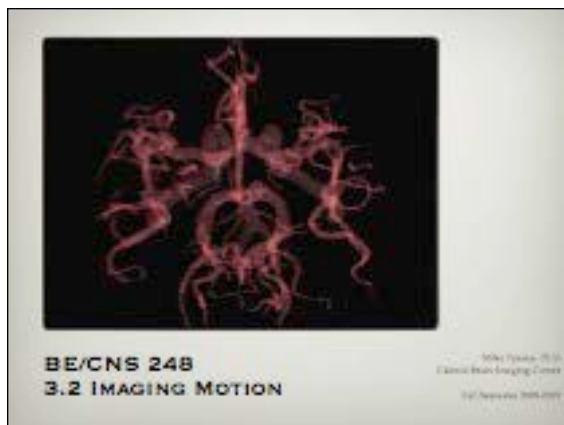


61

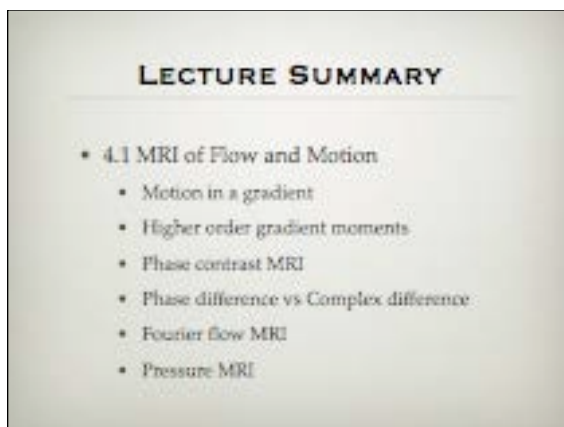
NEXT LECTURE

- MR Image Formation I

62



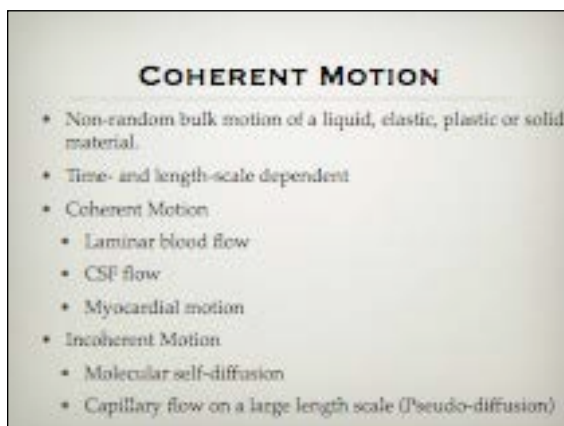
1



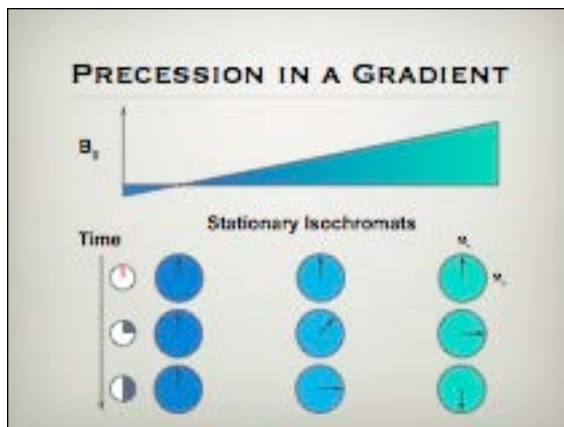
2



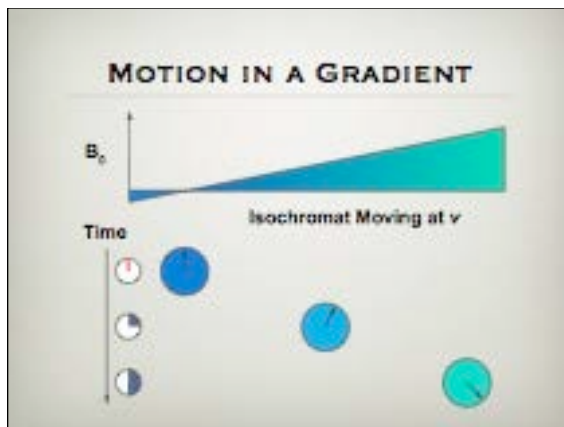
3



4



5



6

HIGHER ORDER MOTION

Maclaurin expansion of position as a function of time

$$x(t) = x(0) + x'(0)t + \frac{1}{2!}x''(0)t^2 + \dots + \frac{1}{n!}x^{(n)}(0)t^n + \dots$$

$x'(0)$	$=$	$v(0)$	Initial Velocity
$x''(0)$	$=$	$a(0)$	Initial Acceleration
$\vdots$			Initial Jerk ...

7

GRADIENT MOMENT EXPANSION

Substitute into 1D phase accumulation equation

$$\begin{aligned} \phi(T) &= \gamma \int_0^T dt G(t) x(t) \\ &= \gamma \left[x(0) \int_0^T dt G(t) + x'(0) \int_0^T dt G(t) t + \dots \right] \\ &= \gamma [x(0)m_0(T) + v(0)m_1(T) + a(0)m_2(T) + \dots] \end{aligned}$$

8

PHASE AND MOTION ORDER

Phase contribution due to
a given order of motion

n^{th} gradient
moment

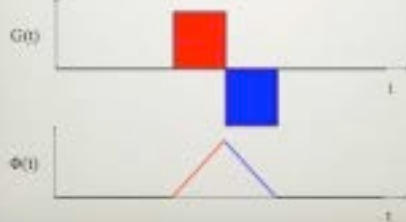
$$\phi_n(T) = \gamma \underbrace{x^{(n)}(0)}_{n^{\text{th}} \text{ time-derivative of displacement}} m_n(T)$$

n^{th} time-derivative
of displacement

9

ZEROth MOMENT

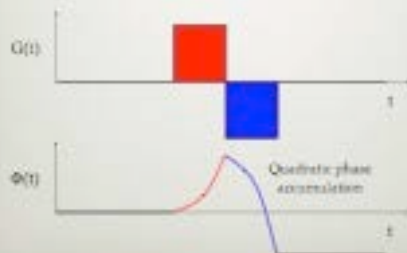
- Area of gradient in pulse sequence diagram.
- Phase evolution of stationary spin.



10

FIRST MOMENT

- Phase evolution of spin with constant velocity.



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GRADIENT ECHO MOMENTS

Basic gradient echo waveform



What are the zeroth and first gradient moments at the time of the echo?

12

GRADIENT ECHO MOMENTS



0th Moment $m_0(TE) = \int_0^{TE} dt G(t) = 0$

1st Moment $m_1(TE) = \int_0^{TE} dt G(t)t$
 $= G\tau^2$

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FLOW COMPENSATION

14

FLOW COMPENSATION

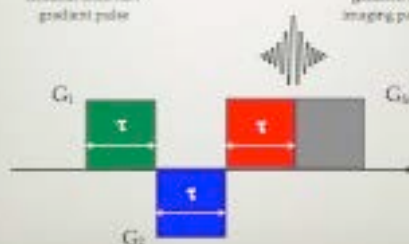
- Can we design a gradient waveform where both zeroth and first order moments are zero at the echo time?
- If this is possible, then any constant motion would have no effect on the phase of the echo.
- Need one more degree of freedom.

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MOTION COMPENSATING GRADIENT WAVEFORM

Additional degree of freedom from new gradient pulse

Frequency encoding gradient fixed by imaging parameters



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MOMENT EQUATIONS

Gradient echo with equal duration gradient pulses

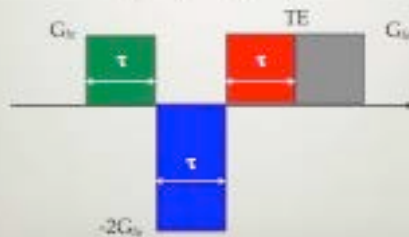
$$m_0(TE) = \tau(G_1 + G_2 + G_{fe})$$

$$m_1(TE) = \frac{1}{2}\tau^2(G_1 + 3G_2 + 5G_{fe})$$

17

MOTION COMPENSATION SOLUTION

$$m_0(TE) = m_1(TE) = 0$$



18

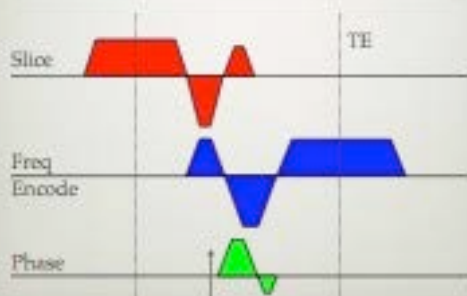
EVEN ECHO FLOW COMPENSATION

First moment nulled intrinsically for even echos in both gradient and spin echo trains



19

FLOW COMPENSATED SLICE SELECTION AND PHASE ENCODING



20



FLOW ENCODING

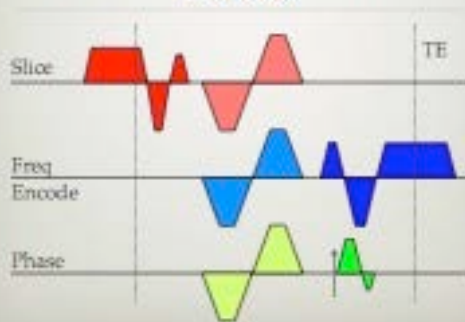
21

PHASE ENCODING OF FLOW

- Can a gradient waveform be designed that gives the echo a phase proportional to velocity in a given direction?
- Requires that zeroth moment is zero and first moment is finite at the echo time.
- More convenient still to have independent control over first moment.

22

BIOPOLAR FLOW ENCODING PULSES



23

BACKGROUND PHASE

- The voxel phase without velocity encoding cannot be assumed to be zero.
- Many contributions to background phase:
 - B_0 inhomogeneity
 - Gradient eddy currents
 - Other orders of motion
 - Deliberate offsetting of echo position
 - —

24

REMOVING BACKGROUND PHASE

General velocity encoded magnetization

$$\hat{\rho} = \rho_0 e^{i(\phi_b + \phi_v)}$$

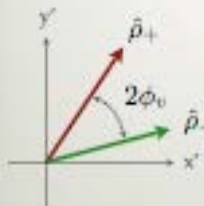
Acquire two images with opposite sign bipolar gradient pulses

$$\hat{\rho}_{\pm} = \rho_0 e^{i(\phi_b \pm \phi_v)}$$

$$\phi_v = \frac{1}{2}(\angle \hat{\rho}_+ - \angle \hat{\rho}_-)$$

25

PHASE DIFFERENCE



Difference in phase of transverse magnetization between positive and negative velocity encoded acquisitions.

26

PRINCIPLE OF PHASE CONTRAST

Background phase arises from B_0 inhomogeneities, echo timing offsets, eddy currents, Maxwell terms, ...



Velocity encoded through plane

Velocity compensated reference

Corrected phase velocity image

27

ENCODING SCHEMES

6 Point Scheme



	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

4 Point Scheme



	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

Scale gradients by $1/\sqrt{3}$

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RECOVERING VELOCITY FROM 4-POINT ENCODING

Use linear combinations of the voxel phase from each acquisition. For example, for the phase encoded z-component of velocity:

$$\phi_z = \frac{1}{2} [(\phi_3 + \phi_4) - (\phi_1 + \phi_2)]$$

29

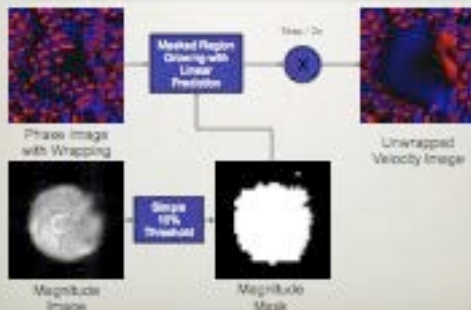
ALIASING VELOCITY : VENC

Phase is modulo 2π so we expect to see phase aliasing if the velocity exceeds a certain value. This critical velocity is called the encoding velocity or V_{enc} .

$$V_{enc} = \frac{\pi}{\gamma m_1 (TE)}$$

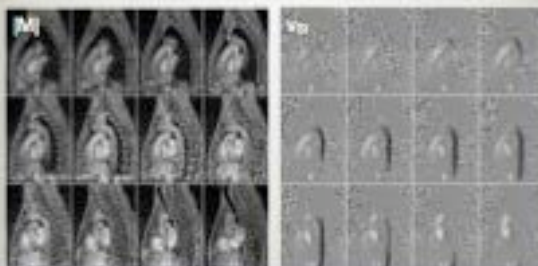
30

PHASE UNWRAPPING



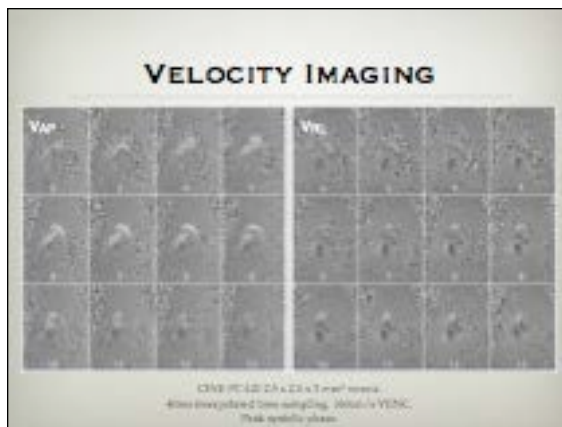
31

VELOCITY IMAGING

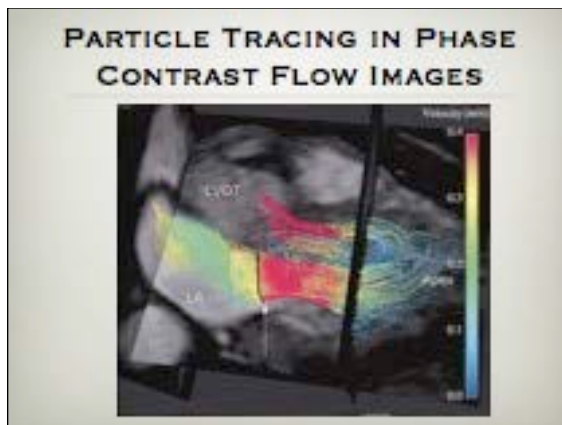


GE 1.5T PCAS 2D x 2D x 2 mm slices.
4000 interpolated lines sampling. Velocity VENC.
Think spatial aliasing.

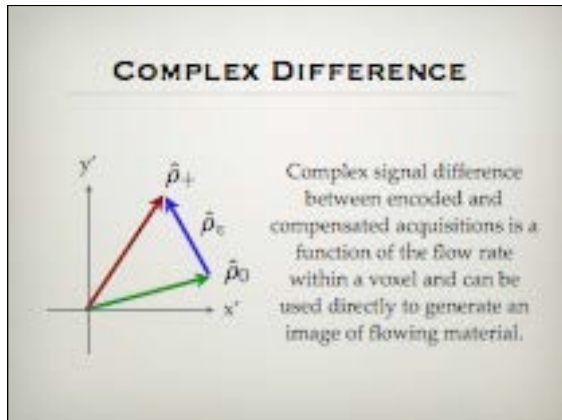
32



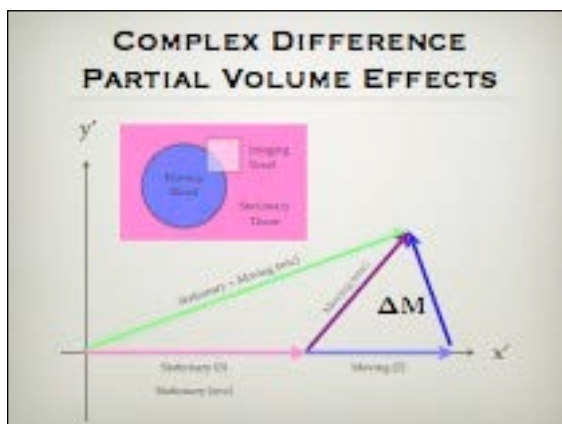
33



34

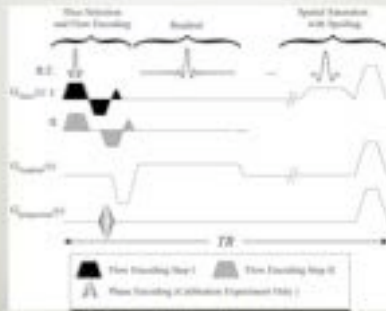


35



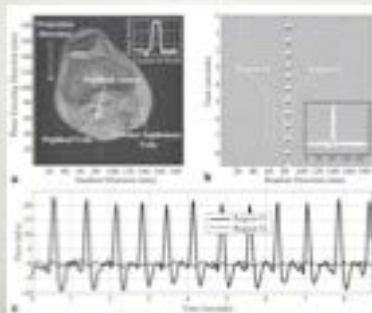
36

COMPLEX DIFFERENCE EXAMPLE



37

COMPLEX DIFFERENCE EXAMPLE



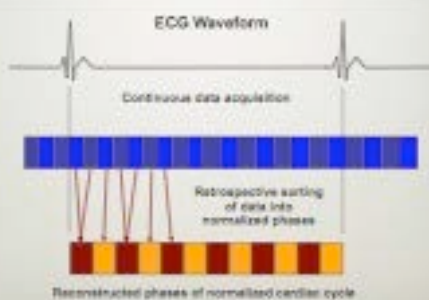
38



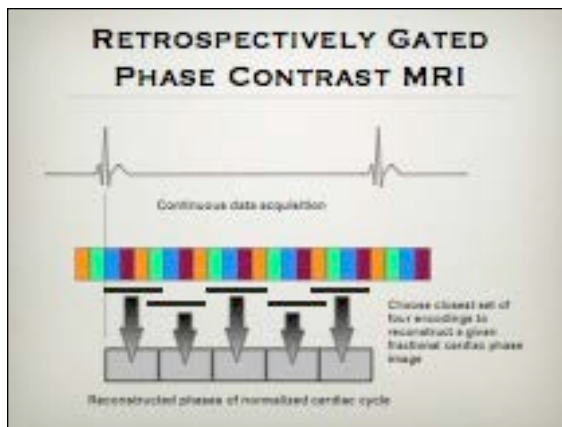
TIME-RESOLVED FLOW MEASUREMENT

39

RETROSPECTIVE GATING



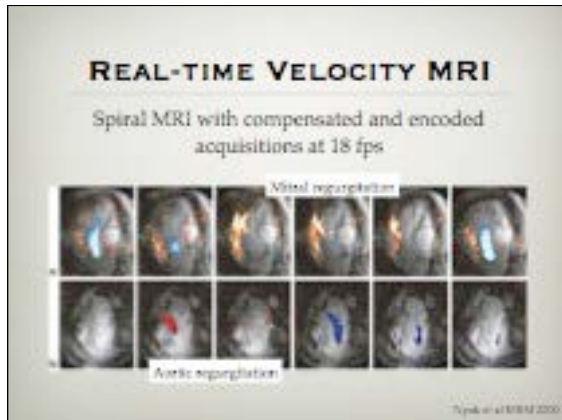
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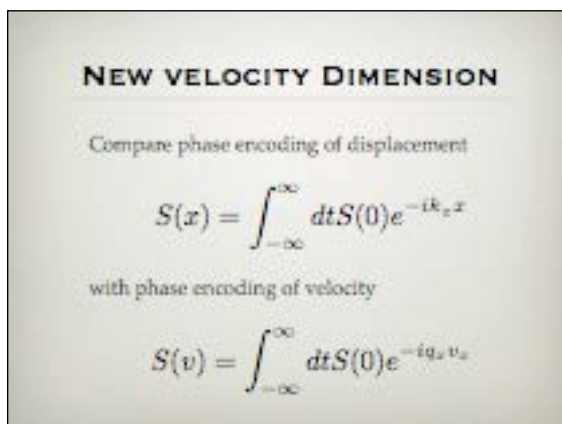
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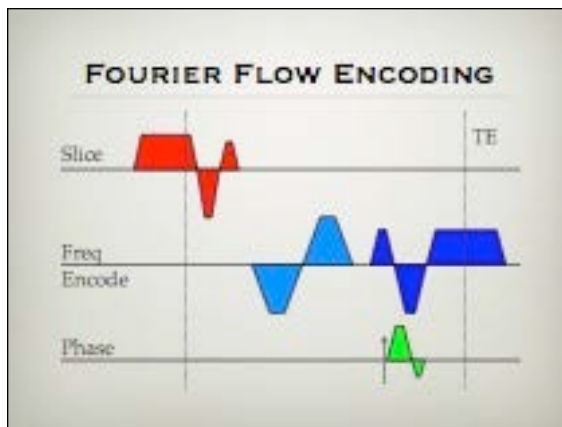
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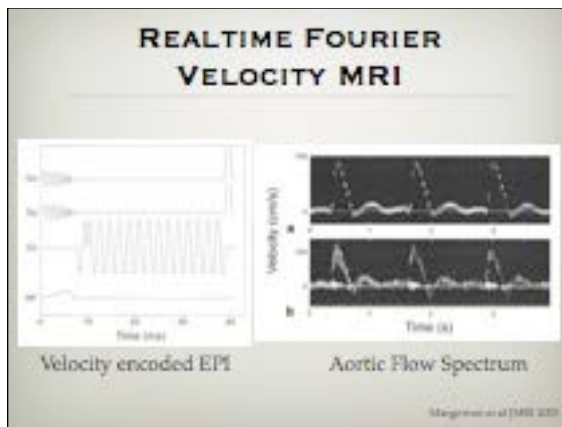
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44



45



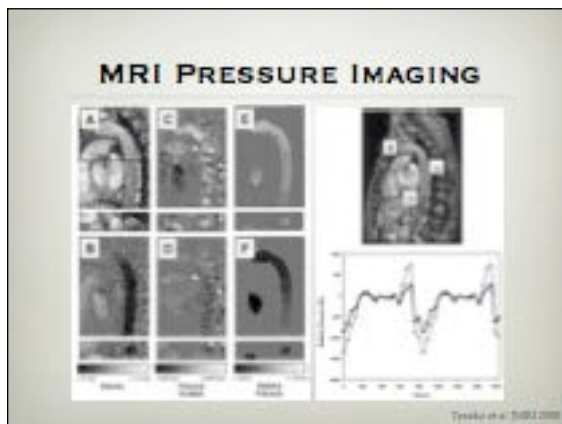
46

PRESSURE CALCULATION

- The Navier-Stokes Equation relates velocity and relative pressure.
- Flow is viscous, time-varying and rotational.

$$-\nabla p = \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \times \nabla \mathbf{u} + \eta \nabla \times \nabla \times \mathbf{u}$$

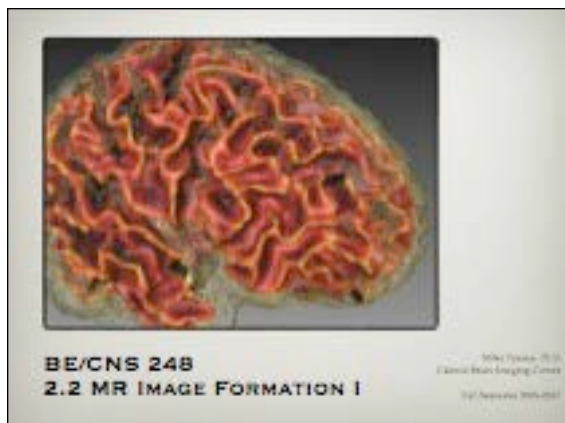
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48

NEXT LECTURE

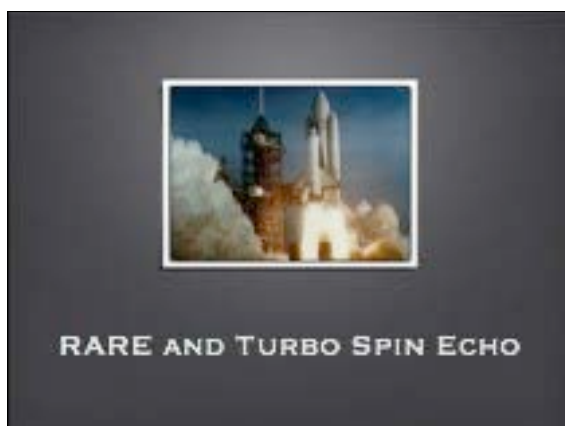
- Diffusion MRI



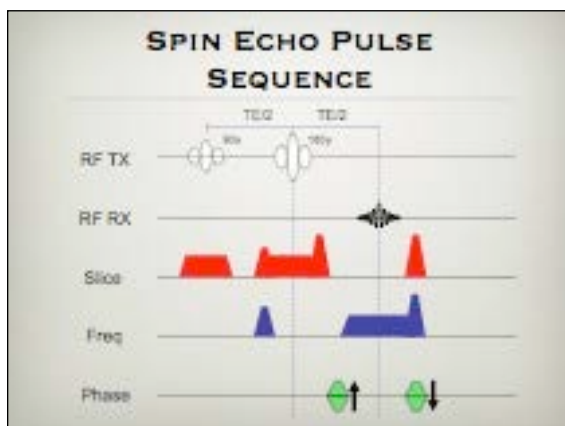
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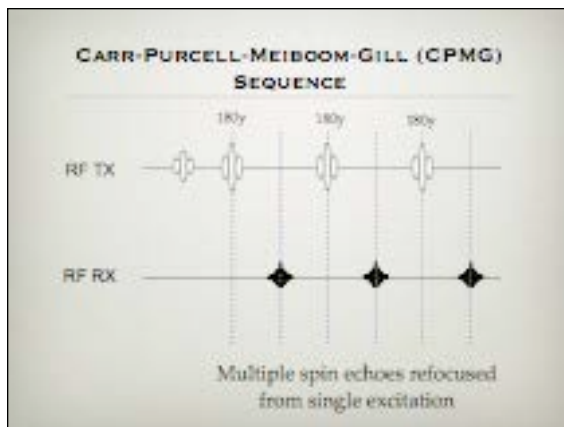
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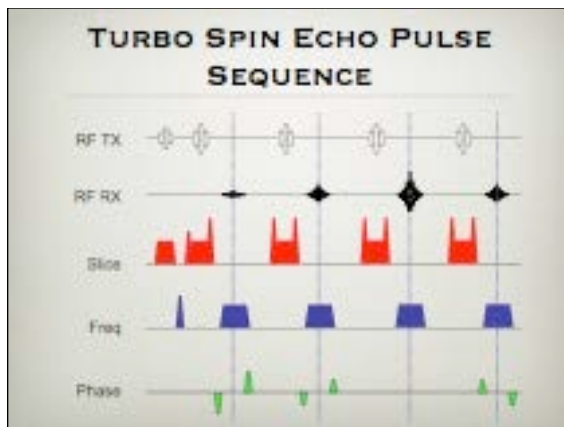
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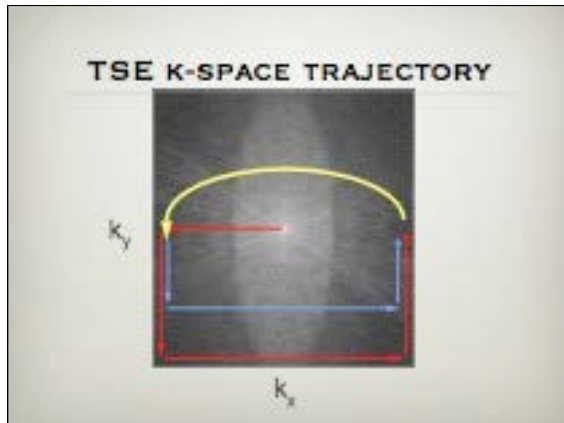
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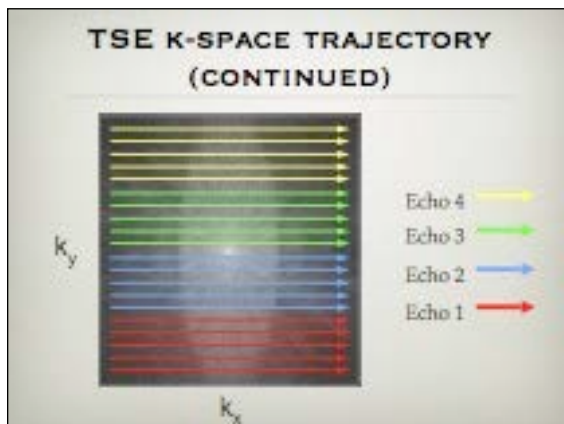
5



6



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8

TURBO SPIN ECHO

- + Key Parameters
 - + TR : Typically long (seconds)
 - + TE : Typically long (10s to 100s ms)
 - + Echo Train Length : Speed factor with side-effects
 - + Typically T_2 -weighted (T_1 -weighted possible)
- + Primary Applications
 - + Rapid T_2 -weighted 2D multislice imaging
- + Artifacts
 - + Motion sensitive (single-shot less sensitive)
 - + Blurring from short T_2 tissues

9

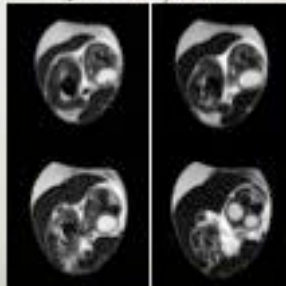
T_2 -WEIGHTED RARE



10

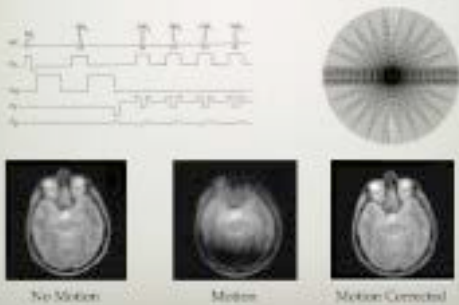
SINGLE-SHOT RARE

Quail Embryo in ovo



11

PROPELLER



12

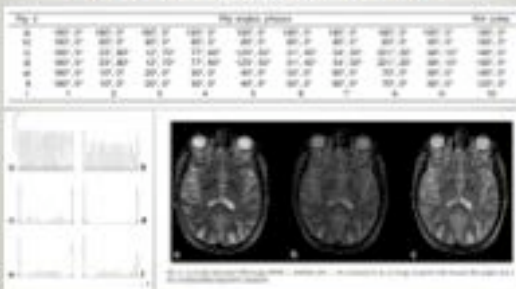
HYPERECHOES

- Originated by Juergen Hennig and Klaus Scheffler
- Reduced RF power deposition.
- SNR largely preserved



13

HYPERECHOES AND TSE



Hennig J, Scheffler K. Hyperchoes. Magn Reson Med 2011; 65:9-12

14

RARE SYNONYMS

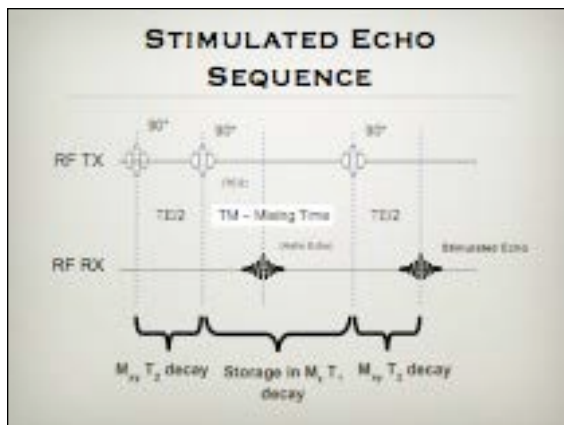
- RARE : Rapid Acquisition with Relaxation Enhancement
- TSE : Turbo Spin Echo
- FSE : Fast Spin Echo
- ssRARE : Single-shot RARE
- HASTE : Half-Fourier Acquisition Single-shot TSE

15

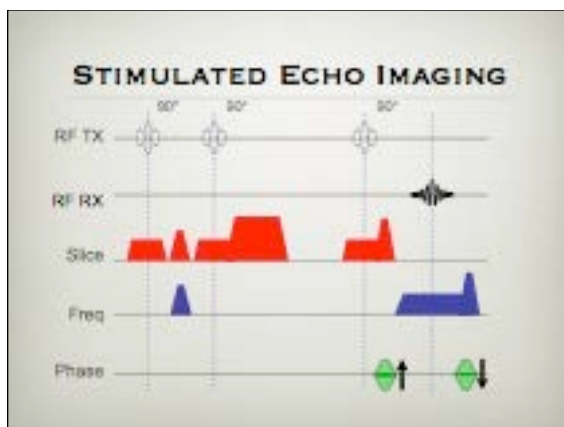


STIMULATED ECHO MRI

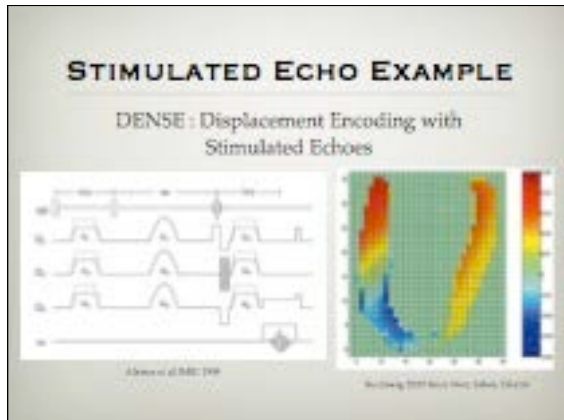
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17



18

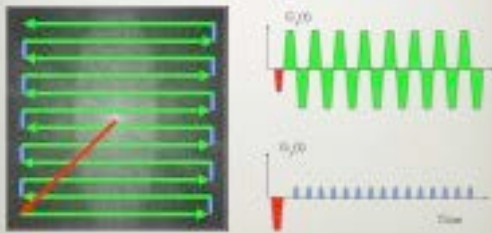


19



20

ECHO-PLANAR IMAGING



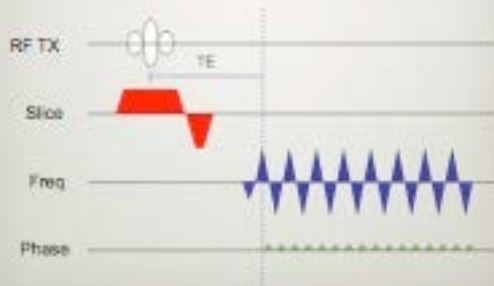
21

EPI PULSE SEQUENCES

- Any transverse magnetization preparation can be imaged using an EPI k-space traversal.
- Gradient echo and spin echo most common preparation methods.
- Other popular sequences:
 - Inversion Recovery Spin-echo EPI
 - Diffusion-weighted Spin-echo EPI

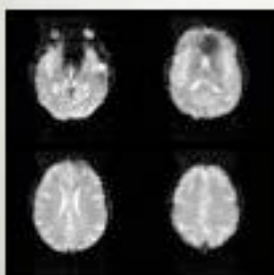
22

FID EPI



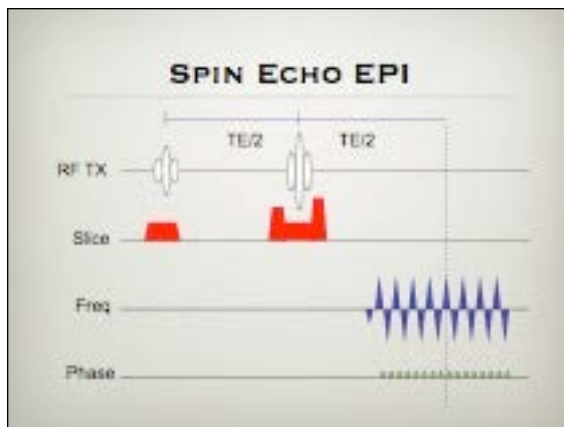
23

FID-EPI



FID-EPI
 Shots : 1
 TR : 3000ms
 TE : 60ms
 Slice : 5mm/2.5mm (18)
 Matrix : 128 x 128
 FOV : 24cm x 24cm
 Time : 6s
 NEX : 1

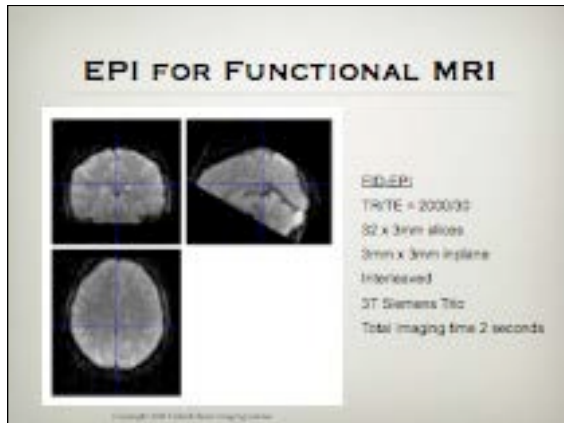
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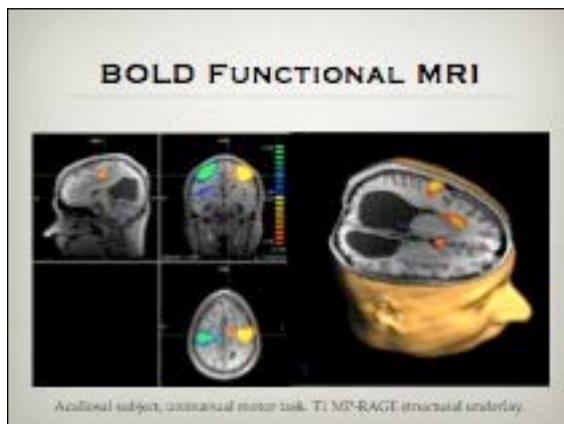
25



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28



ACHILLES' HEEL OF EPI

29

FREQUENCY ENCODING BANDWIDTH



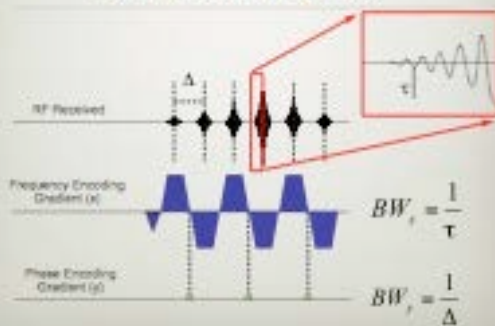
$$BW = \gamma G_x FOV_x$$

$$\left[\frac{Hz}{G} \right] = \left[\frac{Hz}{G} \right] \left[\frac{G}{cm} \right] [cm]$$

$$B_1 = B_0 + G_x x$$

30

EPI BANDWIDTHS



31

STATIC INHOMOGENEITY AND EPI

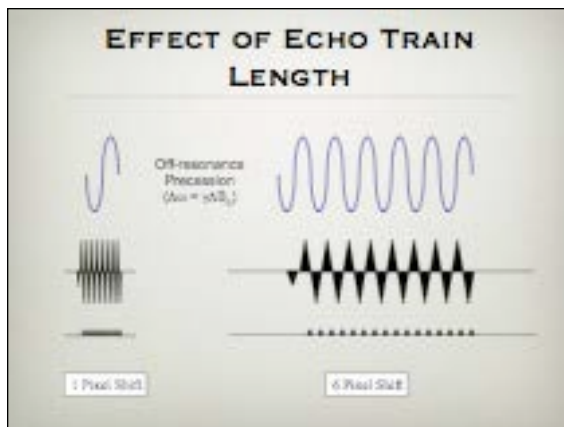
Frequency Encoding Direction



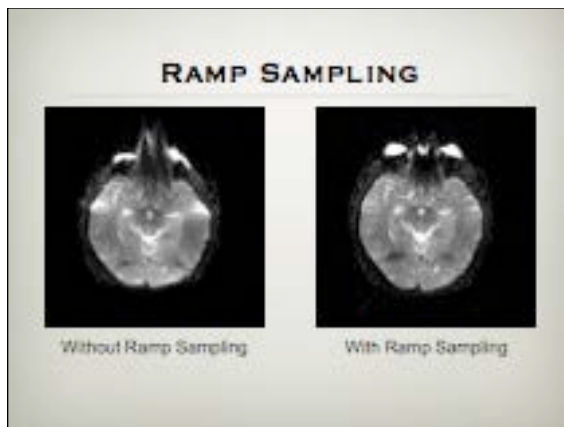
Voxel displacement due to static field inhomogeneity

$$\Delta y = \frac{\gamma \Delta B_0 \cdot FOV_y}{BW_x}$$

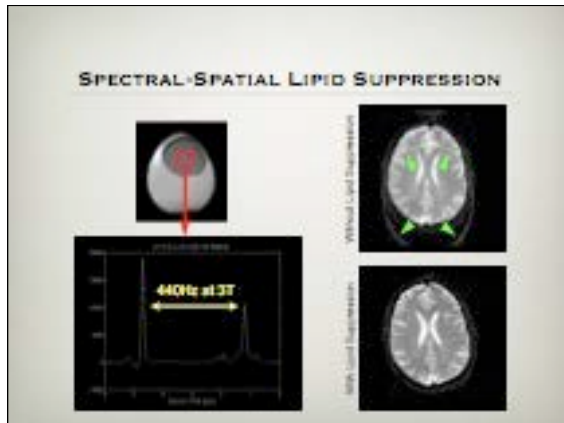
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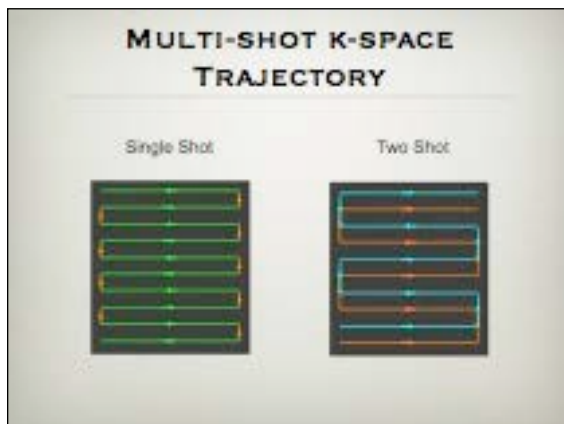
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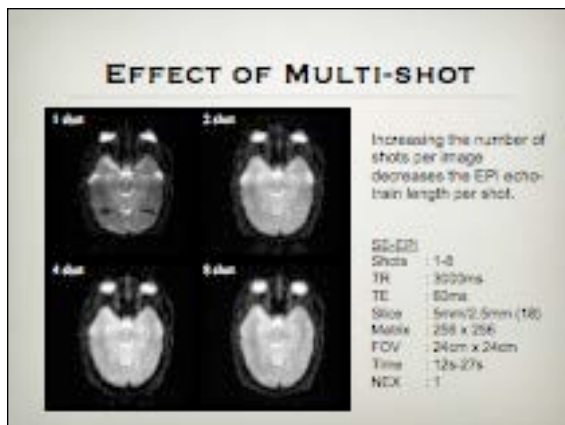
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35



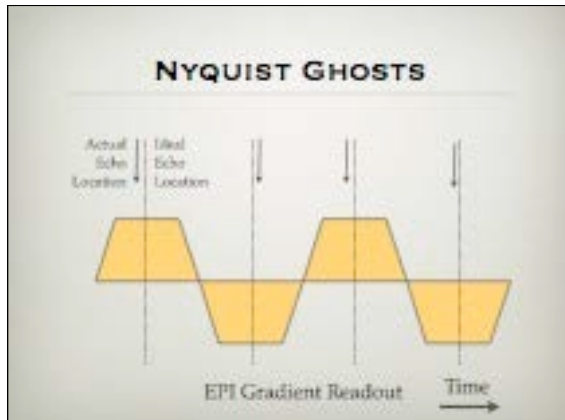
36



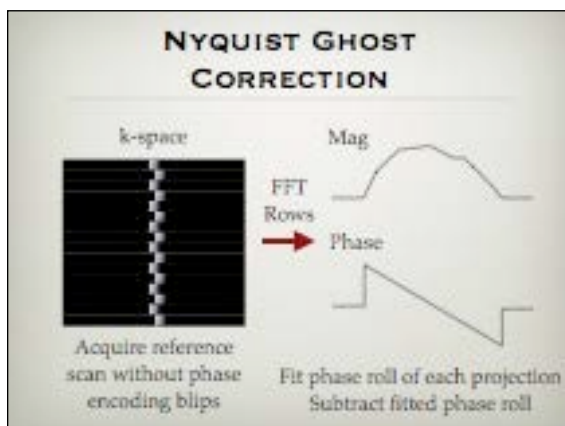
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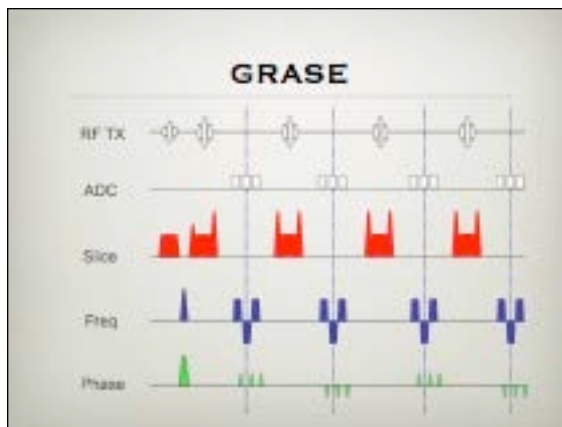
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40



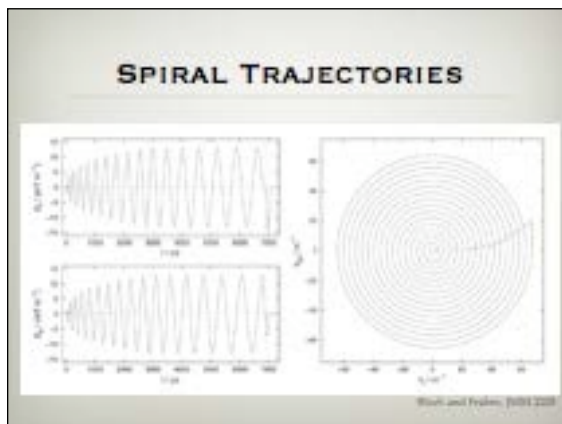
41

- ### APPLICATIONS OF EPI
- Diffusion-weighted Imaging
 - Reduces effect of gross motion
 - Reduces total imaging time
 - Functional BOLD Imaging
 - Allows high temporal resolution
 - Reduces effect of gross motion
 - Reduces total imaging time

42



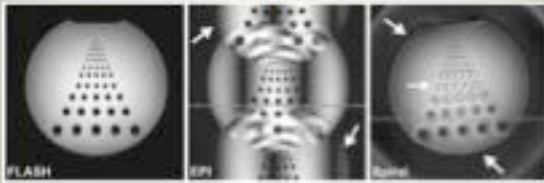
43



44

SPIRAL ARTIFACTS

Imperfections in actual trajectory cause artifacts in both EPI and Spiral MRI



Reich and Taylor, JMRI 2003

45

SPIRAL FEATURES

- Spiral trajectory less sensitive to motion (periodic first order moment nulling).
- Artifacts are more diffuse and peripheral than for Cartesian trajectories.
- Image reconstruction requires regridding to Cartesian grid for FFT.

46

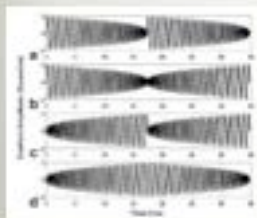
SPIRAL APPLICATIONS: MR CORONARY ANGIOGRAPHY



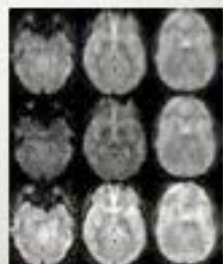
Shen et al, JMRI 1999

47

SPIRAL FOR BOLD FMRI



Li et al Magn Reson Med. 2004;51:125



DSPIN Trajectory

48



MAGNETIZATION PREPARATION

49

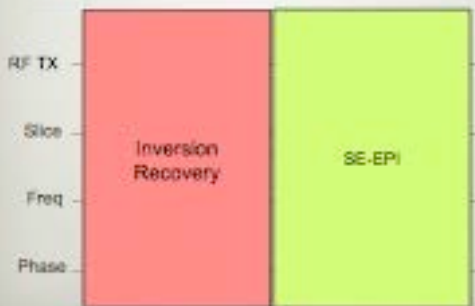
MAGNETIZATION PREPARATION



Longitudinal or transverse magnetization carries magnetization contrast prepared by initial pulse sequence

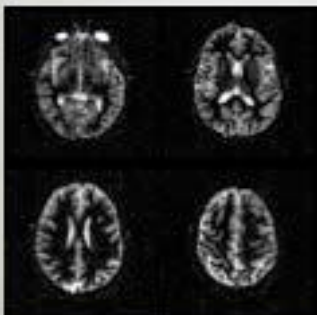
50

INVERSION-RECOVERY EPI



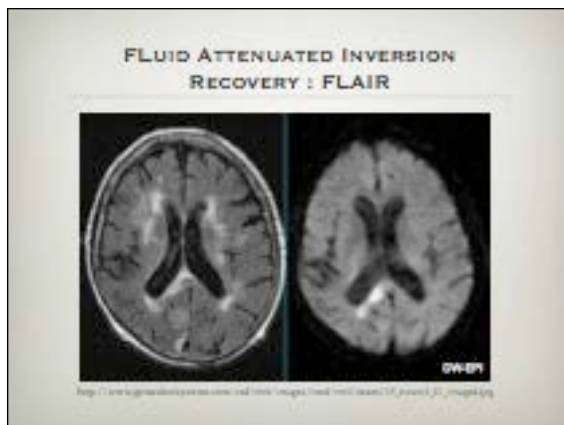
51

IR-EPI IMAGES

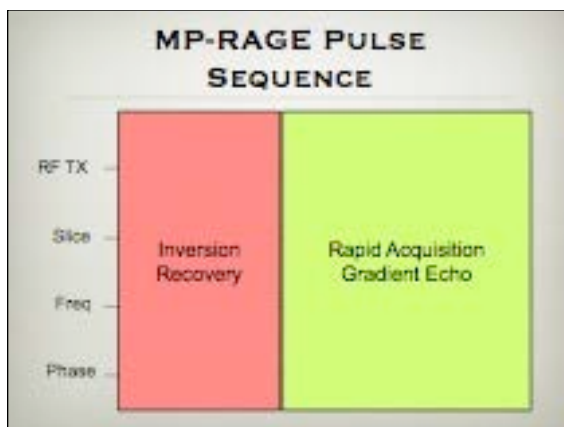


R-SE-EPI
Shots : 1
TI : 400ms
TR : 3000ms
TE : 80ms
Slice : 5mm/2.5mm (18)
Matrix : 128 x 128
FOV : 24cm x 24cm
Time : 12s
NEX : 1

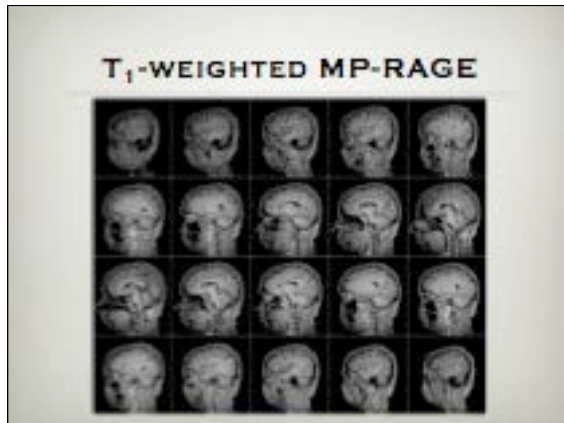
52



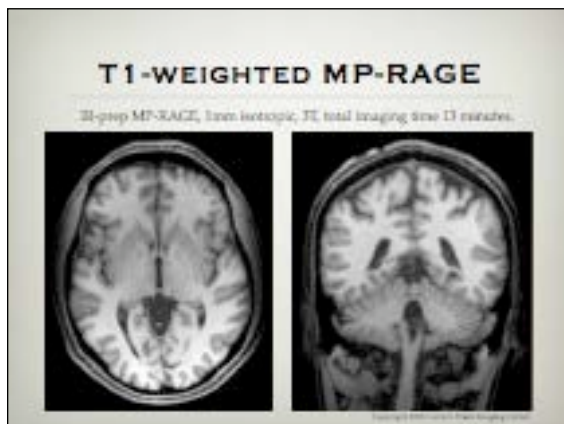
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54



55



56

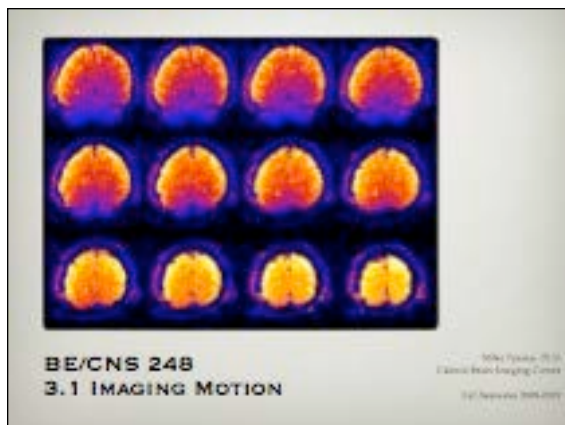


57

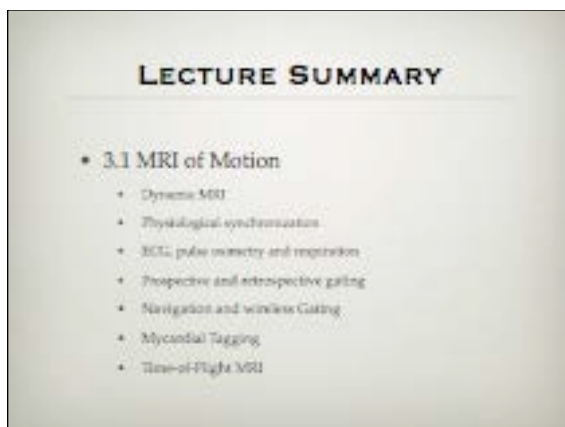
NEXT LECTURE

- MR Image Formation II
- Steady-state MRI

58



1



2

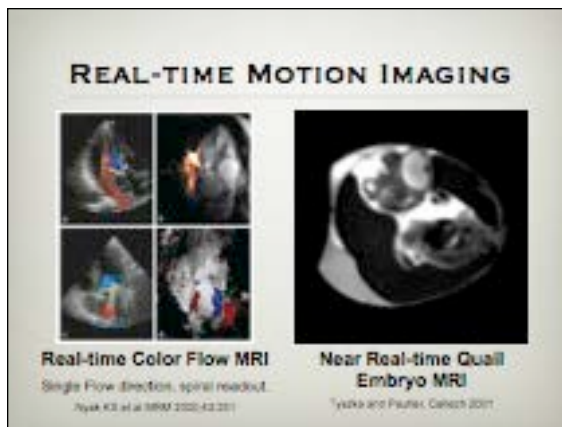


3

PHYSIOLOGICAL MOTION

Periodic	Aperiodic
Myocardial Motion	Development
Respiration	Musculoskeletal Motion
Cardiovascular Flow	Gut Peristalsis
CSF Flow	

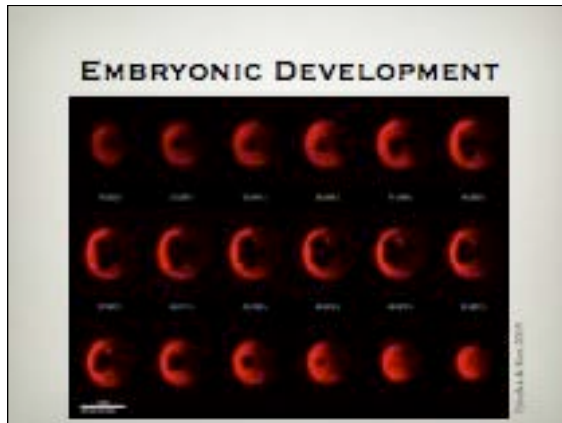
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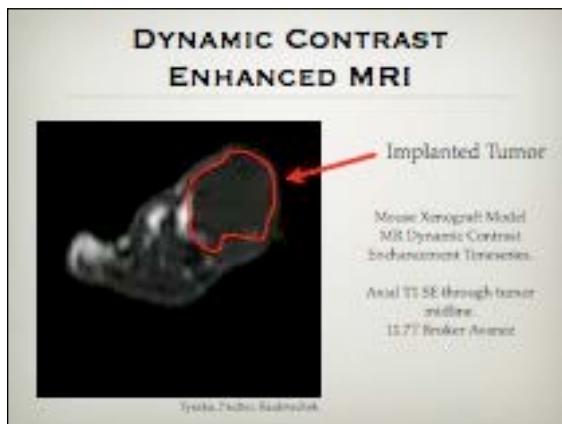
5



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7



8

GUT PERISTALSIS



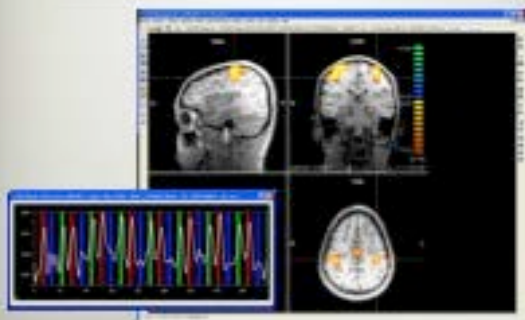
Mouse Xenograft Model
MR DCE bilus passage

Axial T₂-weighted SE
through kidneys

11.7 Bruker Avance

9

FUNCTIONAL MRI



10



PERIODIC PHYSIOLOGICAL MOTION

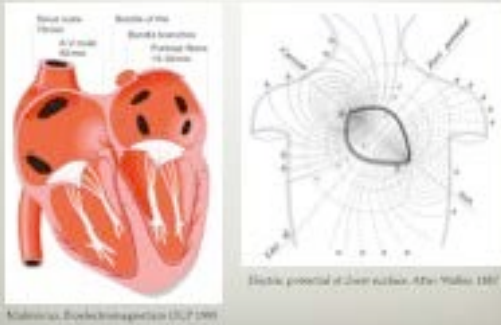
11

MRI AND PERIODIC MOTION

- Duration of MRI experiment typically much longer than period of motion.
- Strobe MR data acquisition to motion period.
- Synchronizing trigger
 - ECG
 - Pulse oximeter (plethysmograph)
 - Respiratory transducer
 - Other signal

12

ELECTRIC FIELD OF THE HEART

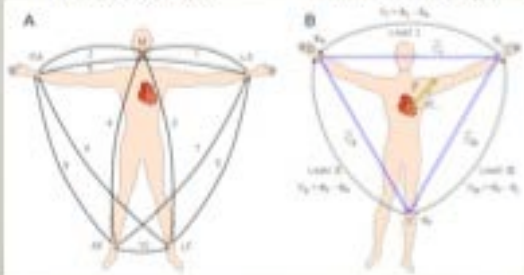


13

ECG LEADS

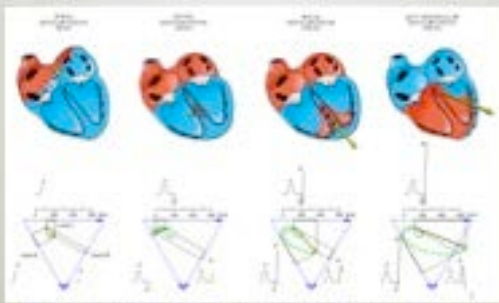
10 Leads of Waller

Einthoven Triangle



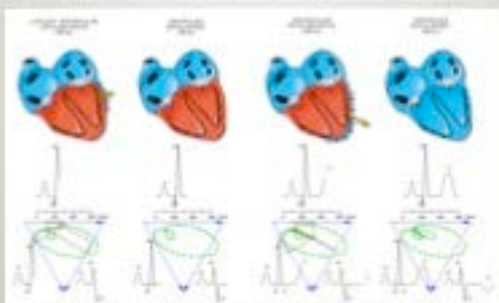
14

THE ECG WAVEFORM



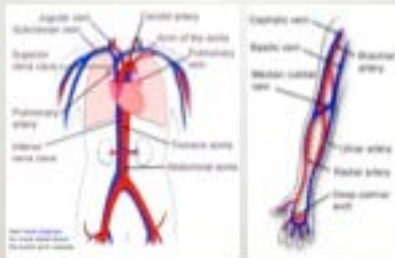
15

THE ECG WAVEFORM



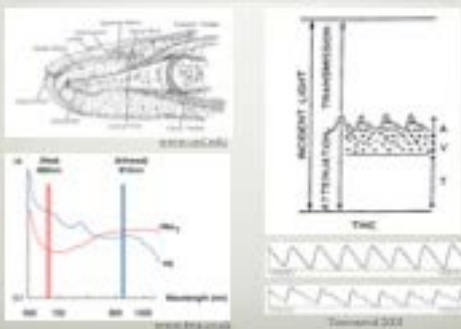
16

PERIPHERAL PULSE



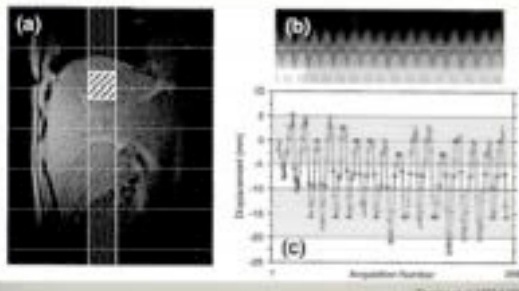
17

PULSE OXIMETRY



18

RESPIRATION

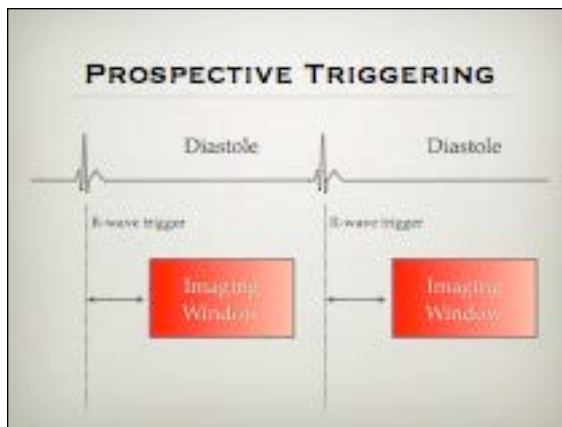


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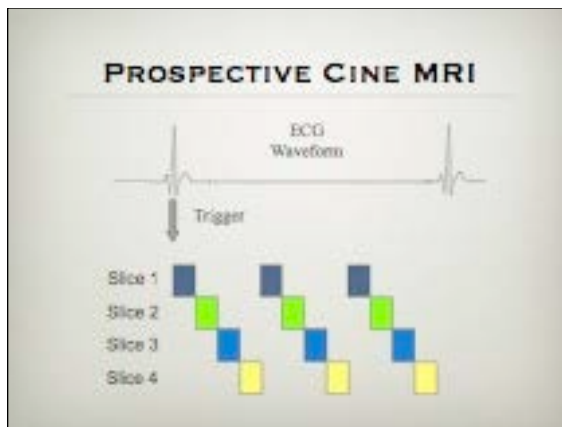


PROSPECTIVE GATING

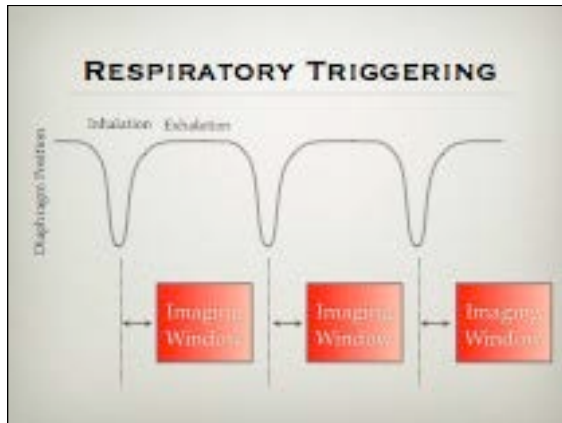
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22

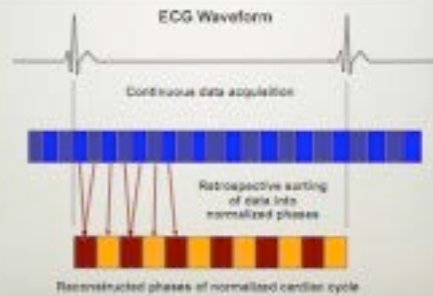


23



24

RETROSPECTIVE GATING



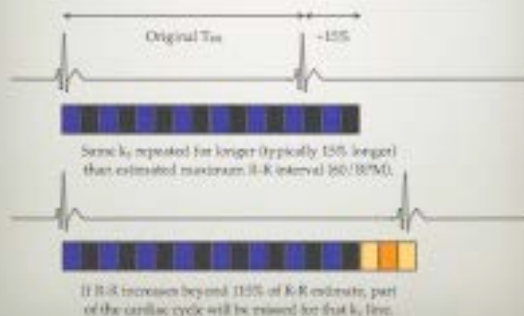
25

RETROSPECTIVE GATING RATIONALE

- Prospective gating not a good match for steady-state imaging.
- Can we preserve steady-state but still obtain cardiac cycle resolved images?
- Acquire continuous MRI and simultaneously record ECG for reference.
- Sort out the mess retrospectively.

26

ARRHYTHMIA IN RETROSPECTIVE GATING



27

GATED OR TRIGGERED MOTION IMAGING



Short Axis Adult Human Heart.
Retrospectively gated.
Images courtesy of Steve Fierbey CBIC 2009

28



OTHER TRIGGER SOURCES

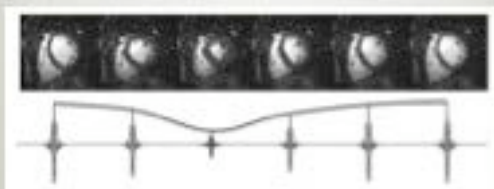
29

IMAGE-BASED GATING

- Detect onset of an event from the image data stream or navigation.
- Volume selective excitation tracks signal in a key region such as the aorta or diaphragm.
- Also called "wireless" gating.

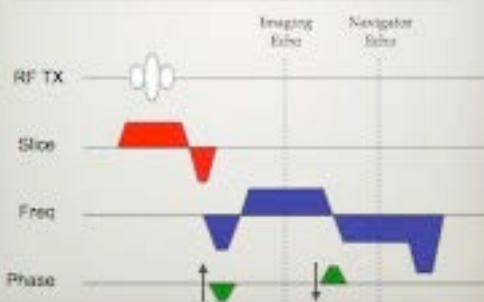
30

PHYSIOLOGICAL MODULATION



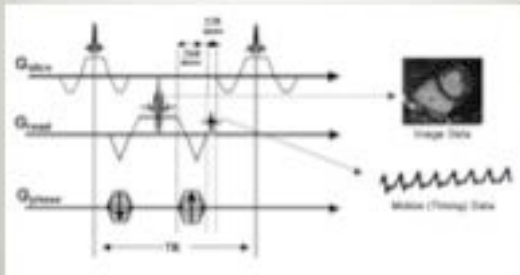
31

NAVIGATION



32

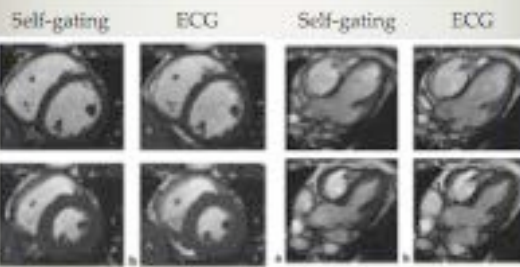
NAVIGATED WIRELESS GATING



© 2008 KJSSA 2008

33

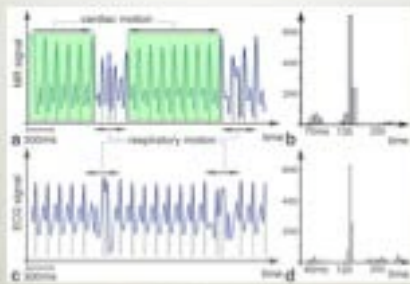
NAVIGATED WIRELESS GATING



© 2008 KJSSA 2008

34

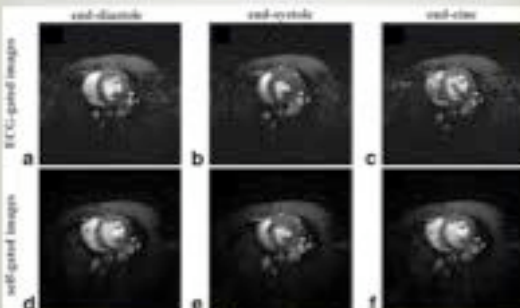
COMBINED CARDIAC + RESPIRATORY GATING



© 2008 KJSSA 2008

35

CARDIAC+RESPIRATORY SELF GATING IN MOUSE



© 2008 KJSSA 2008

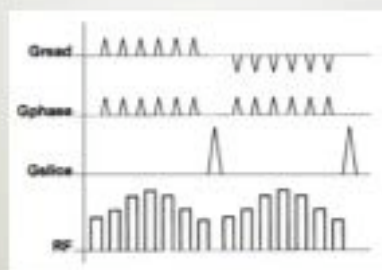
36



MYOCARDIAL TAGGING

37

SPAMM AND DANTE



38

MAGNETIZATION TAGGING



- Longitudinal magnetization tagging at end-diastole
- Image at subsequent phases of cardiac cycle to follow myocardial motion

Typical MRI magnetization tagging sequence

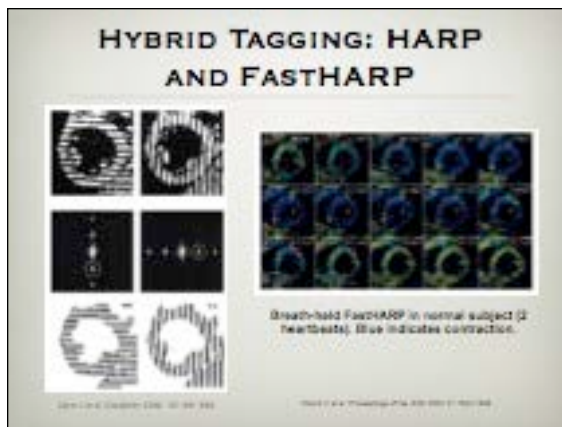
Copyright © 2000 by Lippincott Williams & Wilkins. All rights reserved. Printed in the United States of America. No part of this publication may be reproduced without written permission from the publisher.

39

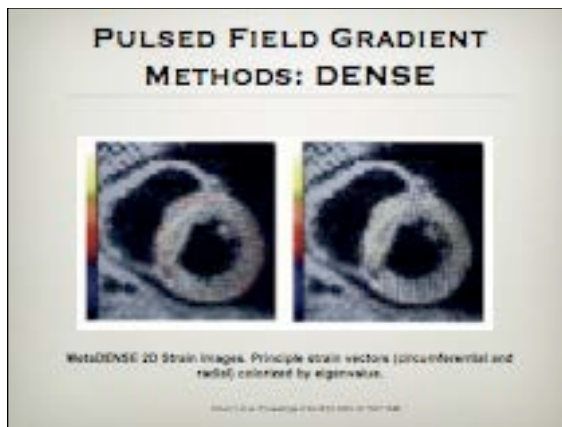
MYOCARDIAL TAGGING



40



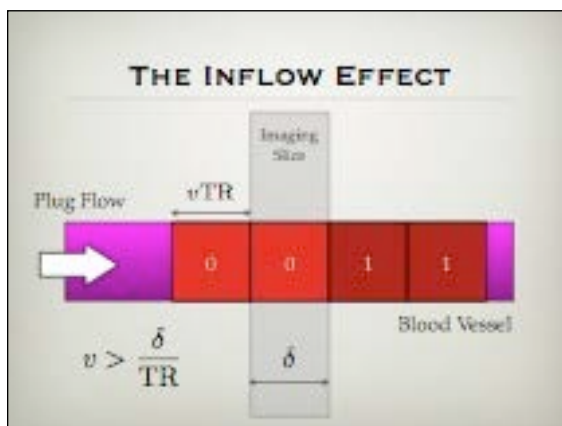
41



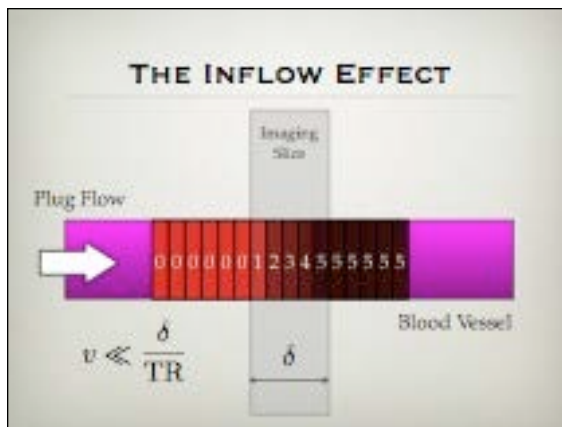
42



43



44



45

TOF CONTRAST EQUATION

Number of TRs or pulses taken by plug flow to cross the slice

$$n = \frac{\delta}{vTR}$$

Steady-state Equilibrium Longitudinal Magnetization

$$M_{ss} = \frac{M_0(1 - E_1)}{1 - q}$$

where

$$q = E_1 \cos \theta$$

46

TOF CONTRAST EQUATION

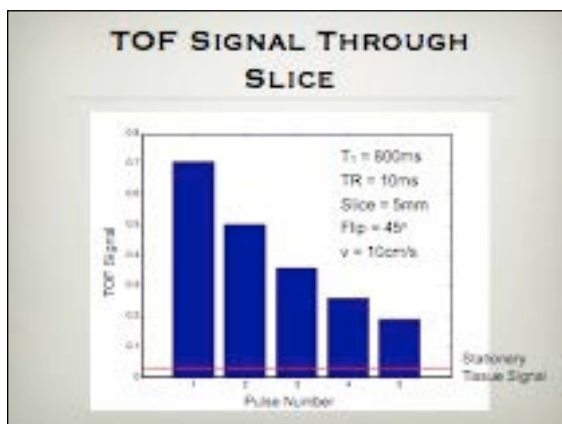
Signal for plug flow within the imaging slice after p pulses

$$S(p > n) = A \frac{\sin \theta}{n} \sum_{m=1}^n (M_{ss} + (M_0 - M_{ss})q^{m-1})$$

$$= A \sin \theta \left(M_{ss} + (M_0 - M_{ss}) \frac{1 - q^n}{n(1 - q)} \right)$$

See Slides 24-5 for full derivation

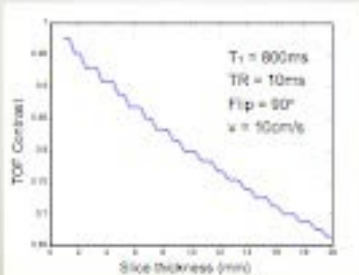
47



48

TOF CONTRAST AND SLICE THICKNESS

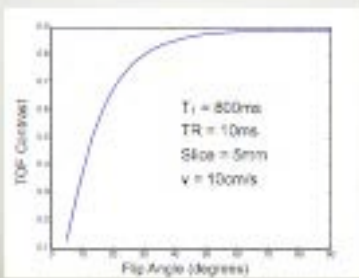
TOF Contrast between plug flow and stationary material



49

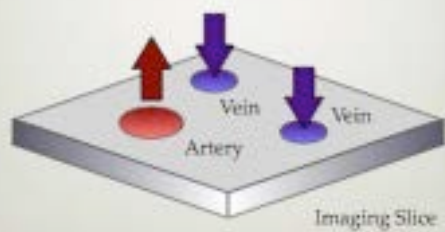
TOF CONTRAST AND FLIP ANGLE

TOF Contrast between plug flow and stationary material



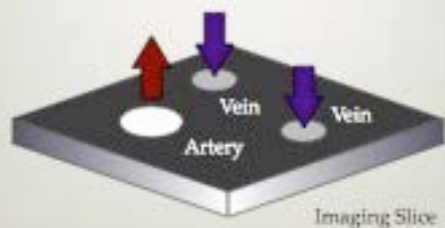
50

DIRECTION-SENSITIVE INFLOW

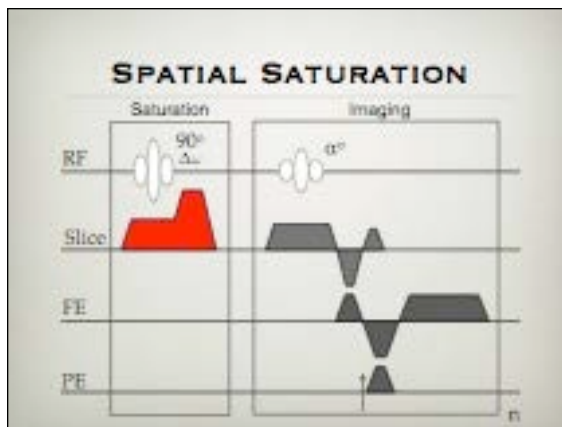


51

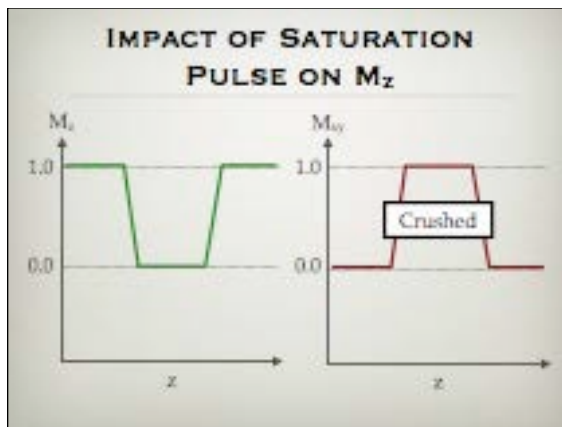
2D TOF INFLOW IMAGING



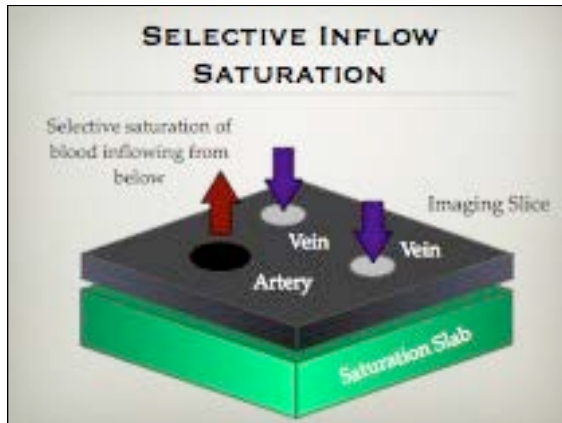
52



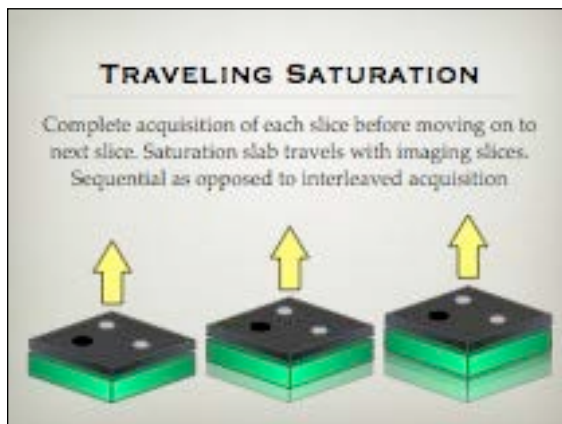
53



54



55



56

3D TIME-OF-FLIGHT



Inflow effect can penetrate substantial distances into a large 3D volume.

3D MP-RAGE showing carotid and vertebral artery inflow effect.

57

3D TOF & RAMPED PULSES



Flip Angle decreases with distance across slab, reducing progressive saturation effects.

58

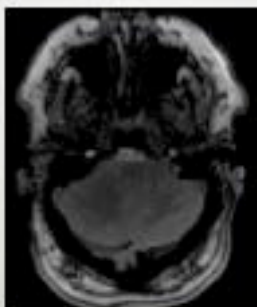
MULTI-SLAB 3D TOF



Overlapped 3D imaging slabs. Saturation slabs not shown.

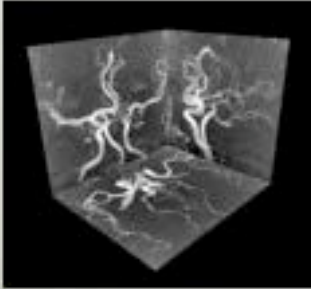
59

3D MULTI-SLAB TOF



60

3D MULTI-SLAB TOF



Maximum intensity
ray-tracing.

3D Multi-slab TOF
at 3 Tesla with
superior inflow
saturation.
Acallosal subject.

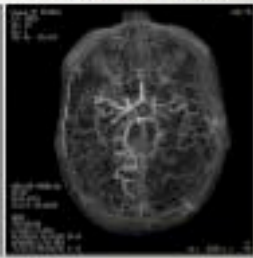
61

ENHANCED TOF ANGIOGRAPHY

3D TOF Without Contrast Agent



3D TOF With Contrast Agent

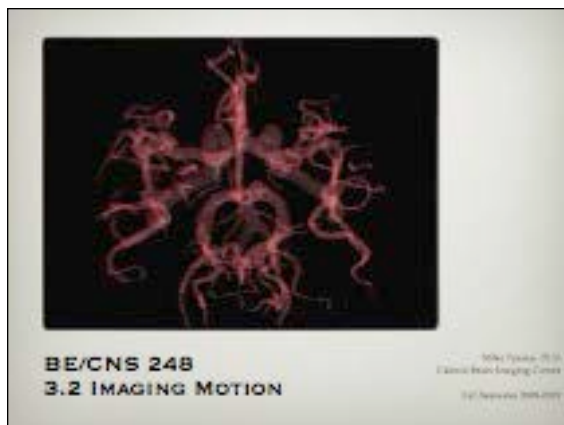


62

NEXT LECTURE

- MRI of Flow and Motion
 - High order gradient moments
 - Phase contrast flow encoding
 - Fourier flow imaging

63



1

LECTURE SUMMARY

- 4.1 MRI of Flow and Motion
 - Motion in a gradient
 - Higher order gradient moments
 - Phase contrast MRI
 - Phase difference vs Complex difference
 - Fourier flow MRI
 - Pressure MRI

2

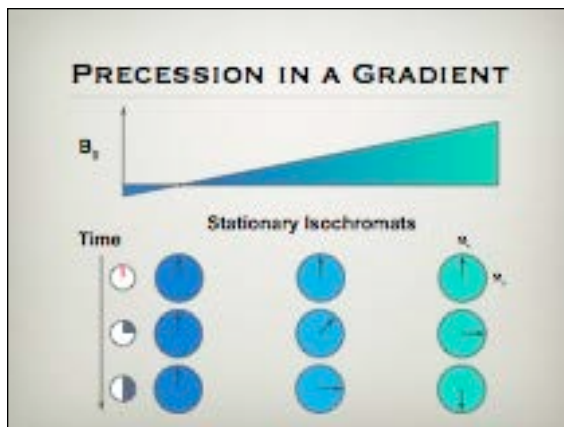
MOTION IN A FIELD GRADIENT

3

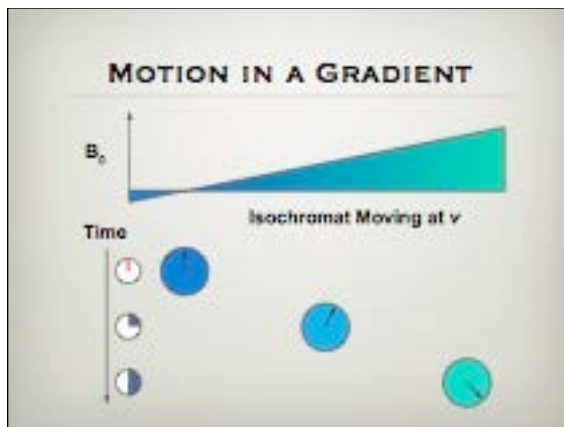
COHERENT MOTION

- Non-random bulk motion of a liquid, elastic, plastic or solid material.
- Time- and length-scale dependent
- Coherent Motion
 - Laminar blood flow
 - CSF flow
 - Myocardial motion
- Incoherent Motion
 - Molecular self-diffusion
 - Capillary flow on a large length scale (Pseudo-diffusion)

4



5



6

HIGHER ORDER MOTION

Maclaurin expansion of position as a function of time

$$x(t) = x(0) + x'(0)t + \frac{1}{2!}x''(0)t^2 + \dots + \frac{1}{n!}x^{(n)}(0)t^n + \dots$$

$x'(0)$	$=$	$v(0)$	Initial Velocity
$x''(0)$	$=$	$a(0)$	Initial Acceleration
$\vdots$			Initial Jerk ...

7

GRADIENT MOMENT EXPANSION

Substitute into 1D phase accumulation equation

$$\begin{aligned} \phi(T) &= \gamma \int_0^T dt G(t) x(t) \\ &= \gamma \left[x(0) \int_0^T dt G(t) + x'(0) \int_0^T dt G(t) t + \dots \right] \\ &= \gamma [x(0)m_0(T) + v(0)m_1(T) + a(0)m_2(T) + \dots] \end{aligned}$$

8

PHASE AND MOTION ORDER

Phase contribution due to
a given order of motion

n^{th} gradient
moment

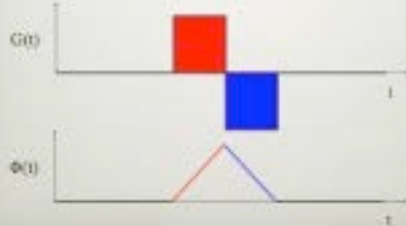
$$\phi_n(T) = \gamma \underbrace{x^{(n)}(0)}_{n^{\text{th}} \text{ time-derivative of displacement}} m_n(T)$$

n^{th} time-derivative
of displacement

9

ZEROth MOMENT

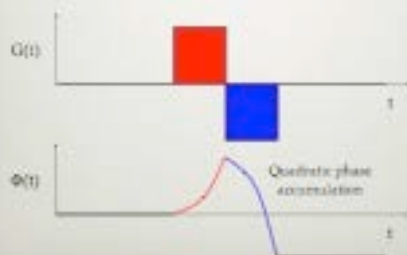
- Area of gradient in pulse sequence diagram.
- Phase evolution of stationary spin.



10

FIRST MOMENT

- Phase evolution of spin with constant velocity.



11

GRADIENT ECHO MOMENTS

Basic gradient echo waveform



What are the zeroth and first gradient moments at the time of the echo?

12

GRADIENT ECHO MOMENTS



0th Moment $m_0(TE) = \int_0^{TE} dt G(t) = 0$

1st Moment $m_1(TE) = \int_0^{TE} dt G(t)t = G\tau^2$

13



FLOW COMPENSATION

14

FLOW COMPENSATION

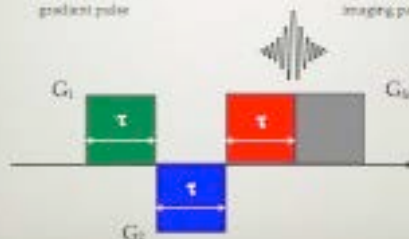
- Can we design a gradient waveform where both zeroth and first order moments are zero at the echo time?
- If this is possible, then any constant motion would have no effect on the phase of the echo.
- Need one more degree of freedom.

15

MOTION COMPENSATING GRADIENT WAVEFORM

Additional degree of freedom from new gradient pulse

Frequency encoding gradient fixed by imaging parameters



16

MOMENT EQUATIONS

Gradient echo with equal duration gradient pulses

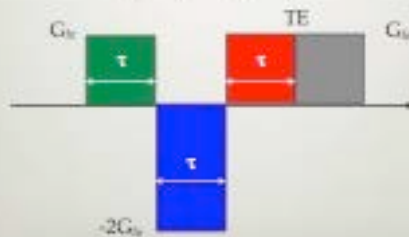
$$m_0(TE) = \tau(G_1 + G_2 + G_{fe})$$

$$m_1(TE) = \frac{1}{2}\tau^2(G_1 + 3G_2 + 5G_{fe})$$

17

MOTION COMPENSATION SOLUTION

$$m_0(TE) = m_1(TE) = 0$$



18

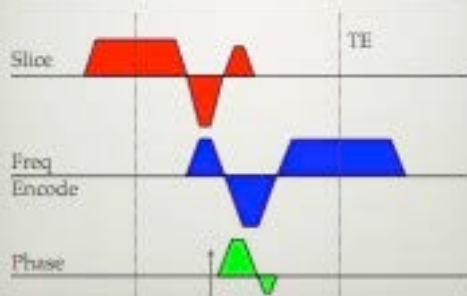
EVEN ECHO FLOW COMPENSATION

First moment nulled intrinsically for even echos in both gradient and spin echo trains



19

FLOW COMPENSATED SLICE SELECTION AND PHASE ENCODING



20



FLOW ENCODING

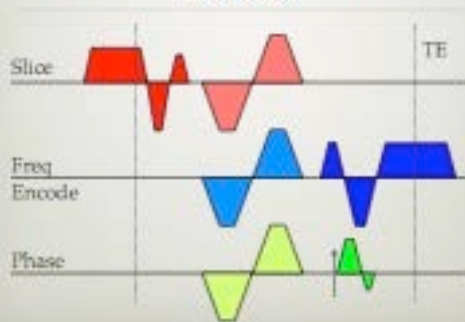
21

PHASE ENCODING OF FLOW

- Can a gradient waveform be designed that gives the echo a phase proportional to velocity in a given direction?
- Requires that zeroth moment is zero and first moment is finite at the echo time.
- More convenient still to have independent control over first moment.

22

BIOPOLAR FLOW ENCODING PULSES



23

BACKGROUND PHASE

- The voxel phase without velocity encoding cannot be assumed to be zero.
- Many contributions to background phase:
 - B_0 inhomogeneity
 - Gradient eddy currents
 - Other orders of motion
 - Deliberate offsetting of echo position
 - —

24

REMOVING BACKGROUND PHASE

General velocity encoded magnetization

$$\hat{\rho} = \rho_0 e^{i(\phi_b + \phi_v)}$$

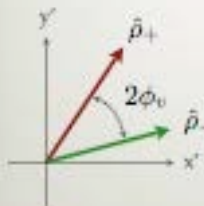
Acquire two images with opposite sign bipolar gradient pulses

$$\hat{\rho}_{\pm} = \rho_0 e^{i(\phi_b \pm \phi_v)}$$

$$\phi_v = \frac{1}{2}(\angle \hat{\rho}_+ - \angle \hat{\rho}_-)$$

25

PHASE DIFFERENCE



Difference in phase of transverse magnetization between positive and negative velocity encoded acquisitions.

26

PRINCIPLE OF PHASE CONTRAST

Background phase arises from B_0 inhomogeneities, echo timing offsets, eddy currents, Maxwell terms, ...



Velocity encoded through plane

Velocity compensated reference

Corrected phase velocity image

27

ENCODING SCHEMES

6 Point Scheme



	1	2	3	4	5	6
1	1	0	0	0	0	0
2	0	1	0	0	0	0
3	0	0	1	0	0	0
4	0	0	0	1	0	0
5	0	0	0	0	1	0
6	0	0	0	0	0	1

4 Point Scheme



	1	2	3	4
1	1	0	0	0
2	0	1	0	0
3	0	0	1	0
4	0	0	0	1

Scale gradients by $1/\sqrt{3}$

28

RECOVERING VELOCITY FROM 4-POINT ENCODING

Use linear combinations of the voxel phase from each acquisition. For example, for the phase encoded z -component of velocity:

$$\phi_z = \frac{1}{2} [(\phi_3 + \phi_4) - (\phi_1 + \phi_2)]$$

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ALIASING VELOCITY : V_{enc}

Phase is modulo 2π so we expect to see phase aliasing if the velocity exceeds a certain value.
This critical velocity is called the encoding velocity or V_{enc}

$$V_{enc} = \frac{\pi}{\gamma m_1 (TE)}$$

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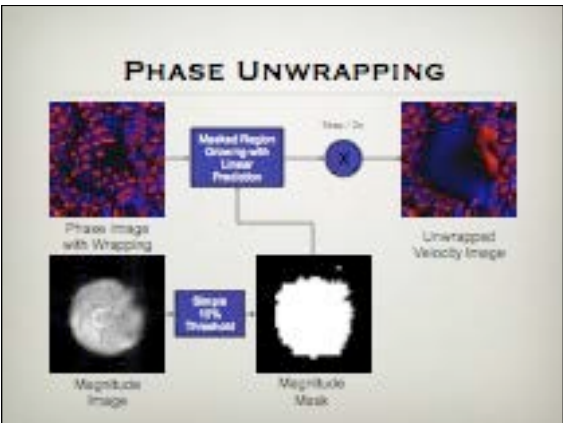
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PHASE UNWRAPPING

```
graph LR; A[Phase image with Wrapping] --> B[Masked Region Growing with Linear Prediction]; A --> C[Magnitude Image]; C --> D[Simple 10% Threshold]; D --> E[Magnitude Mask]; B --> F((x)); E --> F; F --> G[Unwrapped Velocity Image]
```

The diagram illustrates the Phase Unwrapping process. It starts with a 'Phase image with Wrapping' (a colorful, noisy image) and a 'Magnitude Image' (a grayscale image of a circular object). The 'Phase image with Wrapping' is processed by a 'Masked Region Growing with Linear Prediction' block. The 'Magnitude Image' is processed by a 'Simple 10% Threshold' block, resulting in a 'Magnitude Mask' (a binary image of the object). Both the output of the 'Masked Region Growing' block and the 'Magnitude Mask' are combined (indicated by a circle with an 'x') to produce the final 'Unwrapped Velocity Image' (a colorful, smooth image of the object).



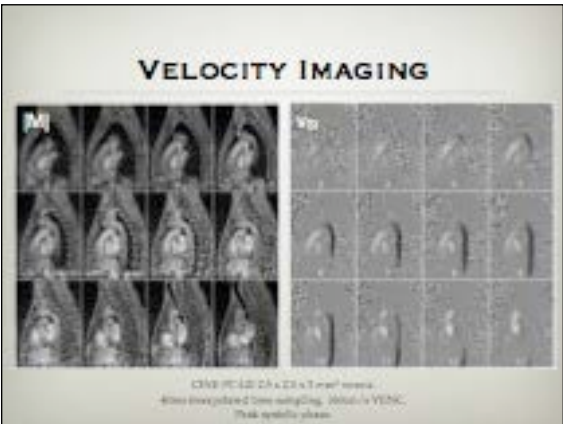
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VELOCITY IMAGING

CINE PC-MRI 24 x 24 x 7 mm³ voxels,
40ms every phase 2 lines using Cine, 30000 x 1 VENC,
100% systolic phase

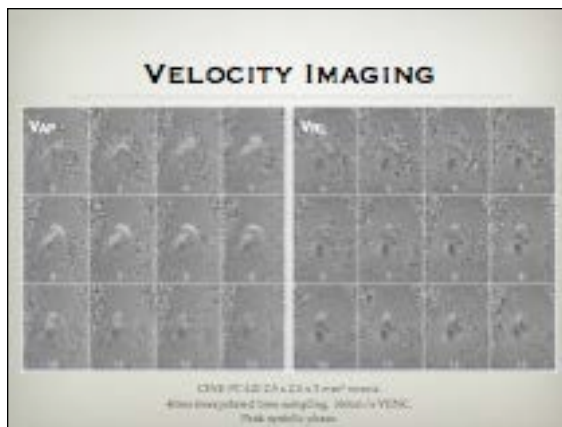


VELOCITY IMAGING

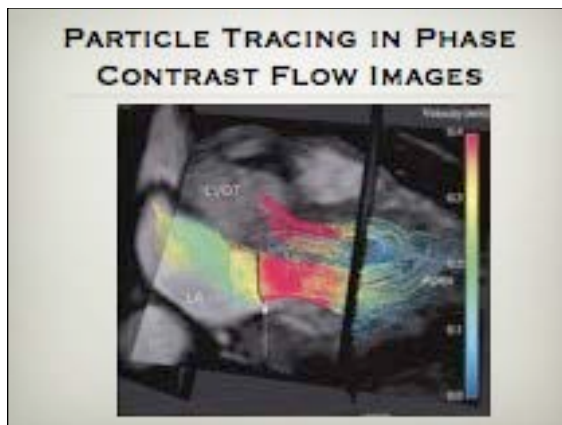
CINE PC-MRI 24 x 24 x 7 mm³ voxels,
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VELOCITY IMAGING

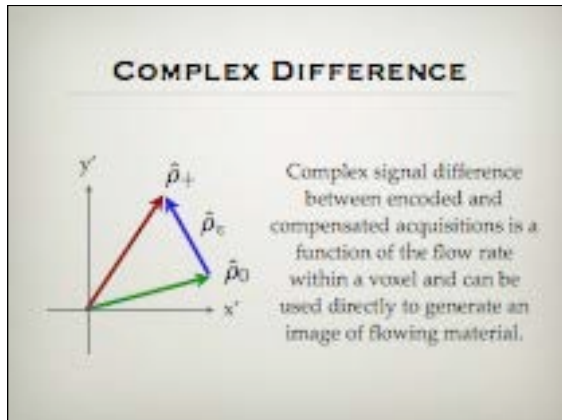
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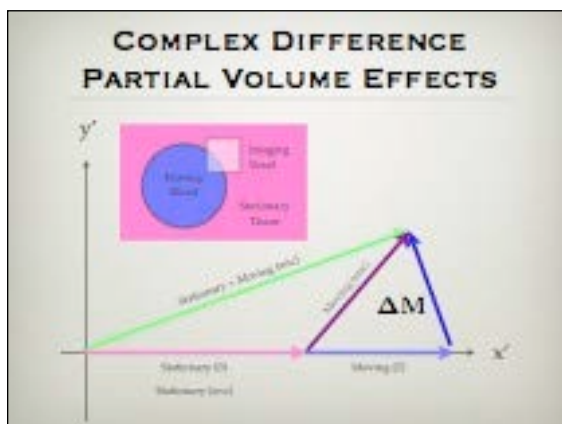
33



34

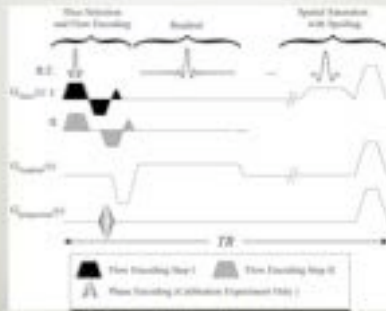


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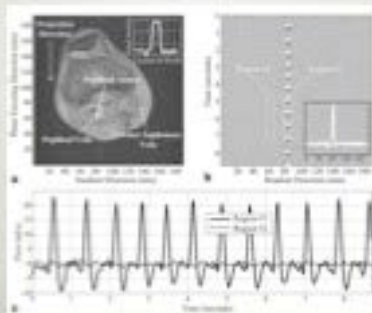
36

COMPLEX DIFFERENCE EXAMPLE



37

COMPLEX DIFFERENCE EXAMPLE



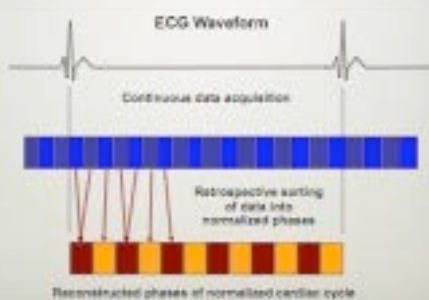
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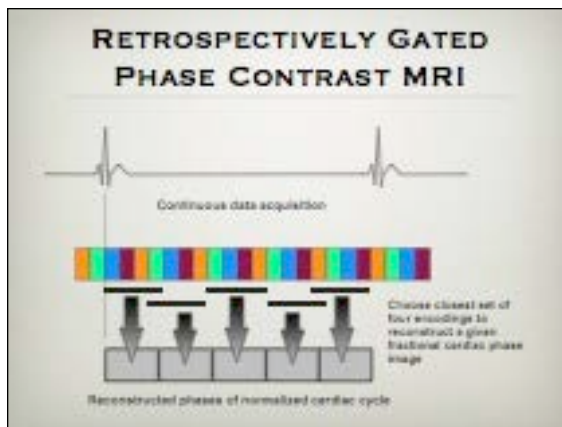
TIME-RESOLVED FLOW MEASUREMENT

39

RETROSPECTIVE GATING



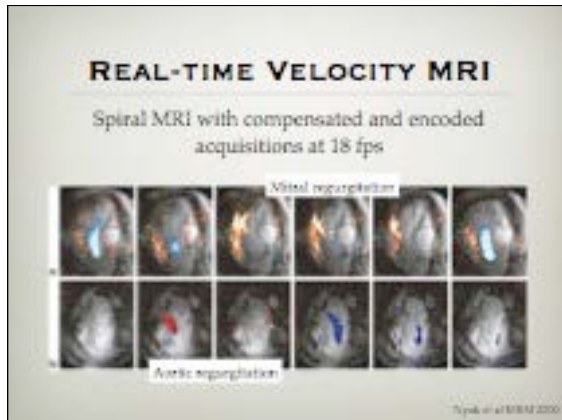
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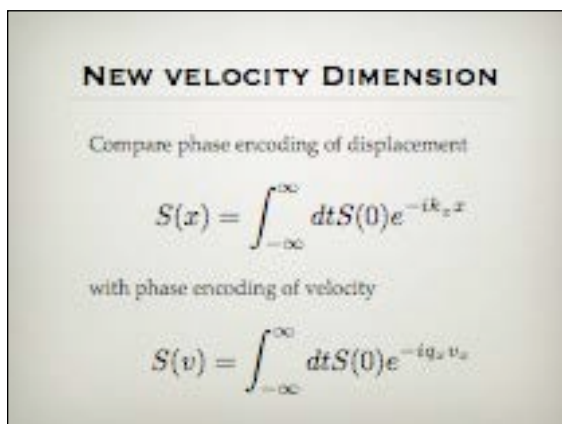
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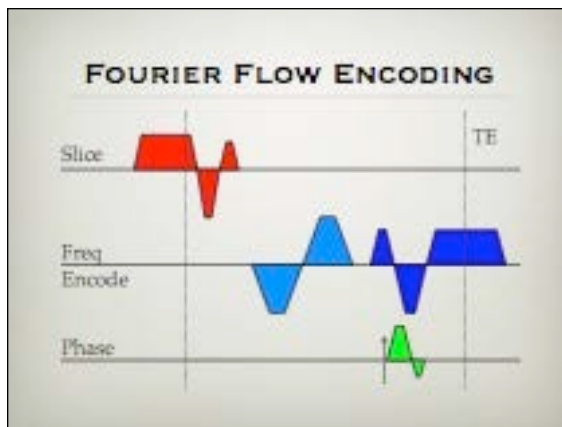
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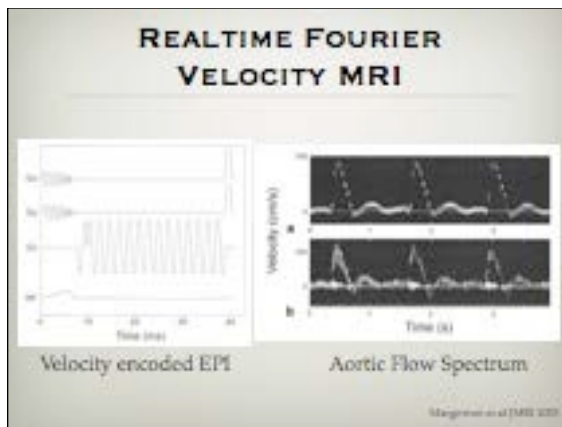
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44



45



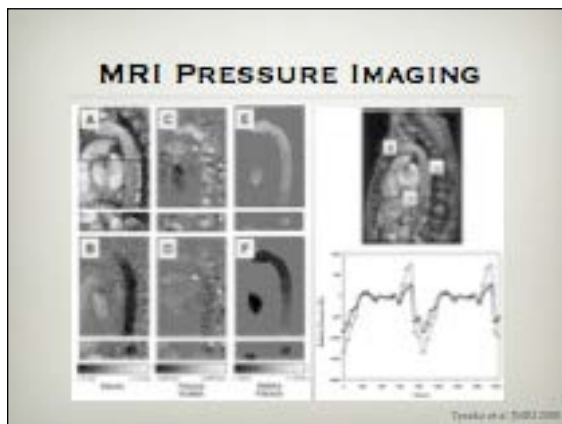
46

PRESSURE CALCULATION

- The Navier-Stokes Equation relates velocity and relative pressure.
- Flow is viscous, time-varying and rotational.

$$-\nabla p = \rho \frac{\partial \mathbf{u}}{\partial t} + \rho \mathbf{u} \times \nabla \mathbf{u} + \eta \nabla \times \nabla \times \mathbf{u}$$

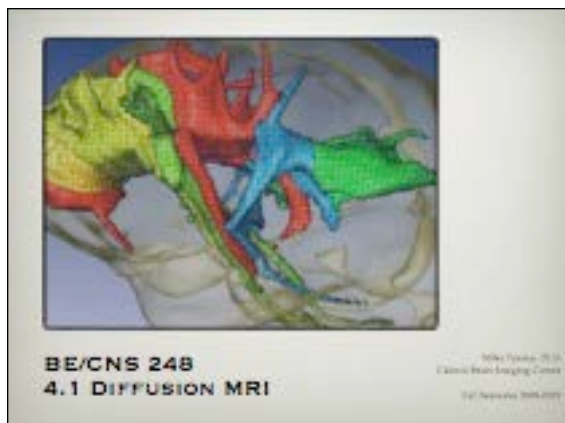
47



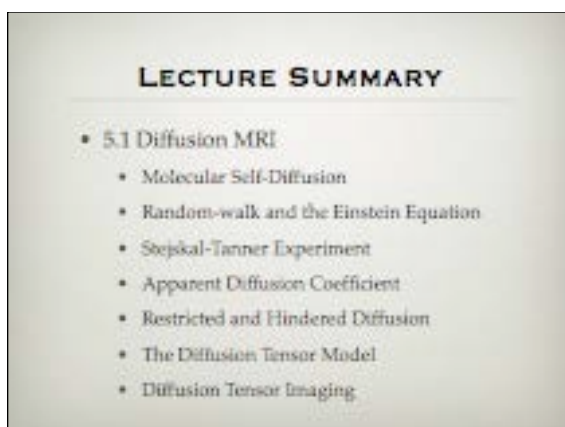
48

NEXT LECTURE

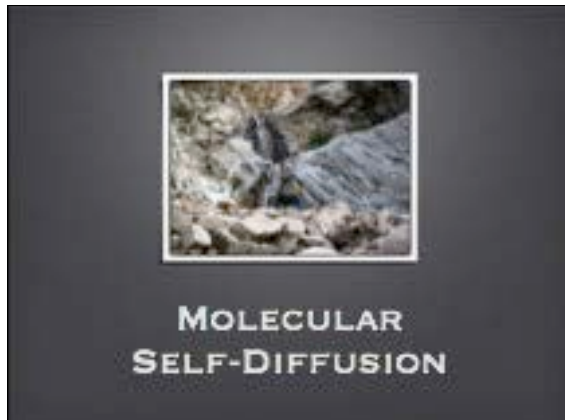
- Diffusion MRI



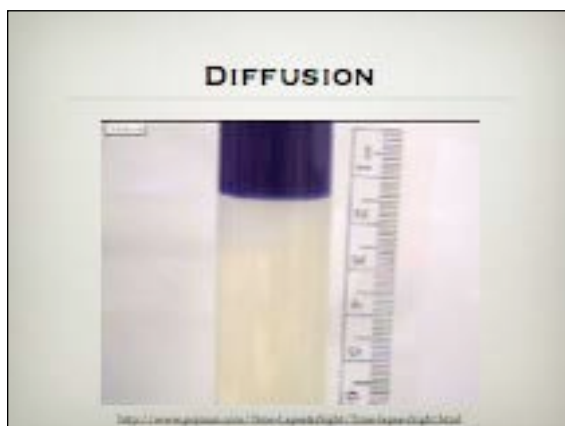
1



2



3



4

EINSTEIN DIFFUSION EQUATION

1D Diffusion

$$\frac{\partial C}{\partial t} = D \frac{\partial^2 C}{\partial x^2}$$

Diffusion Coefficient $\downarrow$ Concentration

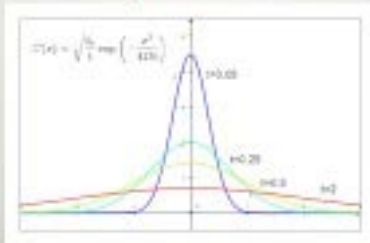
3D Diffusion

$$\frac{\partial C}{\partial t} = D \nabla^2 C$$

5

DIFFUSION EQUATION SOLUTIONS

The 1D spatial concentration distribution is a time-evolving normalized Gaussian



6

MEAN SQUARE DISPLACEMENT

RMS displacement of a molecule

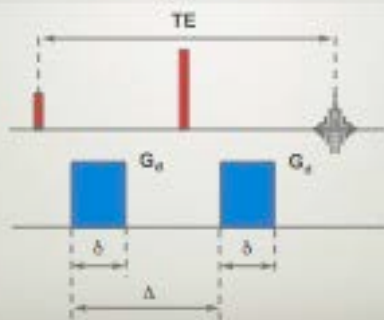
$$\langle x \rangle = 0$$

1D Diffusion $\langle x^2 \rangle = 2Dt$

3D Diffusion $\langle x^2 \rangle = 6Dt$

7

STEJSKAL-TANNER (PGSE) EXPERIMENT



8

SIGNAL ATTENUATION BY DIFFUSION

Incoherent molecular self-diffusion in a gradient causes dephasing of transverse magnetization

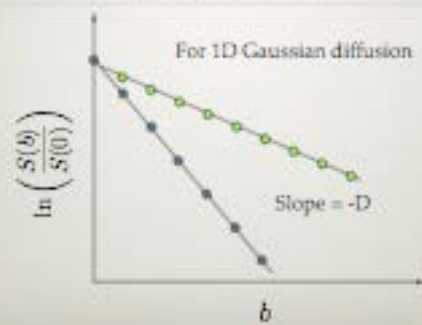
1D Gaussian Diffusion

$$S(b) = S(0)e^{-bD}$$

Where b is a measure of the diffusion weighting generated by the balanced gradient pulses

9

IDEAL DIFFUSION ATTENUATION



10

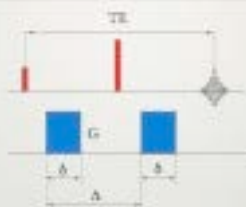
THE B-FACTOR

The b-factor can be calculated from the k-space trajectory:

$$b_x(T) = \int_0^T dt |\gamma k_x(t)|^2$$

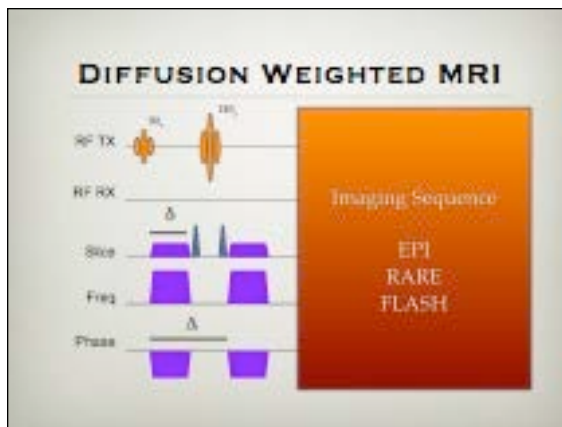
11

B-FACTOR FOR STEJSKAL-TANNER

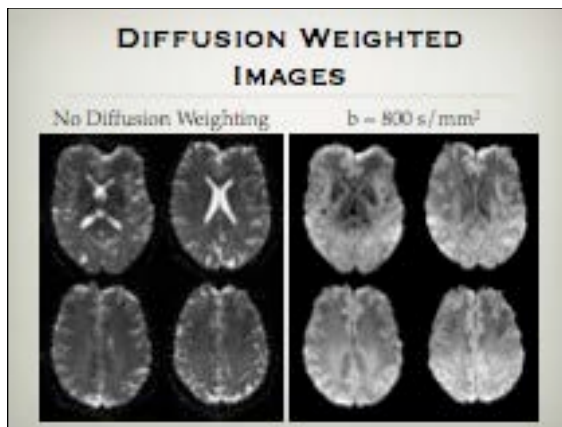


$$b(TE) = (\gamma G \delta)^2 \left(\Delta - \frac{\delta}{3} \right)$$

12



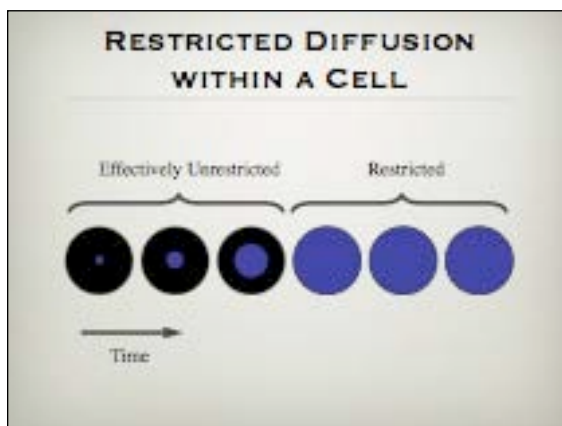
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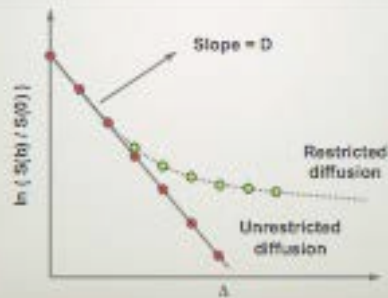


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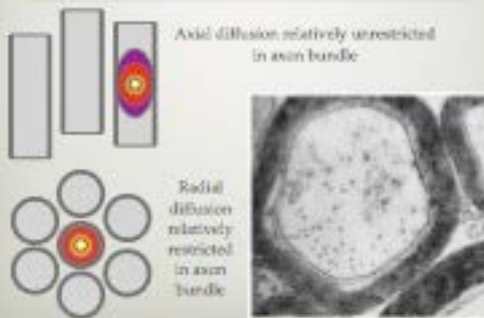
16

UNRESTRICTED AND RESTRICTED DIFFUSION



17

ANISOTROPIC RESTRICTED DIFFUSION



18

FICK'S EQUATION AND THE DIFFUSION TENSOR

Fick's Equation relates diffusive flux (J) to a concentration gradient using the Diffusion Tensor (D)

$$J_i = D_{ij} \frac{\partial c}{\partial x_j}$$

19

WHAT IS A TENSOR?

In mathematics, a **tensor** is (in an informal sense) a generalized linear quantity or geometrical entity that can be expressed as a multi-dimensional array relative to a choice of basis; however, as an object in and of itself, a tensor is independent of any chosen frame of reference. The rank of a particular tensor is the number of array indices required to describe such a quantity. For example, mass, temperature, and other scalar quantities are tensors of rank 0; force, momentum and other vector-like quantities are tensors of rank 1; a linear transformation such as an anisotropic relationship between force and acceleration vectors is a tensor of rank 2.

Wikipedia 2006

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PROPERTIES OF THE DIFFUSION TENSOR

D is an upper triangle
symmetric, positive
definite second rank
tensor $\Rightarrow$ six unknowns

$$D = \begin{pmatrix} D_{11} & D_{12} & D_{13} \\ D_{12} & D_{22} & D_{23} \\ D_{13} & D_{23} & D_{33} \end{pmatrix}$$

D has three orthogonal
positive eigenvectors with
corresponding
eigenvalues

$$D\epsilon_i = \lambda_i \epsilon_i \quad i = 1, 2, 3$$

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DIFFUSION WEIGHTED IMAGES FOR DTI

- Diffusion tensor has six unknowns
- At least six diffusion weighted images
- Diffusion gradient directions sample directional space
- Encoding directions cannot be co-linear
- Additional image with minimal b-value used to eliminate all other weightings (T1, T2, ...)

22

THE B-MATRIX

More generally, for DTI we need to calculate the b-matrix which includes cross terms between the gradient axes.

$$b_{ij}(T) = \gamma^2 \int_0^T dt k_i(t) k_j(t)$$

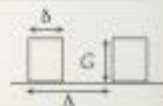
$$S(b_{ij}) = S(0) \exp \left(- \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} D_{ij} \right)$$

23

ANALYTICAL B-MATRICES

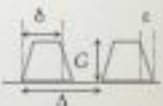
Ideal rectangular pulses

$$b_{ij} = \gamma^2 G_i G_j \delta^2 \left(\Delta - \frac{1}{3} \delta \right)$$



Trapezoidal pulses

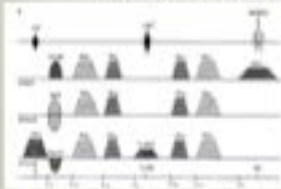
$$b_{ij} = \gamma^2 G_i G_j \left[\delta^2 \left(\Delta - \frac{1}{3} \delta \right) + \frac{1}{30} \epsilon^3 - \frac{1}{6} \delta \epsilon^2 \right]$$



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ANALYTICAL B-MATRIX

Analytical expressions for real pulse sequences are complicated but allow inversion to determine required gradient values for a desired b-matrix.



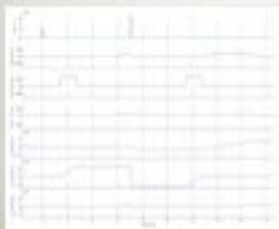
$$b_{ij}(T) = \gamma^2 \int_0^T dt k_i(t) k_j(t)$$

See Mattiello J et al J Magn Reson A 1994;109:131-141

25

NUMERICAL B-MATRIX

$$b_{ij}(T) = \gamma^2 \int_0^T dt k_i(t) k_j(t)$$



Numerical simulation of the b-matrix is easier to implement for different sequences but does not allow direct inversion.

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DIRECTIONAL SAMPLING SCHEMES FOR DTI



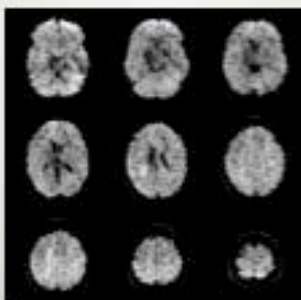
Optimal Icosahedral Sampling:

- (0.0000, 0.0000, 0.0000)
- (0.8507, 0.5257, 0.0000)
- (0.8507, -0.5257, 0.0000)
- (0.0000, 0.8507, 0.5257)
- (0.0000, 0.8507, -0.5257)
- (0.5257, 0.0000, 0.8507)
- (-0.5257, 0.0000, 0.8507)

Example icosahedral directional sampling pattern for DTI. Only half (6) of the vertices are used.

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6-DIRECTION DWI (ICOSAHEDRAL ENCODING)



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FROM DWIS TO DTI

Reference image (no DW) $S(0)$
 Diffusion weighted images $S(b_{ij})$

$$\log \left(\frac{S(b_{ij})}{S(0)} \right) = \sum_{i=1}^3 \sum_{j=1}^3 b_{ij} D_{ij}$$

29

SOLVING FOR THE TENSOR

Recast as a matrix equation

$$\mathbf{s} = \mathbf{B}\mathbf{d}$$

$$\begin{pmatrix} \log S(b_1) \\ \log S(b_2) \\ \vdots \\ \log S(b_n) \end{pmatrix} = \begin{pmatrix} b_{1,1} & b_{1,2} & b_{1,3} & b_{2,1} & b_{2,2} & b_{2,3} & 1 \\ b_{2,1} & b_{2,2} & b_{2,3} & b_{3,1} & b_{3,2} & b_{3,3} & 1 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ b_{n,1} & b_{n,2} & b_{n,3} & b_{n,1} & b_{n,2} & b_{n,3} & 1 \end{pmatrix} \begin{pmatrix} D_{11} \\ D_{12} \\ D_{13} \\ D_{22} \\ D_{23} \\ D_{33} \\ -\log S(0) \end{pmatrix}$$

Solve using the Moore-Penrose pseudo inverse

$$\mathbf{d} = \mathbf{B}^+ \mathbf{s}$$

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DT MODEL OF RESTRICTED, ANISOTROPIC DIFFUSION



Actual Diffusion
Restriction



Tensor Model

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DIFFUSION TENSOR ELLIPSOIDS



Orientation and length of ellipsoid axes correspond to eigenvectors and eigenvalues of the diffusion tensor.

32

DIFFUSION TENSOR ROTATIONAL INVARIATES

Independent of the measurement frame of reference

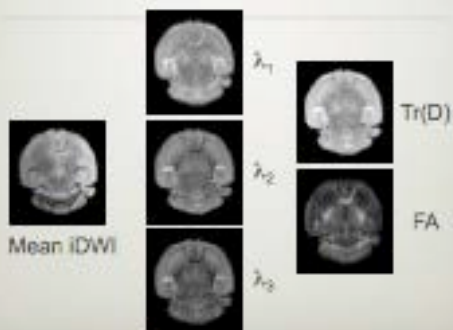
Eigenvalues
Eigenvectors $D\mathbf{x}_i = \lambda_i\mathbf{x}_i$

Trace[D] $\text{Tr}[\mathbf{D}] = \sum_{i=1}^3 \lambda_i$

Fractional
Anisotropy $FA = \frac{\sqrt{3}}{2} \sqrt{\frac{(\lambda_1 - \lambda_2)^2 + (\lambda_2 - \lambda_3)^2 + (\lambda_1 - \lambda_3)^2}{\lambda_1^2 + \lambda_2^2 + \lambda_3^2}}$

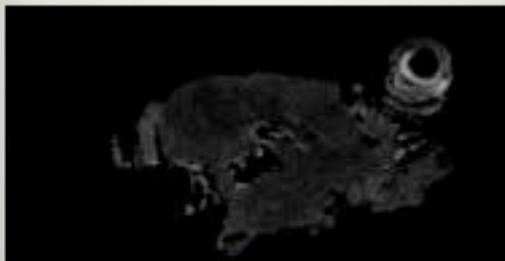
33

DIFFUSION TENSOR INVARIANTS



34

FRACTIONAL ANISOTROPY OF THE MOUSE BRAIN

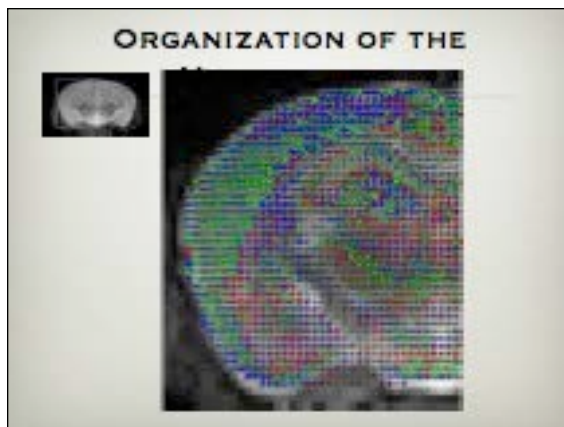


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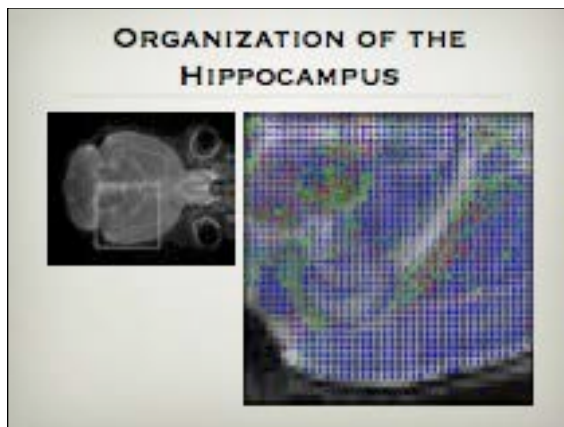
THREE-DIMENSIONAL FRACTIONAL ANISOTROPY



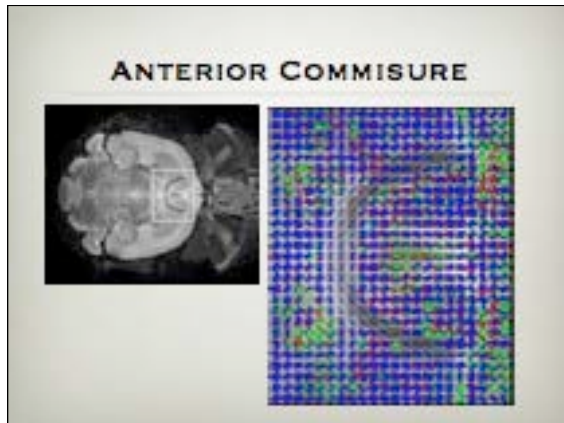
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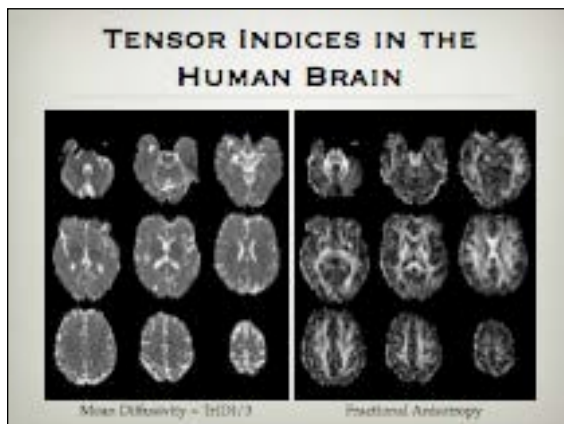
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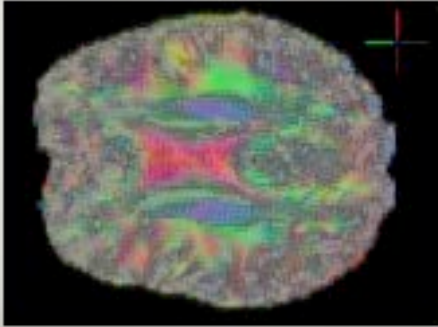


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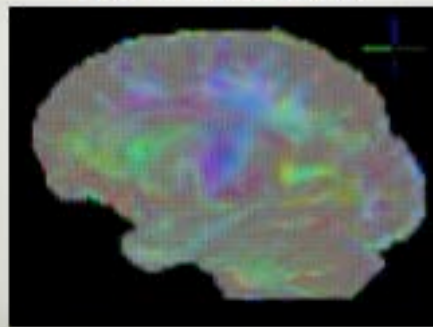
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**TENSOR VISUALIZATION IN
THE HUMAN BRAIN**



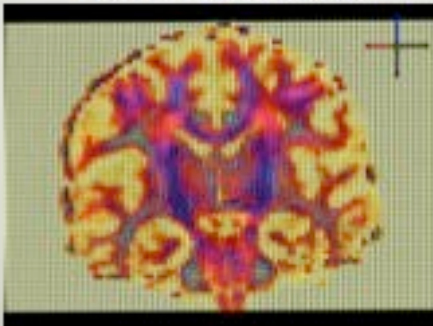
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**TENSOR VISUALIZATION IN
THE HUMAN BRAIN**



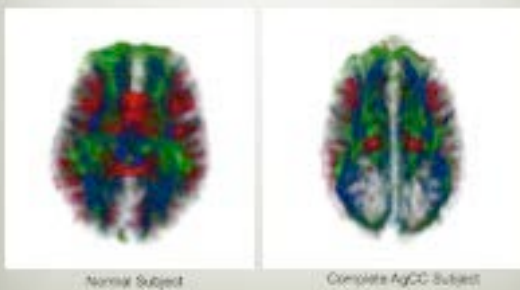
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**TENSOR VISUALIZATION IN
THE HUMAN BRAIN**



43

**DIRECTIONAL CODING
RENDERING IN AGCC**

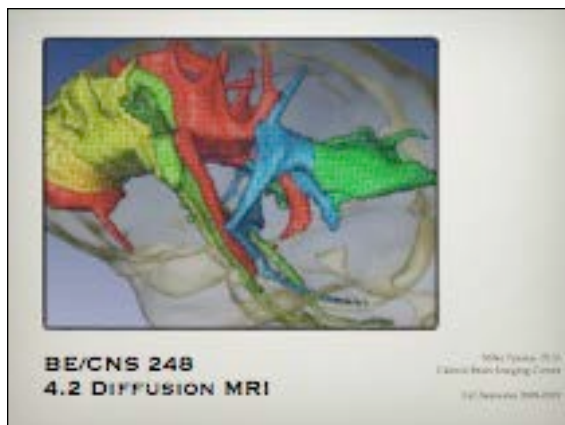


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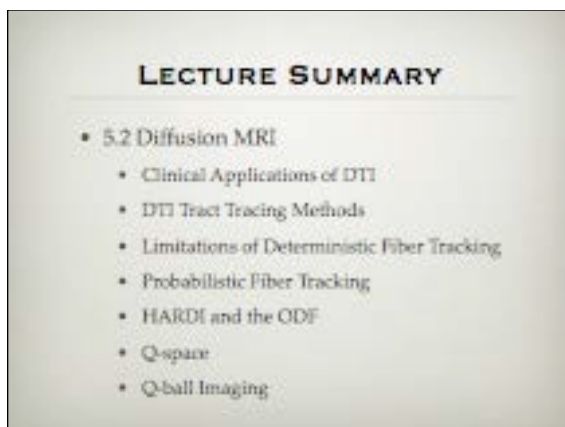
NEXT LECTURE

- Diffusion MRI
- Diffusion Tractography

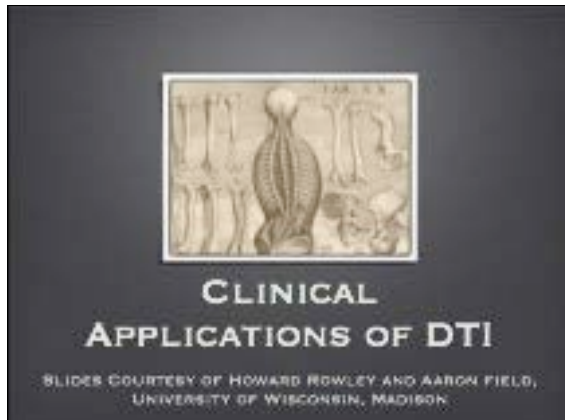
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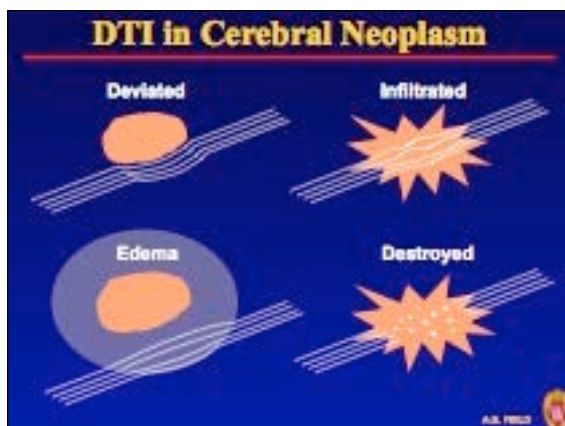
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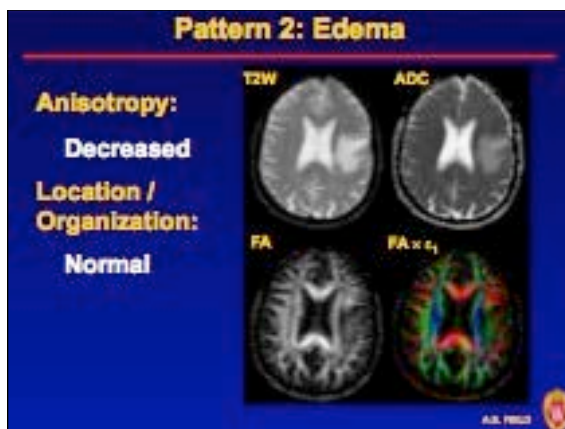
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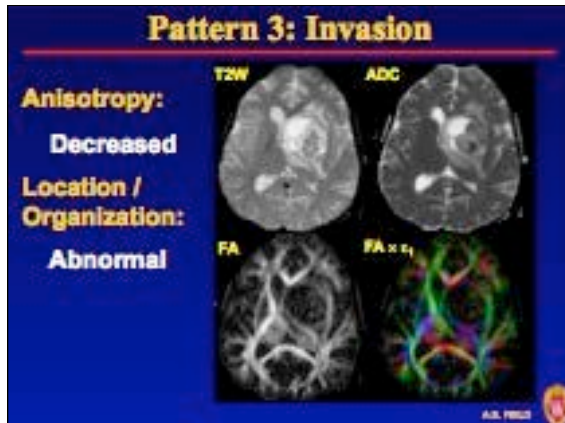
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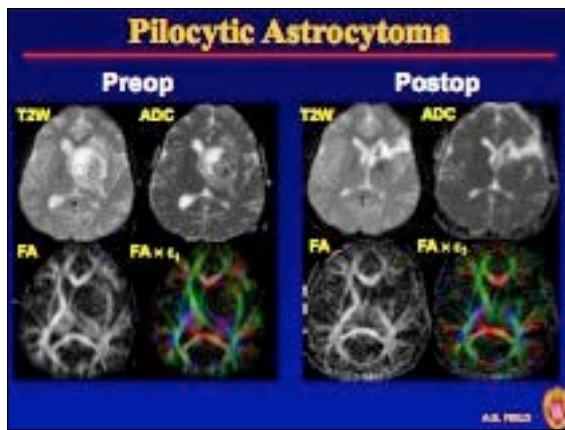
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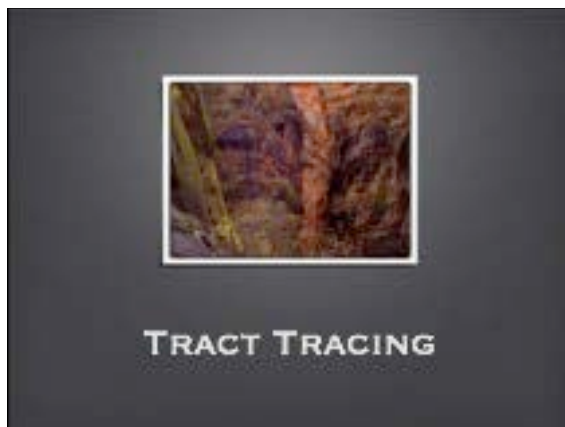
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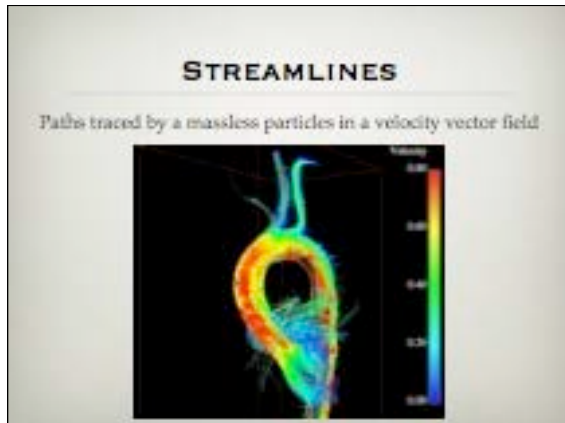
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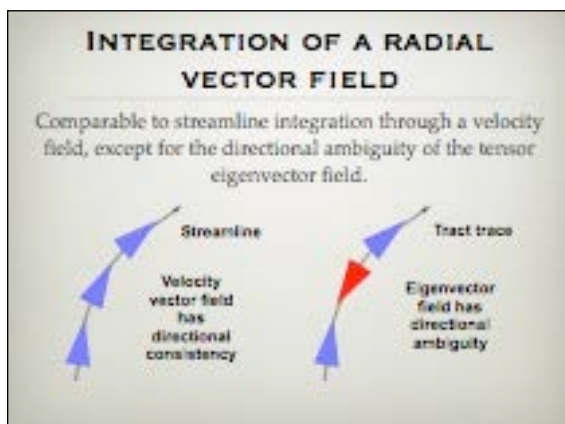
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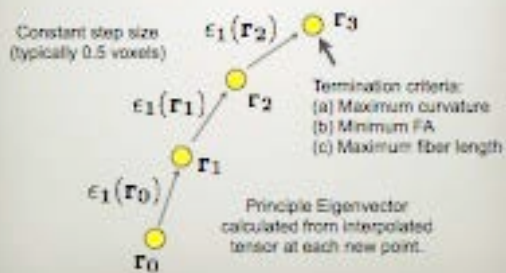
12

FIBER TRACKING ALGORITHMS

- Wide variety of deterministic fiber tracking algorithms
- Most based on finite difference integration of the principle eigenvector or equivalent estimate of maximum diffusion direction
- Not specific to axons - can be performed in any fibrous tissue, including skeletal muscle and myocardium.

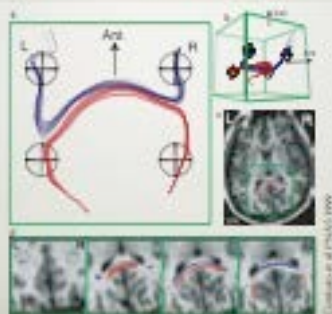
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STREAMLINE TRACT TRACING (STT)



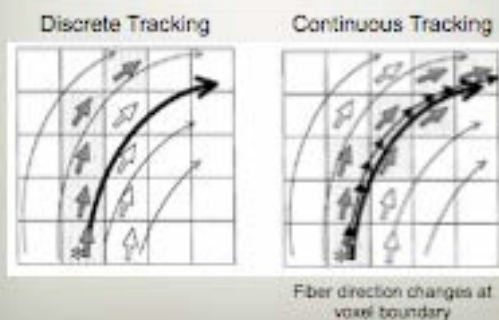
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STREAMLINE TRACT TRACING (STT)



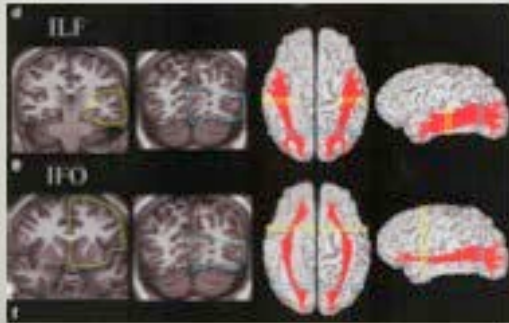
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FIBER ASSIGNMENT BY CONTINUOUS TRACKING (FACT)



16

FIBER ASSIGNMENT BY CONTINUOUS TRACKING (FACT)



17

TENSOR DEFLECTION

Addresses propagation difficulties in areas of low fractional anisotropy.



Tensor deflects fiber vector towards principle eigenvector.

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TENSORLINE ALGORITHM

Weighted combination of STT and Tensor Deflection proposed by Weinstein et al

$$\mathbf{v}_{out} = (f\mathbf{e}_1) + (1-f)((1-g)\mathbf{v}_{in} + g\mathbf{D}\mathbf{v}_{in})$$

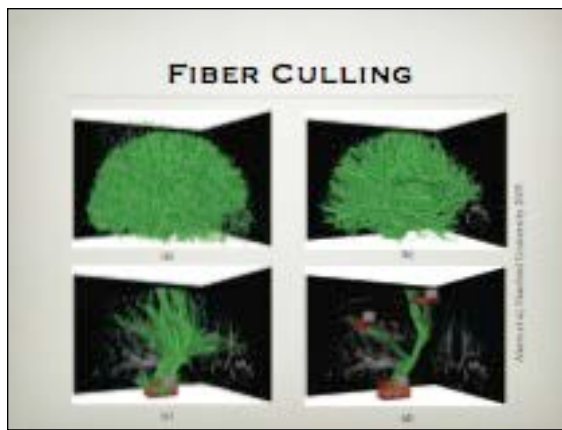
f and g are user defined in range 0..1

19

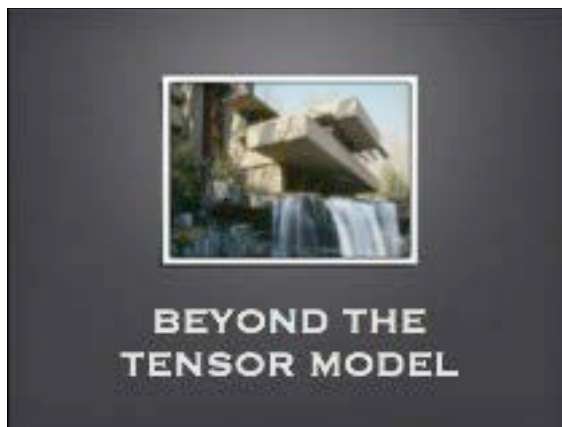
TENSORLINE VISUALIZATION



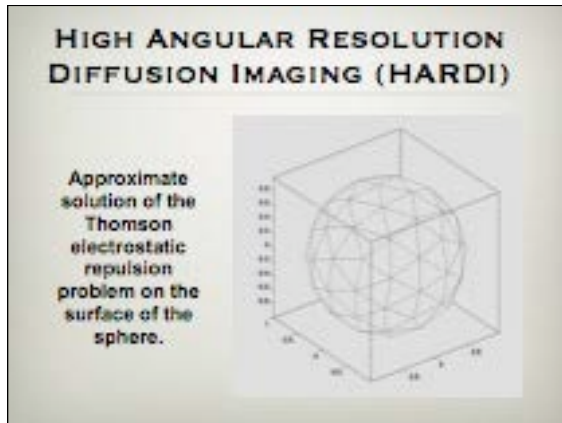
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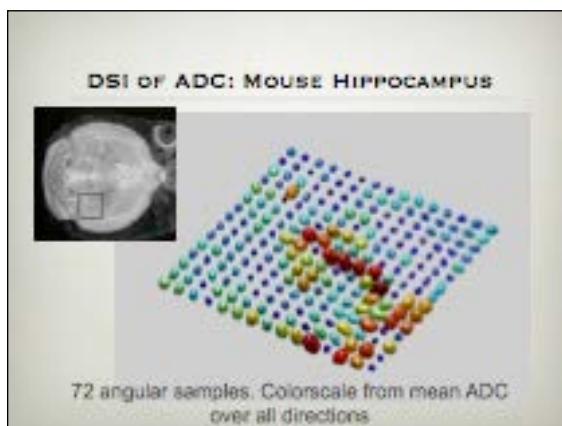
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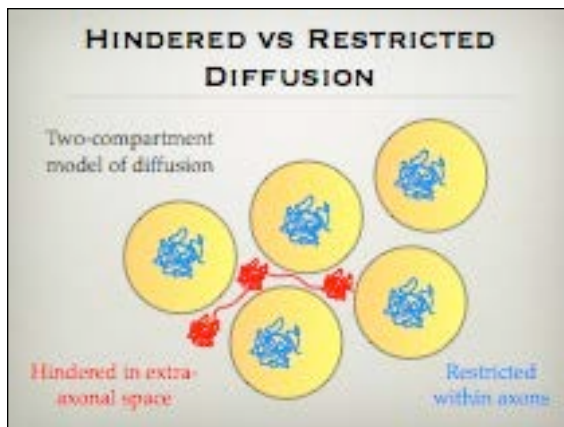
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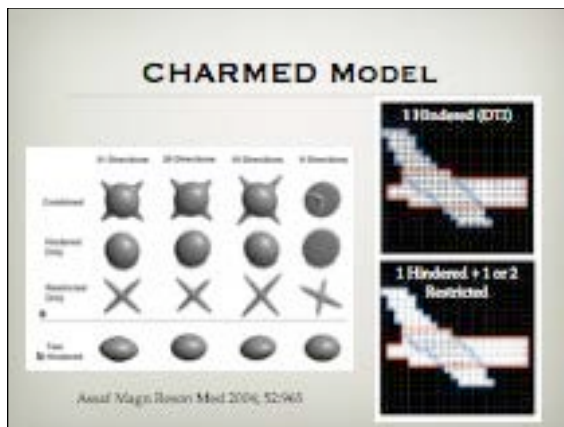
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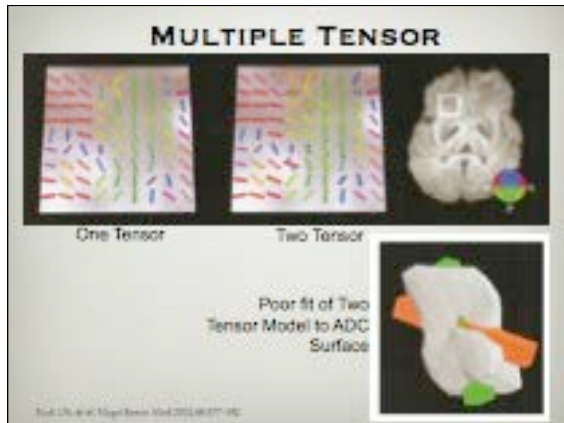
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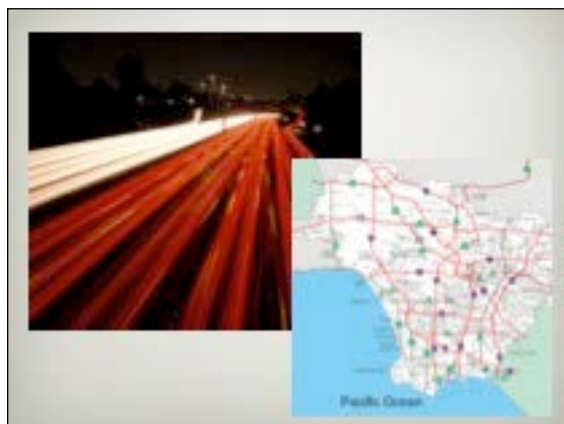
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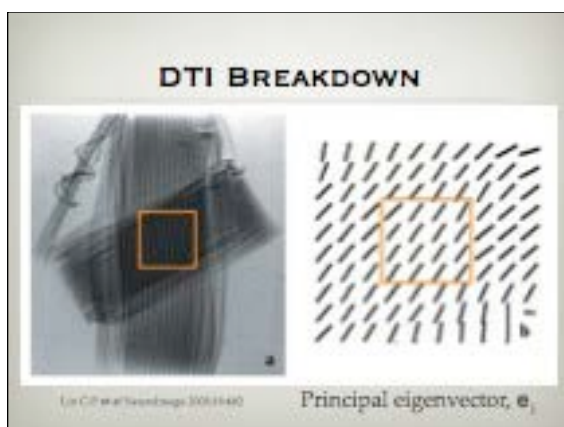
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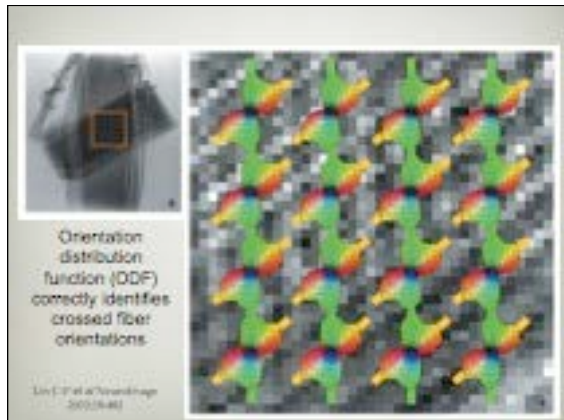
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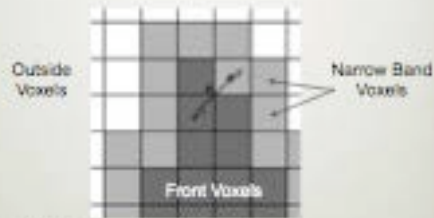


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FAST MARCHING TRACTOGRAPHY

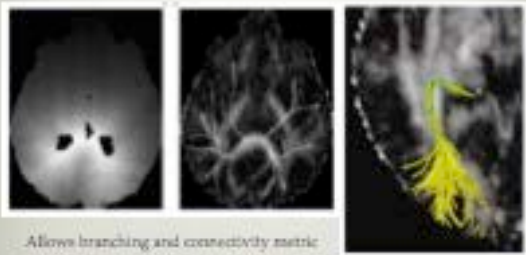


Transition Velocity

$$v(r) = \frac{1}{1 - \min(|e_1(r) \cdot n(r)|, |e_1(r) \cdot e_1(r')|)}$$

33

FAST MARCHING TRACTOGRAPHY

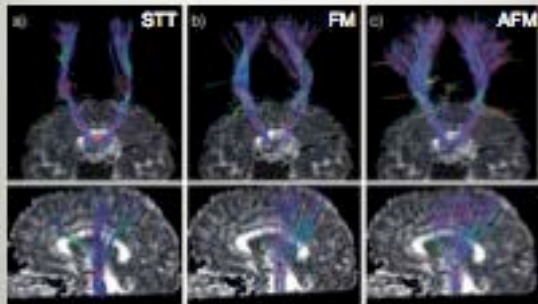


Allows branching and connectivity metric

Chen et al. ISMRM, 2002, 10:1505

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FAST MARCHING TRACTOGRAPHY



35

BEHRENS MODEL OF ANISOTROPIC TISSUE DIFFUSION

Normalized MR intensity for i th encoding

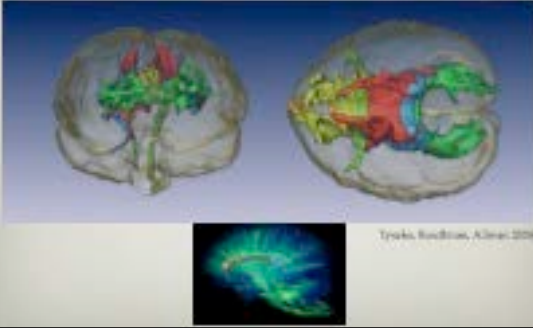
$$\frac{S_i(n, B)}{S_0} = (1 - f) \exp(-b_0 b) + f \int_{-1}^1 \int_{-\pi}^{\pi} \exp(-b_0 \cos^2 \theta) \exp(-b_0 \cos^2 \theta) \exp(-b_0 \cos^2 \theta) d\theta d\phi$$

Fiber orientation distribution

- Partial volume effects modeled as mixture of isotropic diffusion and a distribution of fiber orientations.
- Monte-Carlo Markoff Chain PFT with partial volume effects
- Computationally very intensive.
- See Behrens et al Magn Reson Med 2003.

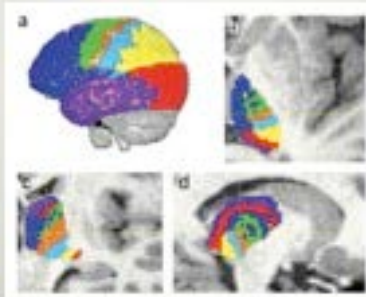
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GORILLA PFT: CALLOSUM TO CORTEX



37

SEGMENTATION BY CORTICAL CONNECTIVITY



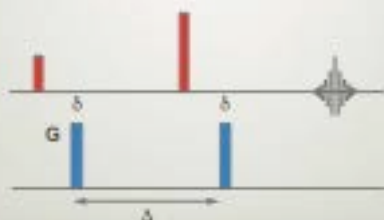
38

Q-SPACE

39

Q-SPACE

Limit of the Stejskal-Tanner experiment when
 $\delta \ll 1, \Delta$



40

Q: THE DIFFUSION WAVENUMBER

For the Stejskal-Tanner, aka the Pulsed Field Gradient (PFG), sequence we define a new inverse space coordinate:

$$q(T) = \frac{\gamma}{2\pi} \int_0^T dt G(t) = \frac{1}{2\pi} \gamma G \delta$$

Note that q is a function of G and δ only. Displacement during the gradient pulse is assumed to be negligible.

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DISPLACEMENT PROBABILITY AND Q-SPACE

We can relate the normalized attenuation to the conditional probability density of a molecule moving from one position to another during Δ

$$E(q, \Delta) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} P(r_2, \Delta | r_1, 0) P(r_1) e^{iq(r_2 - r_1)} d^3 r_2 d^3 r_1$$

$$E(q, \Delta) = \frac{S(q, \Delta)}{S(0, \Delta)}$$



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DISPLACEMENT PROPAGATOR

If we assume local homogeneity, then the probability density function depends only on the relative displacement, r

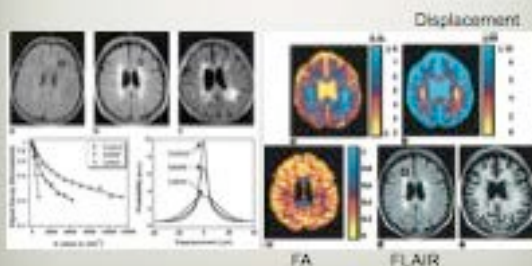
$$P(r, \Delta | 0, 0) = P(r_2, \Delta | r_1, 0)$$

Easy to show that this PDF can be calculated by inverse FT of the normalized attenuation

$$P(r, \Delta | 0, 0) = \int_{-\infty}^{\infty} d^3 q E(q, \Delta) e^{-iq \cdot r}$$

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Q-SPACE PROPAGATORS IN PRACTICE



44

PROPERTIES OF Q-SPACE PROPAGATOR

- No diffusion model assumed.
- Measures probability density of a molecule displacing by a given amount in a given time.
- Time-consuming to acquire data in more than one q-space dimension

45

Q-BALL IMAGING

46

ORIENTATIONAL DISTRIBUTION FUNCTION

Radial projection of the 3D displacement probability function

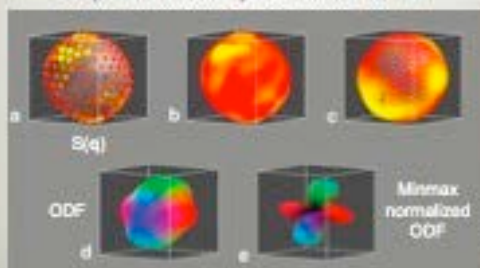


$$\psi(\mathbf{u}) = \int_0^\infty P(r\mathbf{u}) dr$$

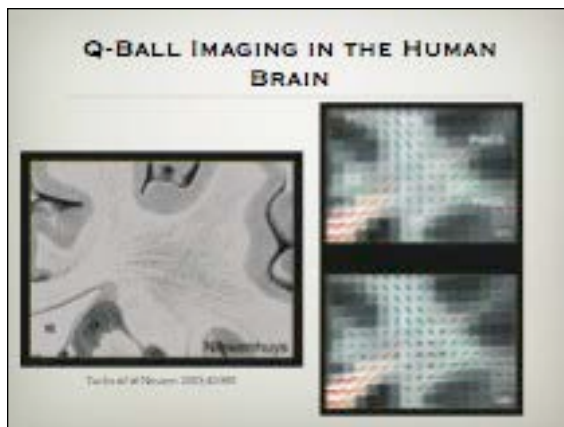
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FUNK-RADON TRANSFORM

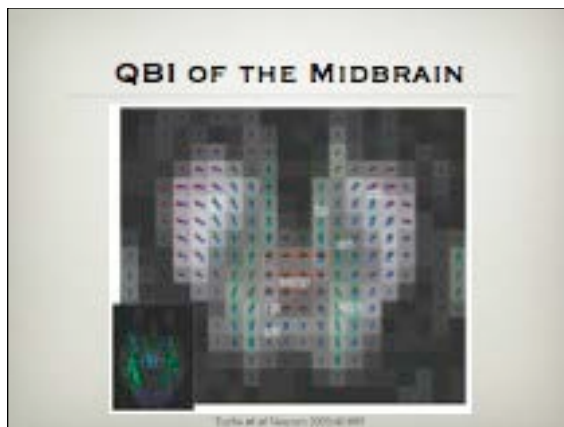
Tuch showed that the diffusion weighted signal on a shell in q-space is closely related to the ODF by the Funk-Radon transform.



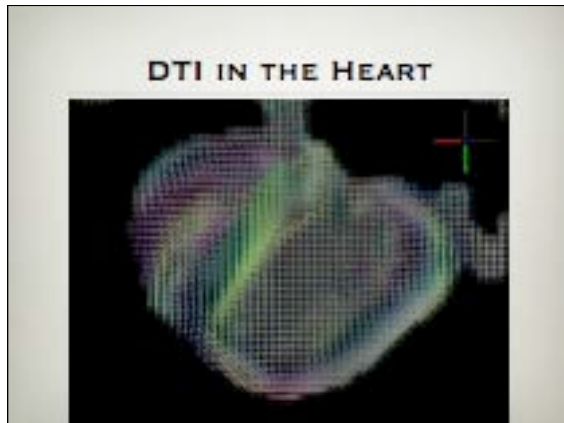
48



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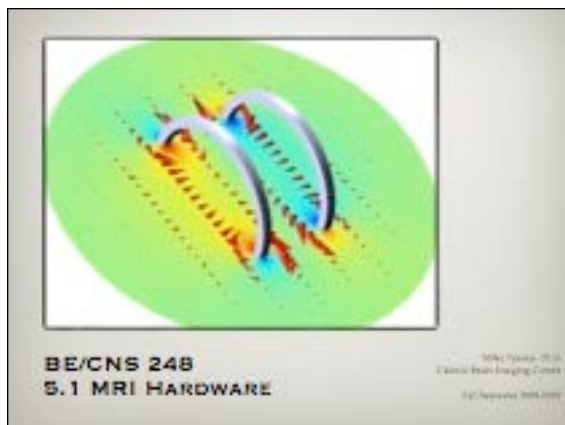
51



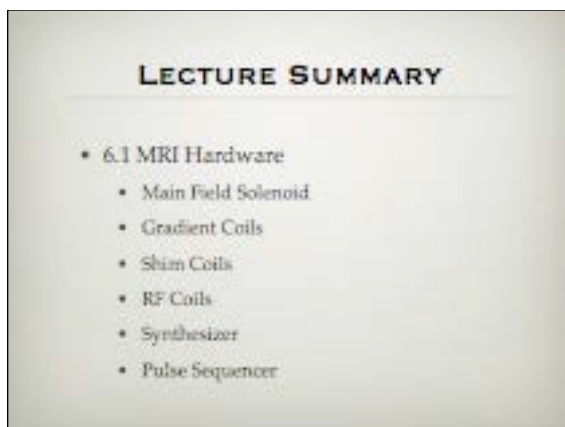
52

NEXT LECTURE

- Magnetic Resonance Engineering



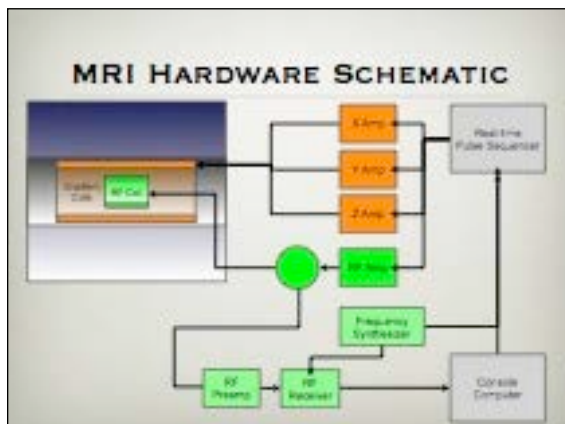
1



2



3



4



MAIN FIELD SOLENOID

5

MAGNET SOLENOID



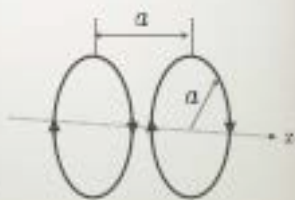
6

NMR SOLENOID CUTAWAY



7

HELMHOLTZ PAIR



8

HELMHOLTZ FIELD ON AXIS

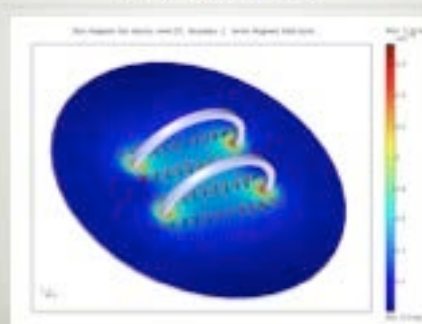
Axial magnetic field

$$B_z = \frac{\mu_0 I}{2a} \left[\frac{1}{(\gamma^2 + \gamma + 5/4)^{3/2}} + \frac{1}{(\gamma^2 - \gamma + 5/4)^{3/2}} \right]$$

where $\gamma = \frac{z}{a}$

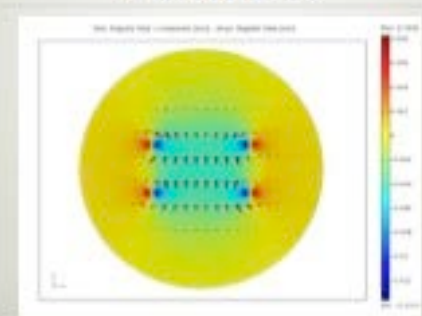
9

HELMHOLTZ FIELD UNIFORMITY



10

HELMHOLTZ FIELD UNIFORMITY



11

HELMHOLTZ PAIR SETS

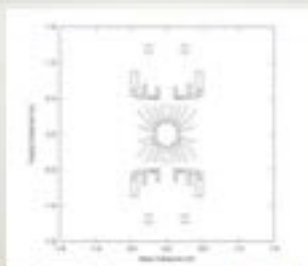
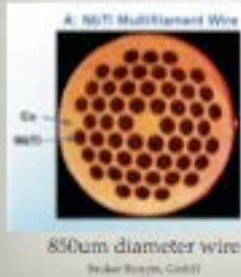


Figure 1. The coil configuration of the magnet design. Also shown are the 40-80-gauss contour lines in the central region of the magnet.

12

SUPERCONDUCTORS FOR MRI



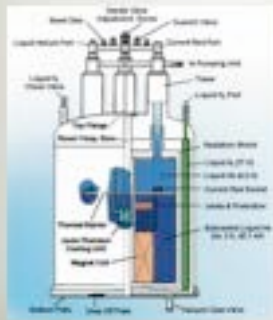
Niobium-Titanium

Class II Superconductor
Transition temperature : 10K
Critical field : 15T

Material of choice for high-field superconducting magnets due to mechanical and magnetic performance advantages.

13

DUAL SKIN CRYOSTATS



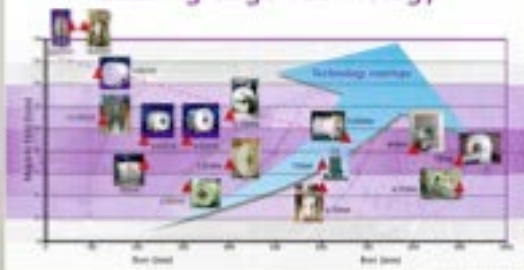
Liquid N_2 boils off preferentially (sacrificial cryogen) in order to reduce liquid He boil off.

Typical MRI cryostat will need N_2 fills weekly to monthly with He fills monthly to quarterly.

14

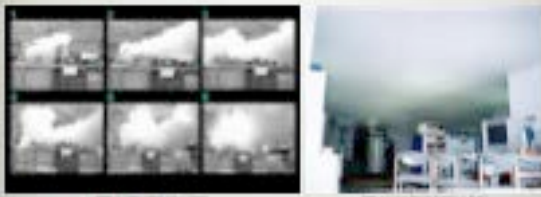
HIGH-FIELD MAGNETS

Leading Edge Technology



15

QUENCHING



Catastrophic breakdown of superconductivity causes collapse of magnetic field. Stored energy dumped as heat into cryogens. Huge volume of gas released.

16



GRADIENT COILS

17

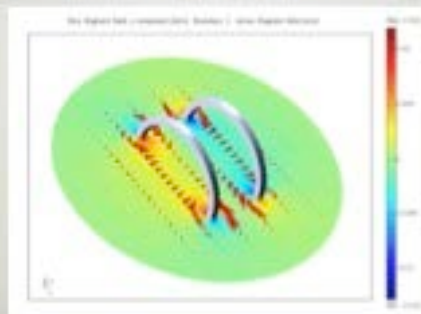
AXIAL FIELD GRADIENTS

The Maxwell Pair
(not to be confused with the Helmholtz Pair)



18

MAXWELL PAIR GRADIENT



19

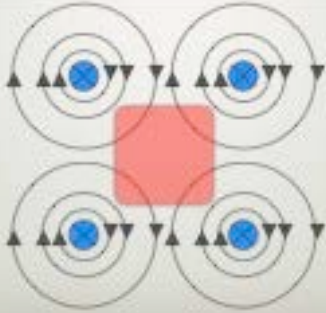
TRANSVERSE FIELD GRADIENTS

All you need are two wires



20

IMPROVING THE GRADIENT



21

THE GOLAY SET

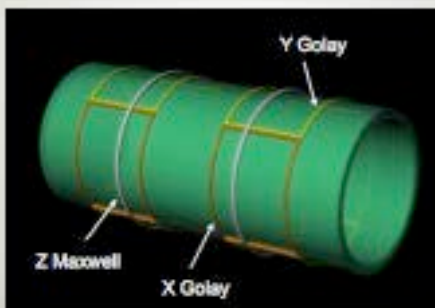
Cylindrical version of four parallel wires



Optimally: $L = 3.5a$ $d = 0.775a$ $\Phi = 120^\circ$

22

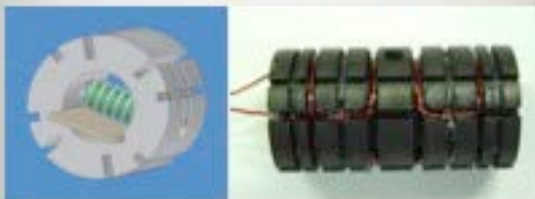
SHIM HARDWARE



23

THREE-AXIS GRADIENTS

11mm Maxwell-Golay Design and Fabrication

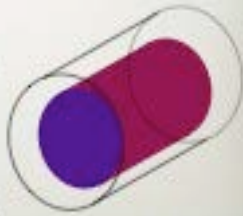


24

THE TARGET-FIELD METHOD

Solve for the current density, $J(z, \theta)$ on the surface of a cylinder of radius a , which generates the required $B_z(z, \theta)$ on the surface of a target cylinder, radius $c < a$.

Integrate current density to determine contours of current density along which windings can be wrapped

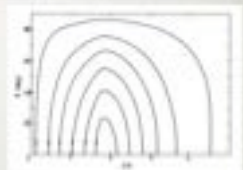


Tucker R. Target Field Method. J Phys D 1995;19:L147 for full method.

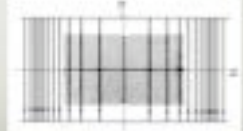
25

THE TARGET-FIELD METHOD

Quadrant of winding pattern for TFM transverse coil winding



Longitudinal gradient TFM winding pattern



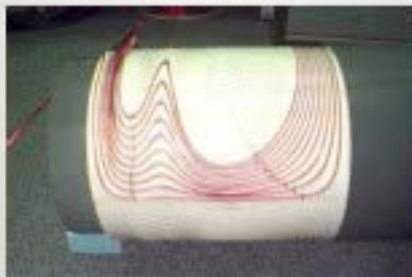
26

TARGET FIELD DESIGNS



27

MINIMUM INDUCTANCE DESIGNS



Conrad R. Sam BM. Constrained Least Squares Minimum Inductance Gradient Coil Design. Magnetic Resonance in Medicine 39:375-378, 1998

28

GRADIENT AMPLIFIERS



High current - hundreds of amps

Fast slew rate - 100ns full range

DC-100s KHz bandwidth

Would make excellent stereo amplifiers.

Conversely, can play music through the gradient set (worst loud speaker ever designed)

29



SHIM COILS

30

SHIM COILS

- Generate additional fields to compensate for B_0 field inhomogeneities in the sample.
- Primary imaging gradients often used to generate linear shim fields (gradient shimming).
- Higher order spatial shim fields also used for improved shimming.

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SHIMMING



Zero shims
 $sd(B_0) = 100\text{Hz}$



Optimized shims
 $sd(B_0) = 25\text{Hz}$

32

Legendre Polynomials

Shim Name	Legendre Polynomial	
X	x	1 st Order
Y	y	
Z	z	
XY	xy	2 nd Order
XZ	xz	
YZ	yz	
X ² -Y ²	$x^2 - y^2$	
Z ²	$z^2 - 1/2 (x^2 + y^2)$	3 rd Order
Z ³ X	$z^3 - 3/4 z (x^2 + y^2)$	
Z ³ Y	$z^3 - 3/4 z (x^2 + y^2)$	
XYZ	xyz	
Z(X ² -Y ²)	$z (x^2 - y^2)$	
X ³	$x^3 - 3xy^2$	
Y ³	$y^3 - 3xz^2$	
Z ³	$z^3 - 3/2 z (x^2 + y^2)$	

<http://www.oxford-journals.org/doi/full/10.1093/mrm/29.1.1>

33



34

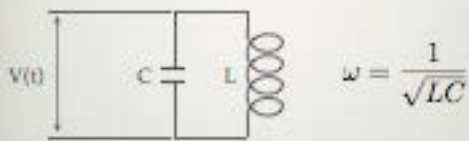


35



36

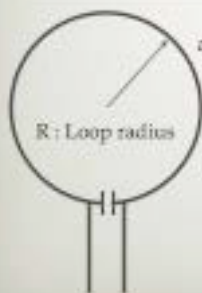
RESONANT CIRCUITS



At resonance, tank circuit has very high impedance and draws little current. Current oscillates between capacitor and inductor.

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SIMPLE LOOP COIL



a : Wire radius

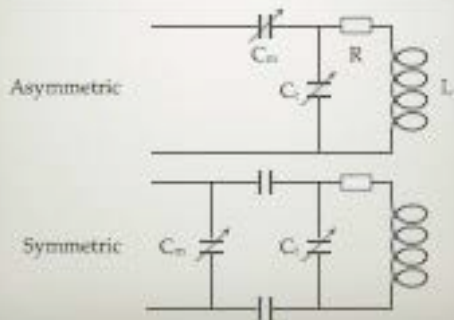
R : Loop radius

$$L = N^2 R \mu_0 \mu_r \left[\ln \left(\frac{8R}{a} \right) - 2 \right]$$

For example: A 5cm radius loop of 2mm radius wire in air has an inductance of about 2nH

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TYPICAL TUNING CIRCUITS



39

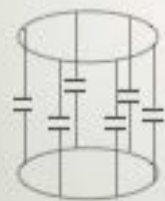
QUALITY FACTOR

- Energy lost per cycle
- FWHM of coil resonance mode
- Typically in range 100-1000 for MRI coils

$$Q = \frac{L\omega_0}{R}$$

40

BIRDCAGE COILS



Low Pass Birdcage



High Pass Birdcage

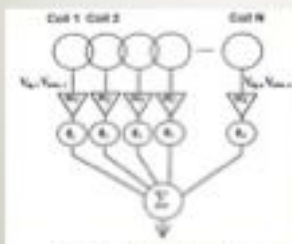
41

PHASED ARRAYS



42

PHASED-ARRAYS

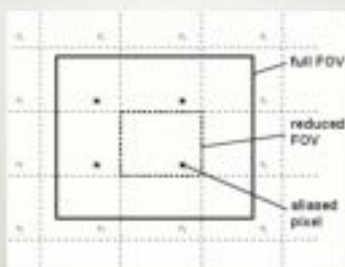


Weight and Gain/LOSS in Shared HW

Signal from an array of receive coils is weighted by a complex factor prior to summation.

43

IMAGE FOLDOVER



44

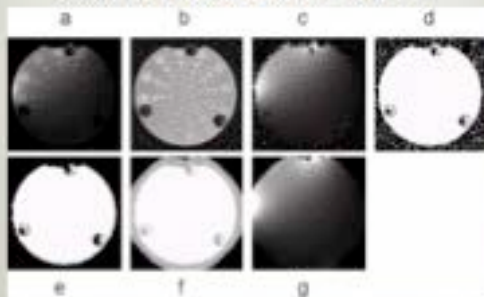
SENSE

SENSE: Sensitivity Encoding for Fast MRI



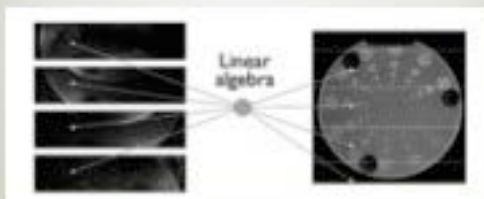
45

COIL SENSITIVITY MAPS



46

SENSE LINEAR ALGEBRA



47

SENSE UNFOLDING

The key to signal separation lies in the fact that in each single-coil image, signal superposition occurs with different weights according to coil sensitivity. Consider one point in the reduced FID: and the corresponding set of points in the full FID. Let n_r denote the number of points superimposed and n_c the number of coils used. Assemble in the vector $\mathbf{g}$ the complex image values the chosen point has in the intermediate images. The complex coil sensitivities of the n_r superimposed positions form an $n_r \times n_c$ sensitivity matrix $\mathbf{S}$:

$$S_{ij} = s_{ij} e^{i\phi_{ij}} \quad (1)$$

where the subscripts i, j count the coils and the superimposed points, respectively. s_{ij} denotes the position of the point i and s_{ij} is the spatial sensitivity of the coil j . The sensitivity matrix is used to calculate the unfolding matrix $\mathbf{U}$:

$$\mathbf{U} = \mathbf{S}^H \mathbf{S}^{-1} \mathbf{S}^H \mathbf{S}^{-1} \quad (2)$$

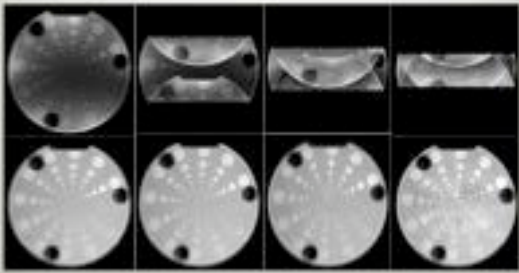
where the superscript H indicates the transposed complex conjugate, and $\mathbf{S}^{-1}$ is the $n_r \times n_c$ receiver noise matrix, which describes the levels and correlation of noise in the receiver channels. Using the unfolding matrix signal separation is performed by:

$$\mathbf{v} = \mathbf{U} \mathbf{g} \quad (3)$$

where the resulting vector's real length n_r and holds separated pixel values for the originally superimposed positions. Repeating this procedure for each point in the reduced FID a two-dimensional full FID image is obtained.

48

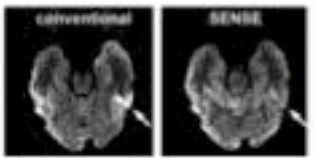
SENSE LIMITATIONS



49

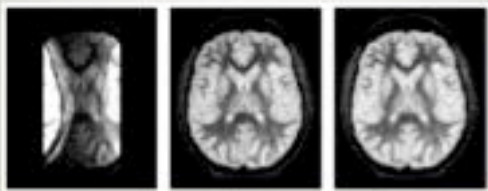
EPI AND SENSE

Short echo trains
improve diffusion
imaging.



50

BRAIN IMAGING

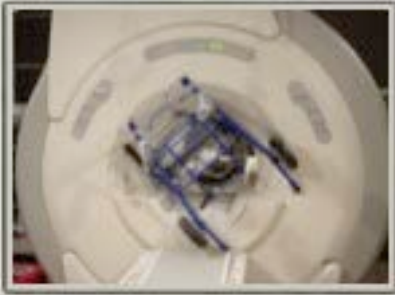


51

NEXT LECTURE

- MRI Bioeffects and Safety
 - Static magnetic field
 - Gradient switching
 - RF heating
 - Other effects

52



BE/CNS 248
6.2 MRI BIOEFFECTS

10/10/2008 10:10
CNS/BE/248/2008/10/10
10/10/2008 10:10

1

LECTURE SUMMARY

- 6.2 MRI Bioeffects and Safety
 - Static field effects
 - Forces on ferromagnetic materials
 - Rapid gradient switching
 - Peripheral nerve stimulation
 - Acoustic noise
 - Claustrophobia
 - Screening questionnaires
 - Human Subjects Protection

2



STATIC MAGNETIC FIELDS

3

FRANZ ANTON MESMER

1734-1815

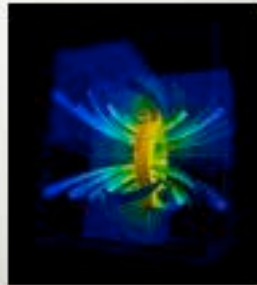
- German physician practiced in Vienna
- Therapeutic system based on manipulation of magnetic fluids in the body.
- "Animal Magnetism"
- Early attempt to exploit magnetic bioeffects.



4

STATIC MAGNETIC FIELD

- Main cause of MRI accidents
- 3 Tesla = 30,000 Gauss
- Forces and torques on ferromagnetic objects.
- Eddy current effects in non-ferromagnetic conductors.
- Induced potential differences in moving conductive fluids such as blood.



5

MAGNETIC SUSCEPTIBILITY

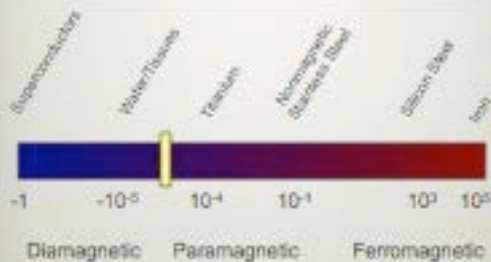
$$\mathbf{M} = \chi \mathbf{H}$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

$$= \mu_0 \mathbf{H} (1 + \chi)$$

6

SUSCEPTIBILITY SPECTRUM



7

FERROMAGNETIC METALS

- Distinguish between ferro-magnetic and non-ferromagnetic metals
- Ferromagnetic:
 - Mild, Tool, HSS Steel
 - Staples, Paper Clips
 - Iron Filings
- Non-ferromagnetic:
 - Gold, platinum, silver
 - Copper, brass, aluminum
 - Many, but not all stainless steels

8

FORCES IN A MAGNETIC FIELD

In general, the force on an object is equal to the gradient of the magnetic potential energy

$$\mathbf{F} = -\nabla U$$

If we can determine U then we can derive the force on a magnetized object in a field.

9

MAGNETIC POTENTIAL ENERGY

For a permanent magnetic dipole brought into a field $\mathbf{B}$

$$U = -\mathbf{m} \cdot \mathbf{B}$$

For a magnetizable object brought into a field $\mathbf{B}$

$$U = -\frac{1}{2} \mathbf{m} \cdot \mathbf{B}$$

10

MAGNETIC FORCE IN A FIELD GRADIENT

For an isotropic object with volume V and field-induced magnetization

$$F_z = \frac{V}{2\mu_0} \chi B_z \frac{\partial B_z}{\partial z} \quad \chi \ll 1$$

$$F_z = \frac{V}{2\alpha\mu_0} B_z \frac{\partial B_z}{\partial z} \quad \chi \gg 1$$

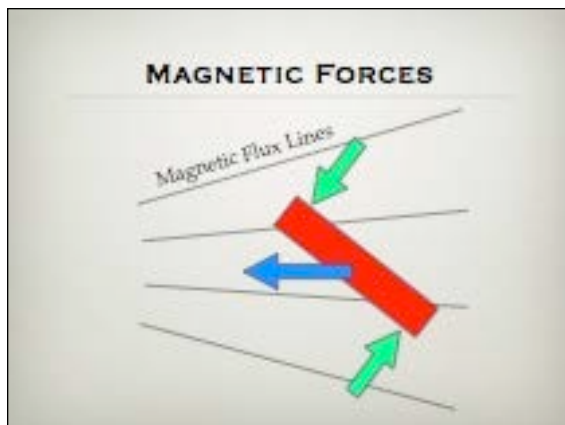
11

TORQUE IN A MAGNETIC FIELD

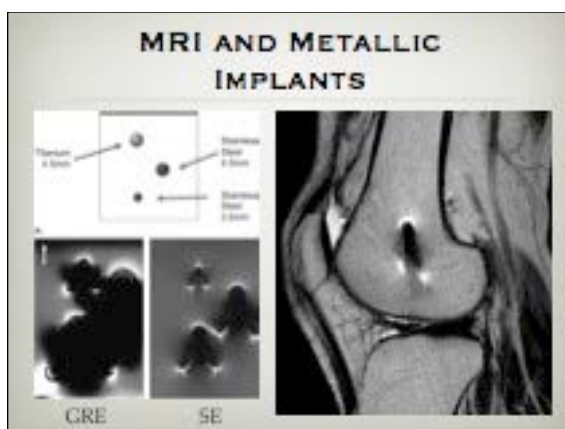
For a small, weakly susceptible object

$$\begin{aligned} \mathbf{T} &= -\frac{\partial U}{\partial \theta} \mathbf{u} \\ &= \mathbf{M} \times \mathbf{B} \end{aligned}$$

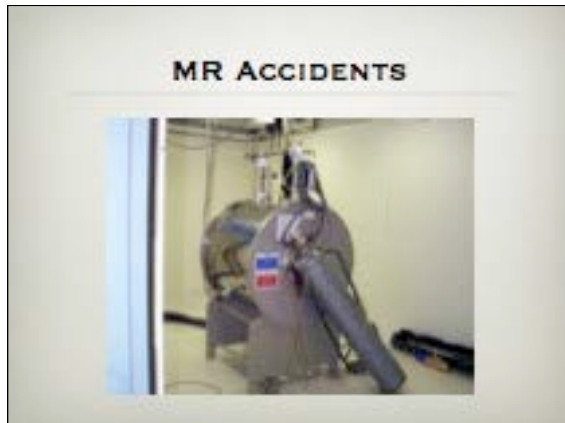
12



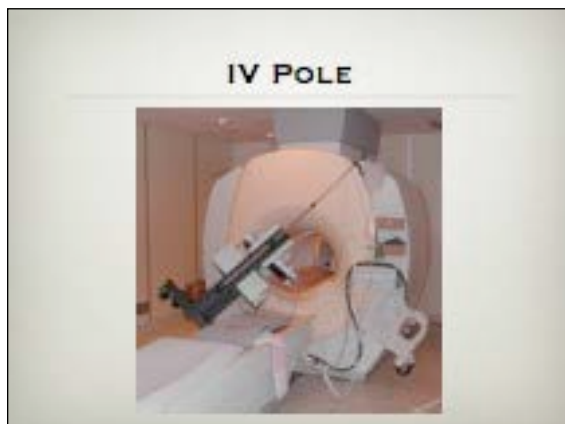
13



14



15



16



17

MRI FATALITIES

The New York Times

Hospital Details Features Leading to MRI Fatality

A poorly trained staff, technical malfunctions and lapses in communication among workers contributed to the death of a 5-year-old boy last month who was struck by an oxygen canister drawn into a magnetic resonance imaging machine at Westchester Medical Center, according to a report by the hospital released today.

The review, required by the state after medical fatalities, was conducted after the death of the boy, Michael Columbeni of Croton-on-Hudson, who was struck by a 5.5-pound metal oxygen tank that was pulled into the machine by its magnetic field as he underwent a test.

By RANDAL C. ARONOLD
Published August 22, 2007

18

RADIOFREQUENCY FIELDS

19

RADIOFREQUENCY FIELD

- 128 MHz RF Electromagnetic Field at 3T
- Tissue heating by current induction
- Specific Absorption of Radiation (SAR)
- 1.3W/kg limited
- RF arcing possible, but very rare.

20

SPECIFIC ABSORPTION RATIO (SAR)

- SAR is defined as the power absorbed per unit mass.
- SAR is the primary focus of RF heating safety limits in MRI.
- Can be simulated by numerical methods.
- Common limit for RF-heavy pulse sequences.

21

ANALYTICAL SAR

$$\text{SAR} = \frac{\sigma |\mathbf{E}|^2}{2\rho} \frac{\tau}{\text{TR}}$$

Diagram showing the components of the SAR equation:

- Conductivity (σ) points to the numerator.
- Electric Field ($\mathbf{E}$) points to the numerator.
- Density (ρ) points to the denominator.
- Equivalent rect RF pulse duration (τ) points to the numerator.
- Repetition time (TR) points to the denominator.

22

TISSUE PROPERTIES FOR SIMULATION

TABLE 1
Tissue Parameters

Tissue type	ρ (g/cm ³)	60 MHz		100 MHz		150 MHz		200 MHz	
		ϵ_r	σ (S/m)	ϵ_r	σ (S/m)	ϵ_r	σ (S/m)	ϵ_r	σ (S/m)
CSF	1.00	80	0.02	80	0.7	80	6.75	80	8.8
sk	1	0	0	1	0	1	0	1	0
muscle	1.04	45	0.60	46	1.3	46	1.3	45	1.3
bone	1.87	16	0.04	15	0.05	16	0.05	15	0.05
fat	0.90	70	0.70	60	0.06	60	0.07	55	0.7
liver	1.05	46	0.15	35	0.05	35	0.06	34	0.06
kidney	1.05	50	1.9	55	1.7	70	1.3	75	1.75
blood	1.05	48	0.40	50	0.50	40	0.50	48	0.6
cartilage	1.20	20	0.06	27	0.20	24	0.03	22	0.05

ϵ_r = relative permittivity σ = electrical conductivity

From: Chen (IEEE TMI 1996)

23

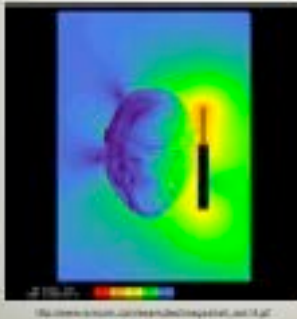
FDTD SIMULATIONS

Solve Maxwell's equations for E and H on a mesh.

$$\begin{aligned}\nabla \times \mathbf{E} &= -\mu_0 \frac{\partial \mathbf{H}}{\partial t} \\ \nabla \times \mathbf{H} &= \epsilon \frac{\partial \mathbf{E}}{\partial t} + \sigma \mathbf{E} + \mathbf{J}^{\text{ex}}\end{aligned}$$

24

MODELING SAR



- New modeling tools from microwave communications R&D.
- Finite Difference Time Domain (FDTD)
- Numerical tissue models and mannequins

25

FDTD OF HEAD COILS

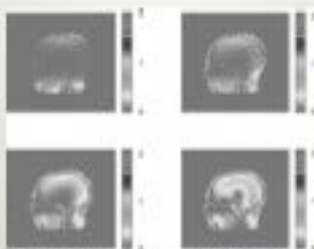


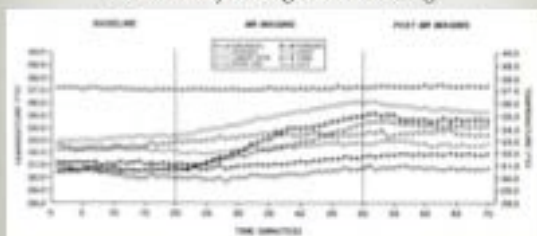
Fig. 4. SAR (W/kg) distribution in the sagittal plane for a standard head coil (left), 100 MHz, 100 MHz, 100 MHz, 100 MHz, and 100 MHz.

From Chen (IEEE TMI 1996)

26

OBSERVED RF HEATING

Whole-body average SAR 2.8W/kg



From IEEE/ITC Magnetic Resonance Procedures, 1996
From IEEE/ITC Magnetic Resonance Procedures, 1996

27

SAR STANDARDS

- US FDA Mean SAR Maxima
 - 4W/kg over whole body over any 15 minute period
 - 3W/kg over head over any 10 minute period
 - 8W/kg in any gram (head and torso) over any 5 minute period
 - 12W/kg in any gram (extremities) over any 5 minute period
- IEC
 - NORMAL OPERATING MODE
 - FIRST LEVEL CONTROLLED OPERATING MODE
 - SECOND LEVEL CONTROLLED OPERATING MODE

28

OPERATIONAL MODES OF AN MR SCANNER

- NORMAL MODE
 - Suitable for all subjects/ patients
- FIRST LEVEL CONTROLLED (FLC)
 - May cause undue physiological stress
- SECOND LEVEL CONTROLLED (SLC)
 - May produce significant risk

29

MRI AND TATOOS



Second-degree burns (arrows) following 1.5T MRI
Wagle WA and Smith M, AJR 2000 176:1795.

30

MR COMPATIBLE SENSORS



- Carbon Fiber Leads
- Twisted Pair
- Fiber Optic Leads
- Image-based Signals

© Kappanetris 2008

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FIELD GRADIENT SWITCHING

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RAPIDLY VARYING FIELD GRADIENTS

- Causes rapid change in B (dB/dt)
- dB/dt increases linearly with distance from gradient isocenter
- dB/dt induces currents
- Peripheral nerve stimulation

33

EMPIRICAL DETERMINATION OF PNS THRESHOLD



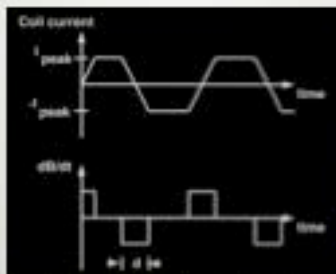
- Volunteer subjects exposed to gradient pulsing.
- Slew durations from 50 μ s to 1000 μ s.
- Subjective rating from 0 (not felt) to 10 (intolerable)

From: Medical PG Magnetic Resonance Procedures: Health Effects and Safety © 2007 CRC Press

See also: Gradient in a Scanning (C) book for 1999-2005

34

PNS TESTING

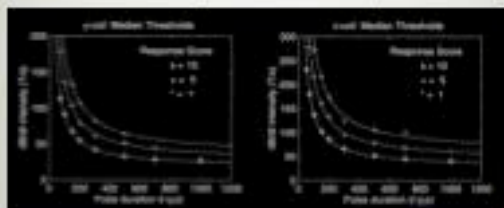


From: Medical PG Magnetic Resonance Procedures: Health Effects and Safety © 2007 CRC Press

35

PNS STIMULATION CURVE

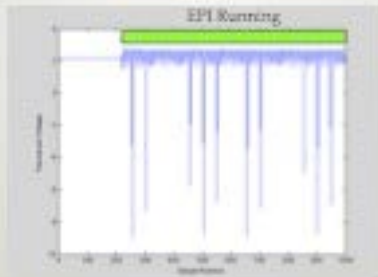
Median responses from 84 subjects



From: Medical PG Magnetic Resonance Procedures: Health Effects and Safety © 2007 CRC Press

36

GRADIENT INTERFERENCE



Piezoresistive strain gage signal during EPI

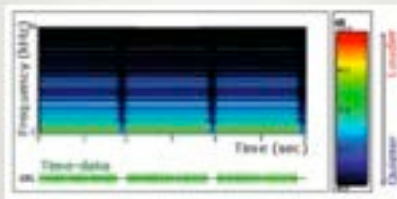
37



OTHER EFFECTS

38

ACOUSTIC NOISE



41 LAM, Mose et al. MRI 2006

Sound pressure levels (SPL) can easily exceed 110dB/A
Noise reduction for subject includes pulse sequence
modifications and acoustic insulation.

39

MODELING GRADIENT NOISE



Finite Element Analysis compared
with experimental measurement.

See G2 et al. MRM 2004 52:11-22



40

CLAUSTROPHOBIA



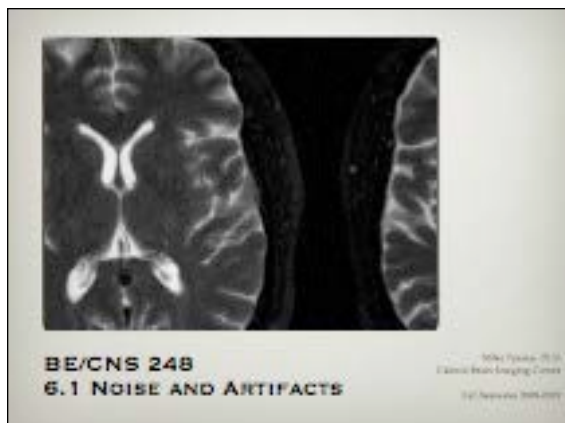
- Reported incidence at clinical sites highly variable: 0.7% to 20%
- Much lower incidence in motivated research subjects.
- Many solutions: open magnets, reassurance, sedation ...

41

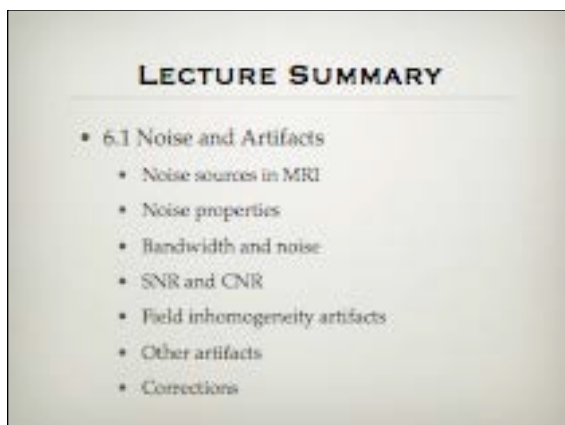
RECOMMENDED READING

- Shellock FG. *Magnetic Resonance Procedures: Health Effects and Safety*. CRC Press
- Jin JM. *Electromagnetic Analysis and Design in Magnetic Resonance Imaging* CRC Press

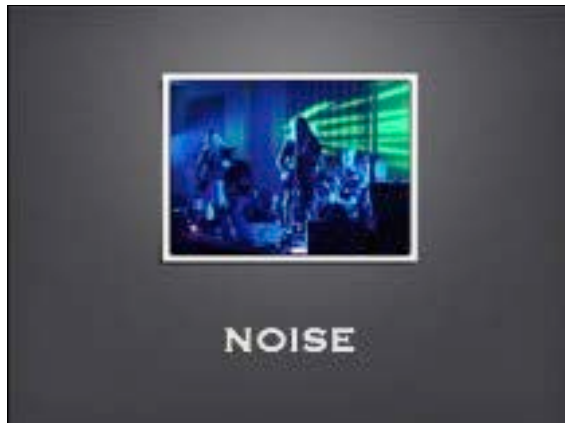
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1



2



3



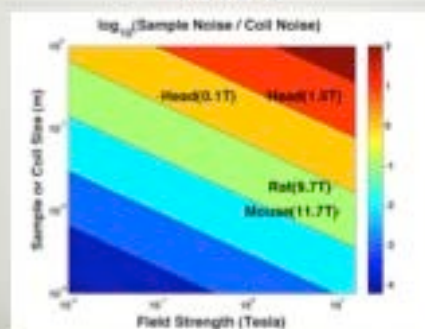
4

SOURCES OF NOISE IN MRI

- Sample or Subject
 - Important at lower fields/larger subjects
- RF coil
 - Important at higher fields/smaller samples
- RF Preamplifier
- RF Receiver Electronics
- External Sources

5

SAMPLE TO COIL NOISE RATIO



6

SIGNAL AND NOISE

Total signal from a voxel is proportional to the voxel volume:

$$\frac{S}{N} \propto \Delta x^3 \sqrt{N_{ox}}$$

$$N_{ox} \propto \frac{1}{\Delta x^6}$$

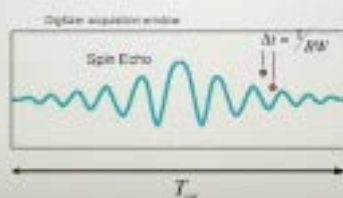
Total imaging time has an inverse sixth power relation to voxel dimension at constant SNR.

7

FREQUENCY ENCODING BANDWIDTH AND NOISE

SNR is proportional to the square-root of the total digitizer acquisition time (c.f. SNR proportional to the square root of the number of signal averages).

$$\frac{S}{N} \propto \sqrt{T_{acq}} = \sqrt{N_s \Delta t} = \sqrt{\frac{N_s}{BW}}$$



8

SNR PER VOXEL

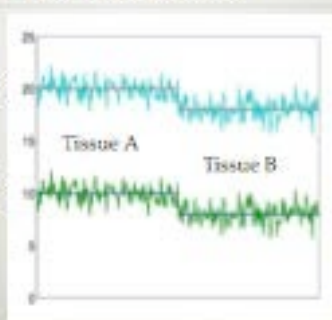
$$SNR/voxel \propto \Delta x \Delta y \Delta z \sqrt{\frac{N_{acq} N_x N_y N_z}{BW_{fe}}}$$

9

SNR AND CNR

SNR = 20
CNR = 2

SNR = 10
CNR = 2



10

*** MORE ON SNR ***

- Full SNR equation (including field strength, etc)
- More on noise spectrum (white, gaussian)
- Prewhitening of NMR noise
- Image example for CNR
- Microscopy vs human MRI SNR
- SNR efficiency

11



DENOISING

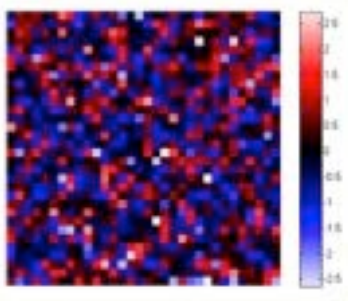
12

NOISE IN MRI

- Typically treated as Gaussian White Noise
- Zero mean, standard deviation σ
- Magnitude image has a Rician noise distribution.
- Non-zero mean, asymmetric distribution.

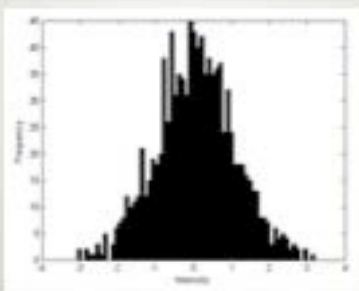
13

GAUSSIAN NOISE



14

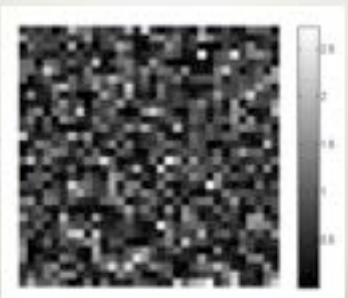
GAUSSIAN NOISE



$N(0,1)$

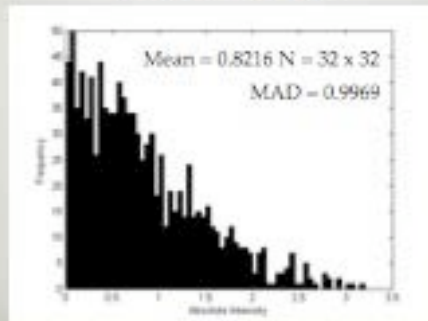
15

RICIAN NOISE



16

RICIAN NOISE



17

ESTIMATING NOISE

MAD : Median Absolute Deviation (from the median)

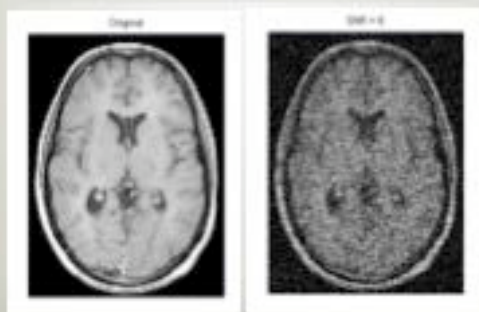
$$MAD(x_i) = \text{med}(|x_i| - \text{med}(x_i))$$

For Gaussian Noise

$$\hat{\sigma}_N = \frac{MAD(x_i)}{0.6745}$$

18

SYNTHETIC GAUSSIAN NOISE



19

GAUSSIAN FILTER DENOISING



20

MEDIAN FILTER DENOISING



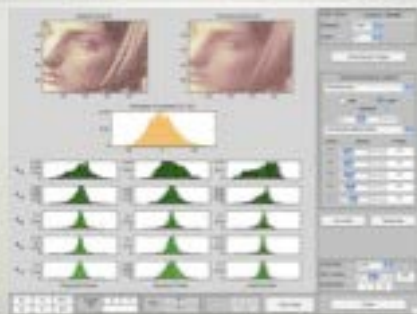
21

WAVELET DENOISING

- Wavelet decompose 2D or 3D image.
- Estimate appropriate threshold at each level of decomposition for removing noise components.
- Recompose image from thresholded wavelet decomposition.

22

EXAMPLE WAVELET DENOISING



23

WAVELET DECOMPOSITION



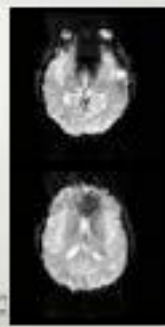
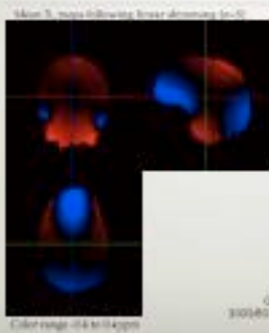
24



ARTIFACTS

25

B_0 INHOMOGENEITY ARTIFACTS



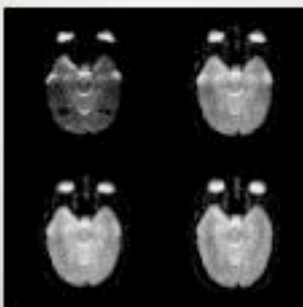
26

FERROMAGNETIC METAL ARTIFACTS

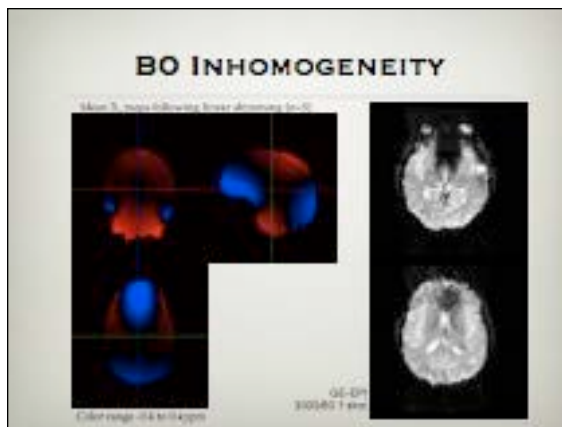


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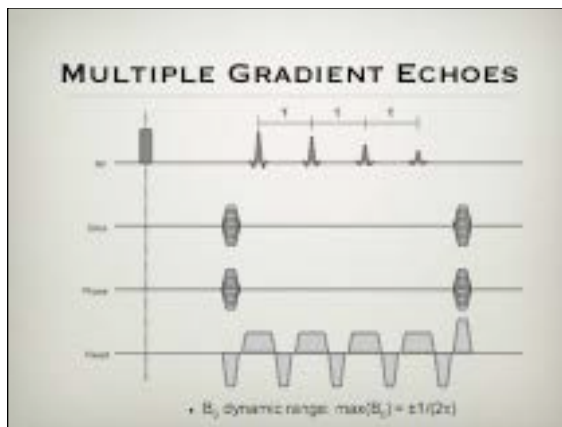
EPI GEOMETRIC DISTORTION



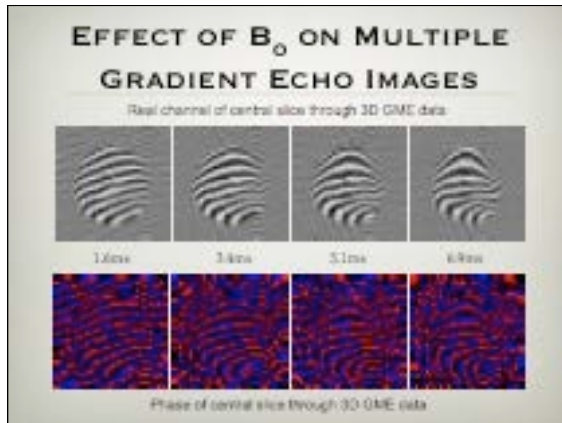
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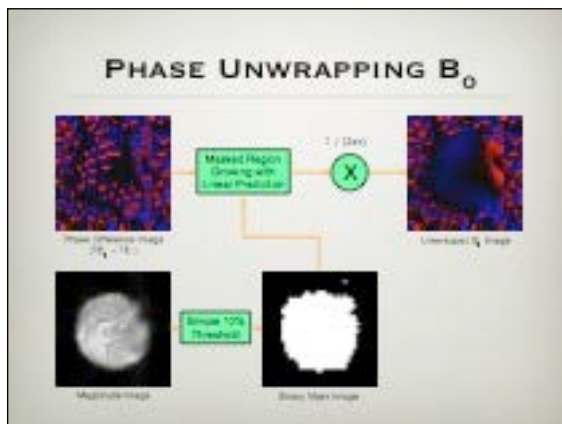
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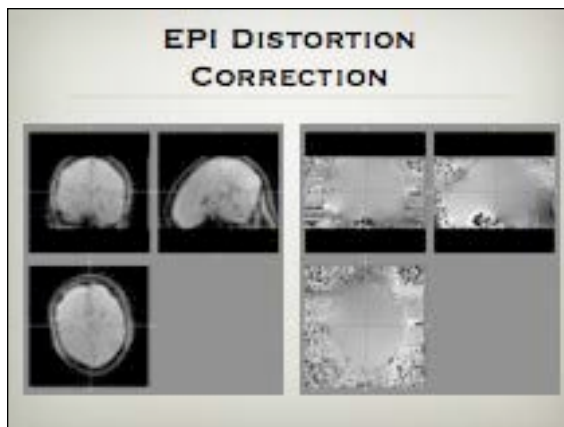
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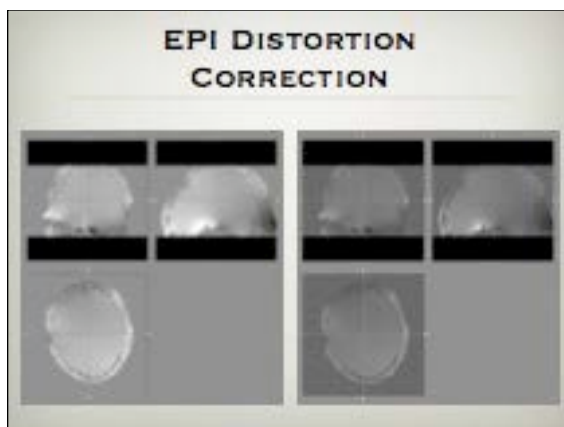
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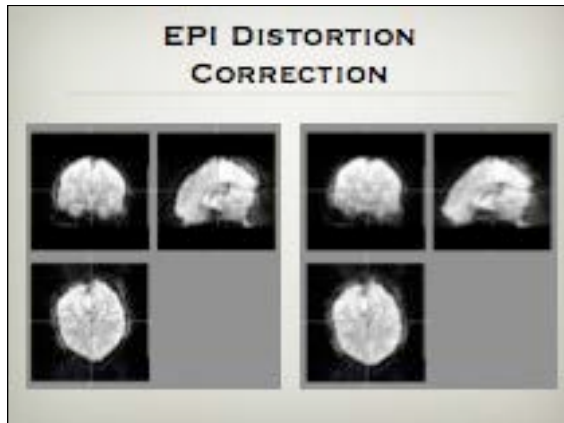
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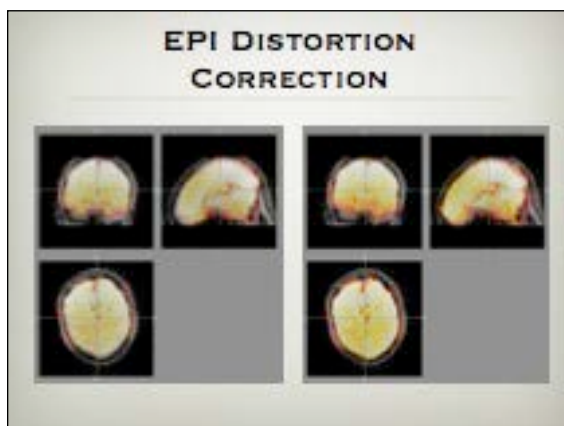
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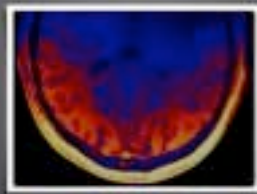
34



35



36



BIAS CORRECTION

37

B₁ INHOMOGENEITY



12T 1mm Surface Coil

3T CP Head Coil

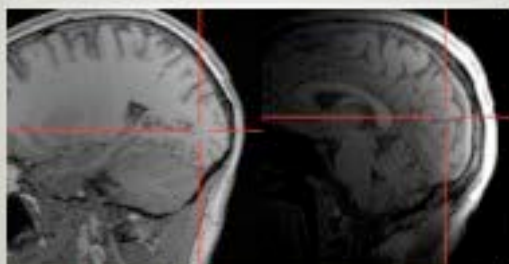
38

SOURCES OF SIGNAL BIAS

- B1 inhomogeneity
 - Transmit coil response
 - Receive coil response
 - High field effects
- B0 inhomogeneity
 - Signal dropout (GE and EPI)

39

APPEARANCE OF B1 INHOMOGENEITY



8-channel Whole-head
Phased Array

8-channel Occipital
Phased Array

40

APPROACHES TO BIAS CORRECTION

- Calibrate spatial response of coil in a phantom.
- Estimate bias from whole image.
- Estimate bias from presumably homogeneous tissue.

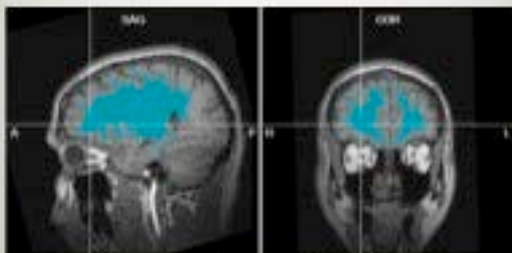
41

SMOOTHNESS ASSUMPTIONS

- Assume that bias field has much lower spatial frequency than tissue structures.
- Estimating bias field from naive smoothing of image data generally fails for real MR data.
- Need to combine with segmentation, allowing bias field estimation within presumably homogeneous tissue.

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TISSUE SPECIFIC BIAS CORRECTION



BrainVoyager: Step 1 - flood fill "white matter"

43

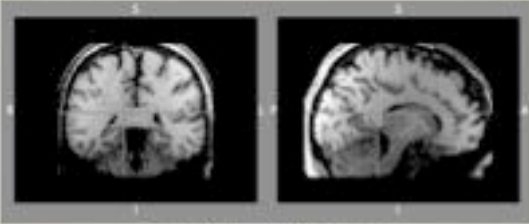
TISSUE SPECIFIC BIAS CORRECTION



BrainVoyager: Step 2 - fit multiplicative 3rd order 3D polynomial (bias field estimate). Repeat until happy.

44

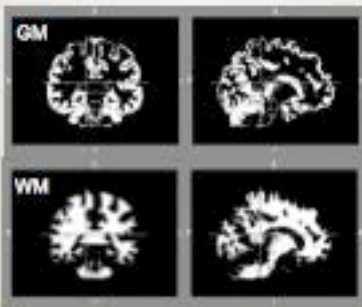
COMBINING SEGMENTATION AND BIAS CORRECTION



Original Biased T1-MPRAGE
SPM5 Segmentation/Bias Correction

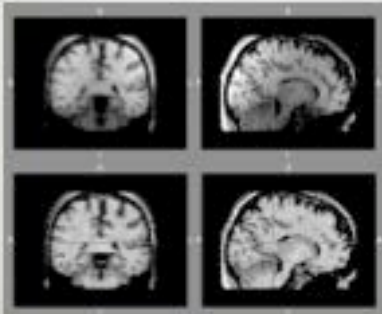
45

SEGMENTATION RESULTS



46

BIAS CORRECTION

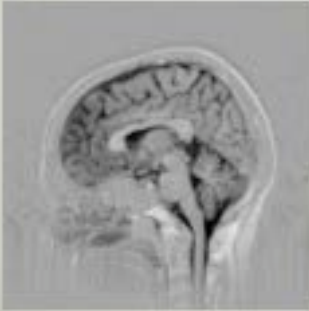


47

OTHER ARTIFACTS

48

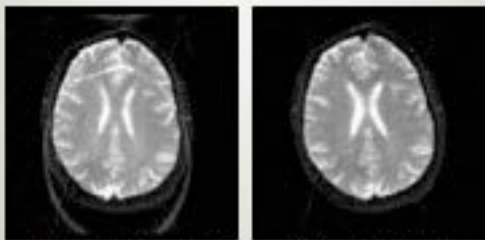
MOTION ARTIFACTS



Arise from departure of actual transverse magnetization phase of moving material from expected phase of stationary material. Ghost artifacts seen in phase encoding direction.

49

CHEMICAL SHIFT ARTIFACT

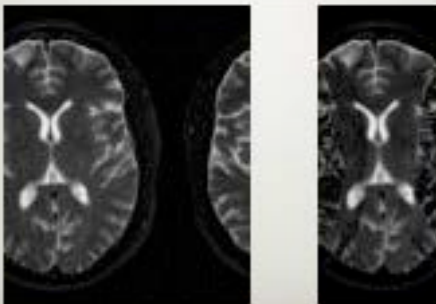


Without Lipid Suppression

With Lipid Suppression

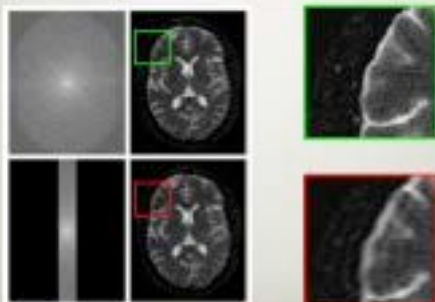
50

PHASE ROLL AND FOLD-OVER

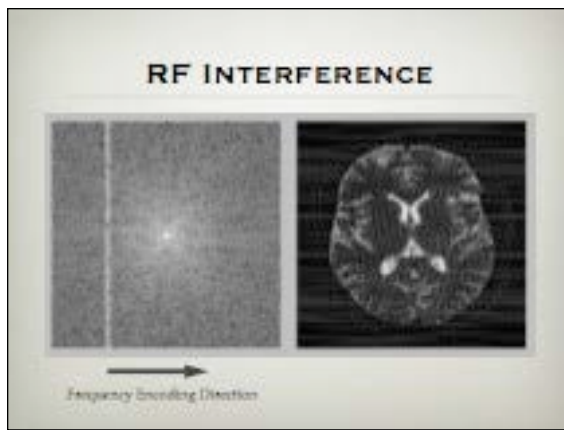


51

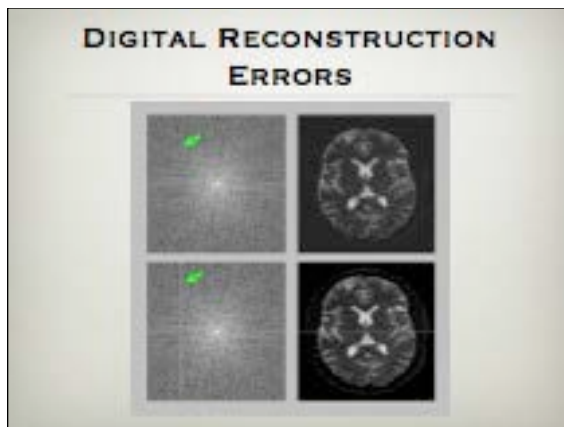
GIBBS RINGING



52



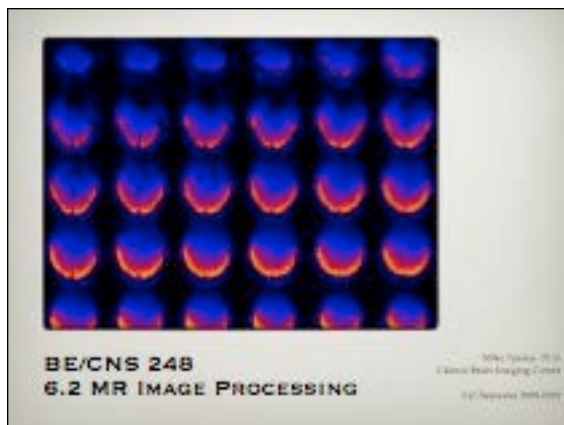
53



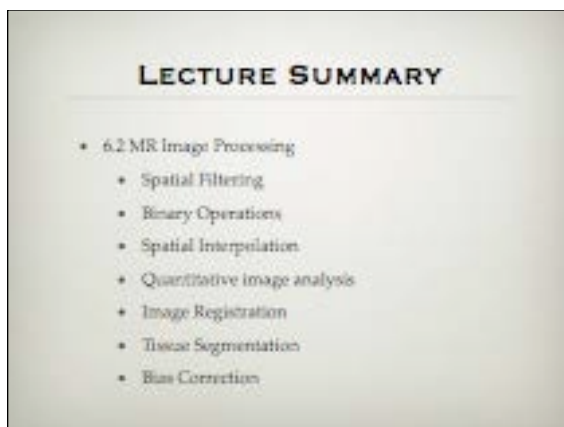
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55



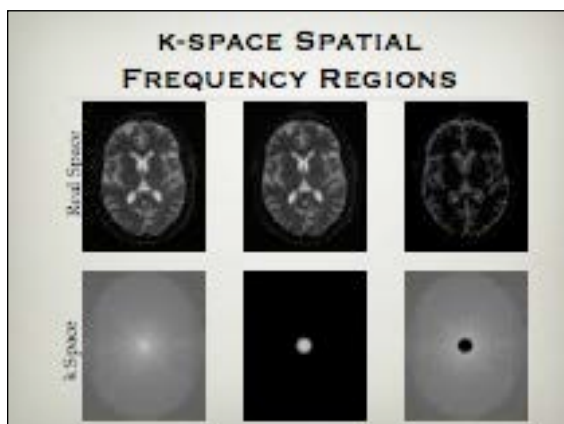
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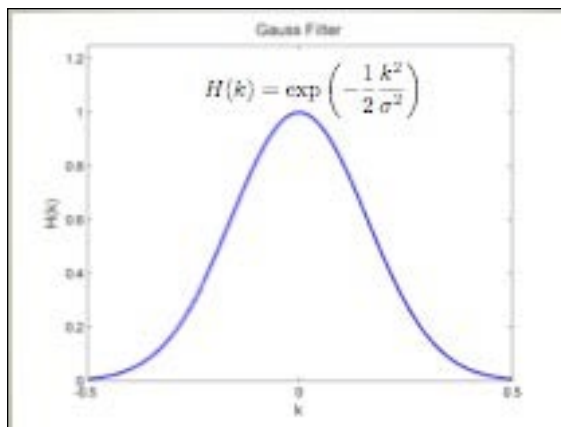
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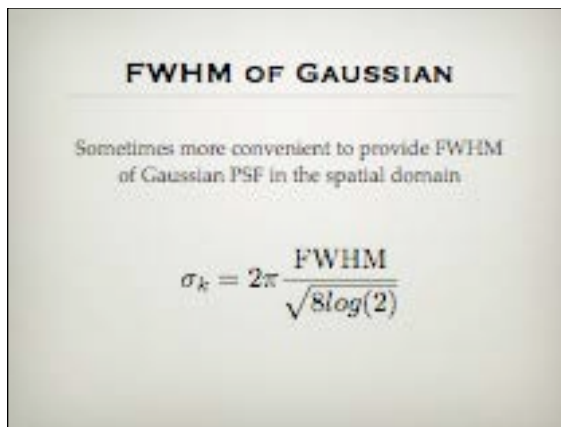
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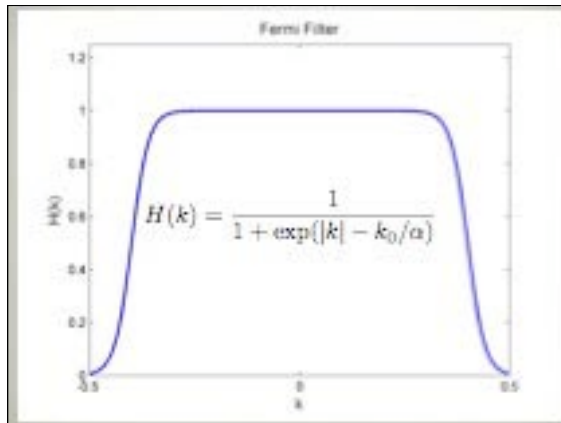
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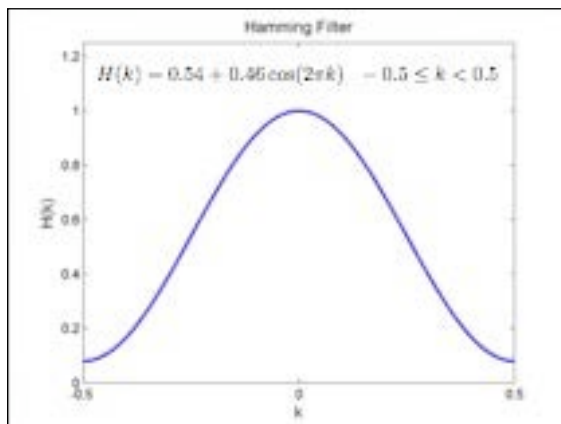
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6

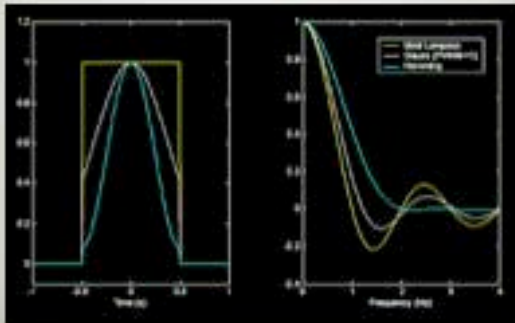


7



8

PSF DUE TO FILTERING



9

SPATIAL DOMAIN FILTERING

- Discrete implementation of convolution filtering in spatial domain.
- 2D or 3D convolution function.
- Linear (eg Gaussian) or non-linear (Median).
- Implemented in a moving mask or kernel.

10

MASK OPERATIONS

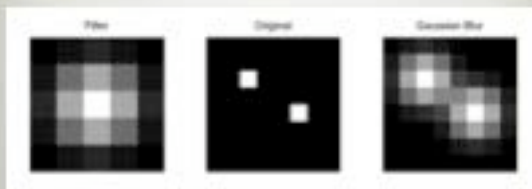
Mask Kernel		Image		New Image																											
<table><tr><td>0</td><td>1</td><td>0</td></tr><tr><td>1</td><td>2</td><td>1</td></tr><tr><td>0</td><td>1</td><td>0</td></tr></table>	0	1	0	1	2	1	0	1	0	*	<table><tr><td>8</td><td>5</td><td>5</td></tr><tr><td>6</td><td>3</td><td>2</td></tr><tr><td>3</td><td>2</td><td>2</td></tr></table>	8	5	5	6	3	2	3	2	2	=	<table><tr><td></td><td></td><td></td></tr><tr><td></td><td>3.5</td><td></td></tr><tr><td></td><td></td><td></td></tr></table>					3.5				
0	1	0																													
1	2	1																													
0	1	0																													
8	5	5																													
6	3	2																													
3	2	2																													
	3.5																														

$$S'(x, y) = \frac{1}{Q} \sum_{k=-m}^{m} \sum_{l=-m}^{m} H(k, l) S(x + k, y + l)$$

$$Q = \sum_{k=-m}^{m} \sum_{l=-m}^{m} H(k, l)$$

11

GAUSSIAN FILTER

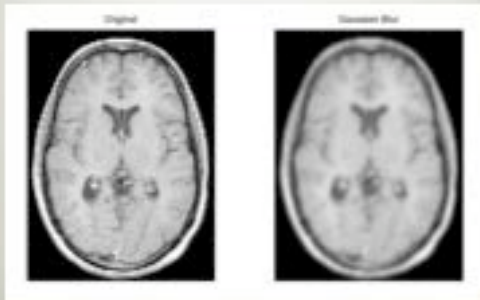


$$H(k, l) = H_0 \exp\left(-\frac{(k^2 + l^2)}{2\sigma^2}\right) \quad k, l = -m, \dots, m \quad m = 1, 2, \dots$$

Radial Gaussian

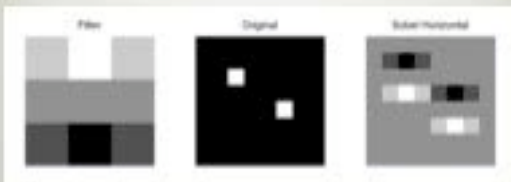
12

GAUSSIAN FILTER



13

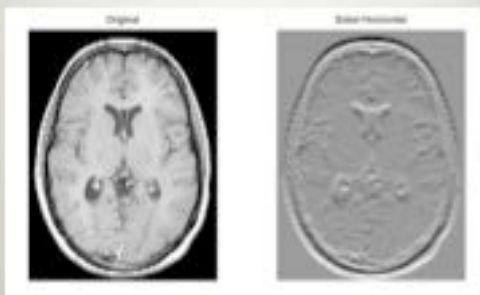
HORIZONTAL SOBEL



Center weighted edge detection.
Compare with Prewitt filter - unweighted
edge detection

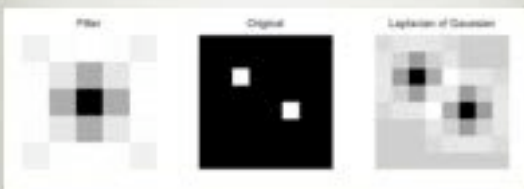
14

HORIZONTAL SOBEL



15

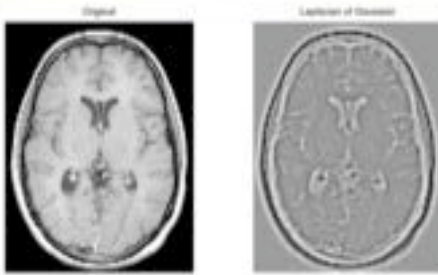
LAPLACIAN OF GAUSSIAN



$$H(x, y) = \nabla^2 \exp\left(-\frac{x^2 + y^2}{2\sigma^2}\right)$$

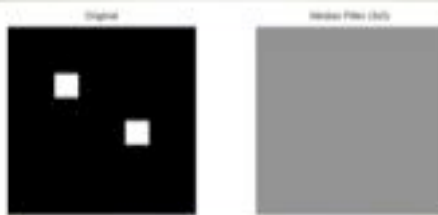
16

LAPLACIAN OF GAUSSIAN



17

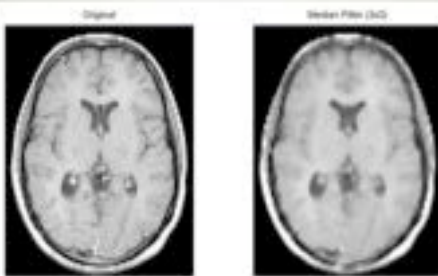
MEDIAN FILTER



Replaces pixel intensity with median of pixel intensities within kernel.
Robust to outliers, Edge preserving, Spike noise filter.

18

MEDIAN FILTER

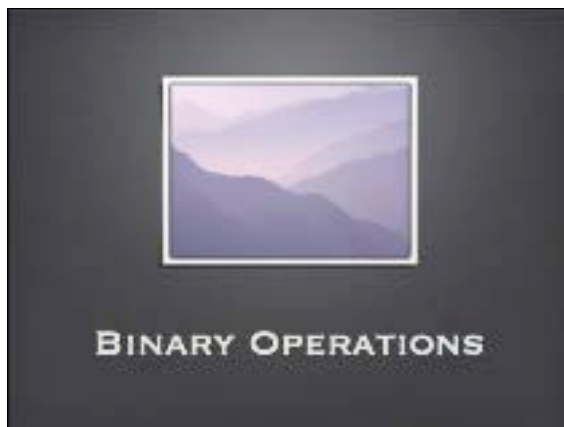


19

MEDIAN FILTER



20



21

MORPHOLOGICAL OPERATIONS

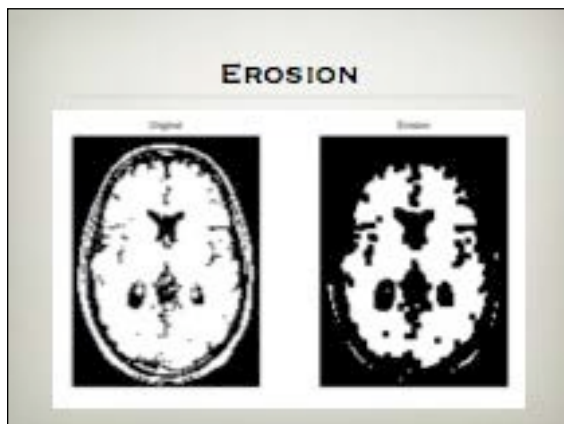
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0	0	0	1	1	0	0	1	1	1
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Original Binary Image Local Maximum
3x3 Square Kernel

22

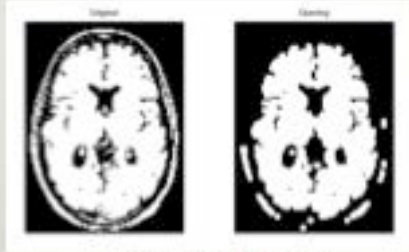


23



24

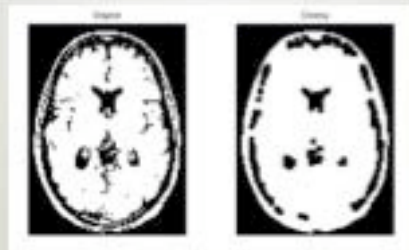
MORPHOLOGICAL OPENING



Erosion followed by dilation.
Cuts bridges and removes small islands on scale of mask.
Approximately preserves object size.

25

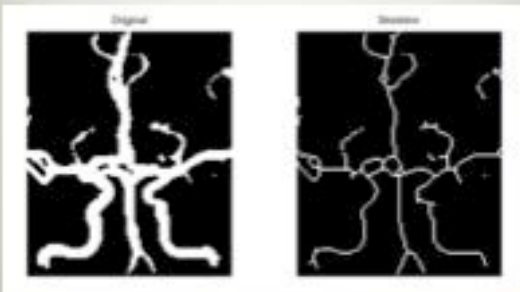
MORPHOLOGICAL CLOSING



Dilation followed by erosion.
Bridges gaps and fills holes on scale of mask.
Approximate size preservation.

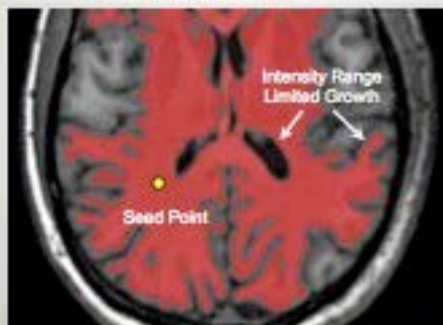
26

SKELETONIZATION

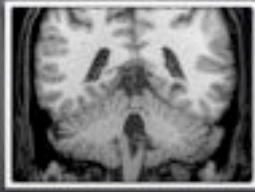


27

REGION GROWING



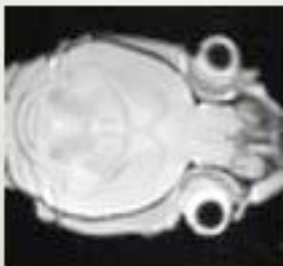
28



QUANTITATIVE IMAGE ANALYSIS

29

RELAXATION IMAGING

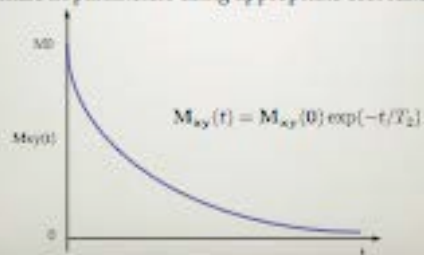


CPMG sequence: One image per echo, TE interval 7ms

30

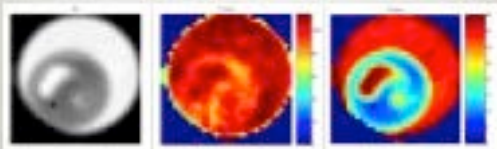
VOXEL-WISE CURVE FITTING

Model signal decay using prior knowledge.
Optimize fit parameters using appropriate cost function.



31

VOXEL-WISE CURVE FITTING



32

DESPOT-1

Make use of flip angle contrast relations for SPGR

$$S_{SPGR} = \frac{M_0(1 - E_1) \sin \alpha}{1 - E_1 \cos \alpha}$$

$$\frac{S_{SPGR}}{\sin \alpha} = E_1 \frac{S_{SPGR}}{\tan \alpha} + M_0(1 - E_1)$$

Linear regression

$$Y = mX + b$$

$$T_1 = -TR / \ln(m)$$

$$M_0 = b / (1 - m)$$

Deony et al. MRM 2003

33

DESPOT-2

Make use of flip angle contrast relation for SSFP

$$S_{SSFP} = \frac{M_0(1 - E_2) \sin \alpha}{1 - E_1 E_2 - (E_1 - E_2) \cos \alpha}$$

$$\frac{S_{SSFP}}{\sin \alpha} = \left(\frac{E_1 - E_2}{1 - E_1 E_2} \right) \frac{S_{SSFP}}{\tan \alpha} + \frac{M_0(1 - E_2)}{1 - E_1 E_2}$$

Linear regression

$$Y = mX + b$$

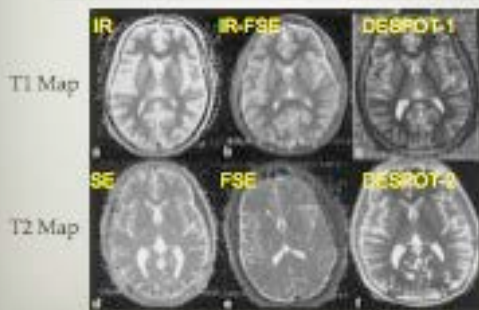
$$T_2 = -TR / \ln \left(\frac{m - E_1}{m E_1 - 1} \right)$$

$$M_0 = b(E_1 E_2 - 1) / (1 - E_1)$$

Deony et al. MRM 2003

34

DESPOT VS OTHER T1 & T2 MAPPING APPROACHES



Deony et al. MRM 2003

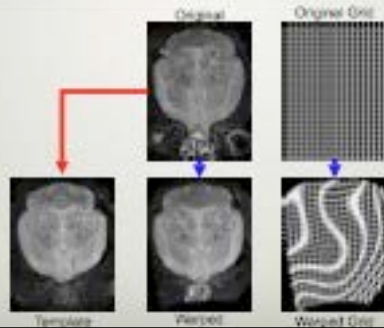
35



IMAGE REGISTRATION

36

DEFORMATION FIELDS



37

LINEAR TRANSFORMS

$$\begin{pmatrix} x' \\ y' \\ z' \\ 1 \end{pmatrix} = \begin{pmatrix} t_{11} & t_{12} & t_{13} & d_x \\ t_{21} & t_{22} & t_{23} & d_y \\ t_{31} & t_{32} & t_{33} & d_z \\ 0 & 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ 1 \end{pmatrix}$$

38

RIGID BODY TRANSFORMS

- Rigid body transform preserves volume and angles.
- Rotation
- Translation

39

ROTATION MATRIX

$$\mathbf{R}_x(\theta) = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos \theta & -\sin \theta & 0 \\ 0 & \sin \theta & \cos \theta & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

40

DISPLACEMENT MATRIX

$$D(p, q, r) = \begin{pmatrix} 1 & 0 & 0 & p \\ 0 & 1 & 0 & q \\ 0 & 0 & 1 & r \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

41

EULER ANGLES

Any arbitrary 3D rotation can be decomposed into three rotations about the z, x and z (again) axes.

$$R(\phi, \theta, \psi) = R_z(\psi)R_x(\theta)R_z(\phi)$$

x-convention described here. There are several competing conventions (eg Landau and Lifschitz 1976, Goldstein 1960 and Tuma 1974).

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ISOTROPIC SCALING

$$S(a) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & a & 0 & 0 \\ 0 & 0 & a & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

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ANISOTROPIC SCALING

$$S(a, b, c) = \begin{pmatrix} a & 0 & 0 & 0 \\ 0 & b & 0 & 0 \\ 0 & 0 & c & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

44

SHEAR TRANSFORM

$$E(\sigma_{xy}, \sigma_{xz}, \sigma_{yz}) = \begin{pmatrix} 1 & \sigma_{xy} & \sigma_{xz} & 0 \\ 0 & 1 & \sigma_{yz} & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

45

AFFINE TRANSFORM

An affine transformation combines rotation, scaling, shear and displacement.

$$T = D E S R = \begin{pmatrix} a_{11} & a_{12} & a_{13} & p \\ a_{21} & a_{22} & a_{23} & q \\ a_{31} & a_{32} & a_{33} & r \\ 0 & 0 & 0 & 1 \end{pmatrix}$$

12 independent parameters.
Straight lines and distance ratios are preserved.

46

NON-LINEAR DEFORMATION

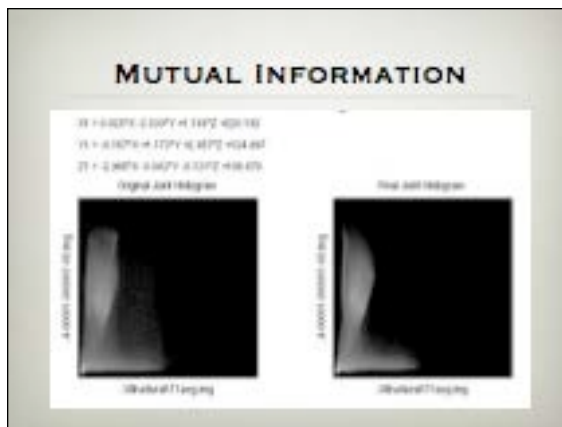


47

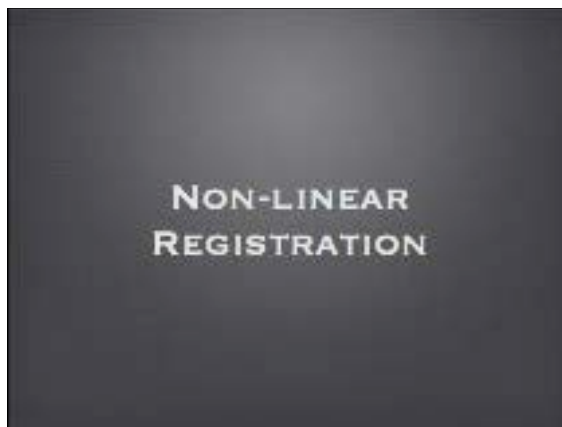
REGISTRATION COST METRICS

- Least-squares difference
- Scaled least-squares
- Correlation
- Mutual Information
- Normalized Mutual Information

48



49



50

NON-LINEAR COREGISTRATION

- Inter-subject registration
- Invoked when affine transformation is insufficiently accurate
- Typically extends an affine transformation with a spatially variant but regularized displacement in each dimension
- Registration to normalized brain spaces

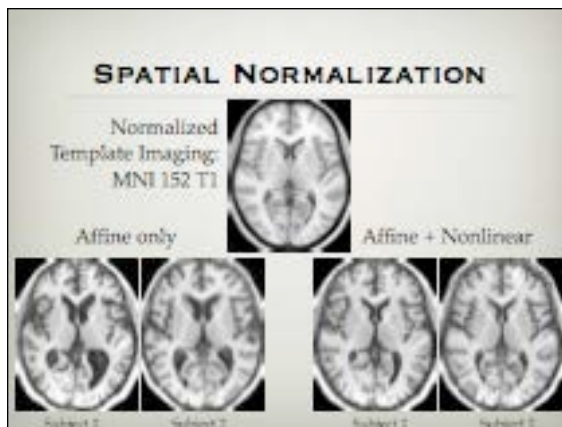
51

NORMALIZED BRAIN SPACES

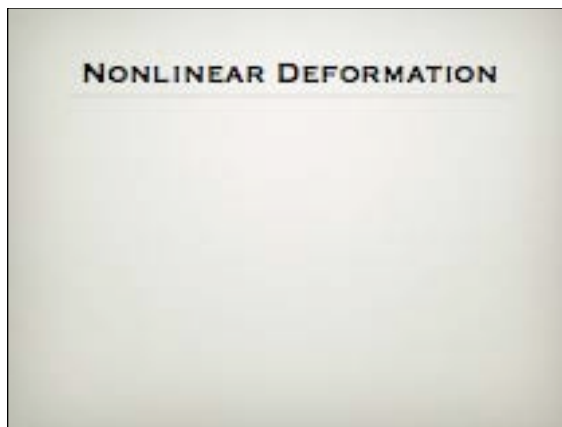
<http://www.bic.mcgill.ca/brain/spaces/>

	original (L)	original (R)	normalized (R)
MNI 305			
ICBM 152			
PED 175			
COLIN 27			

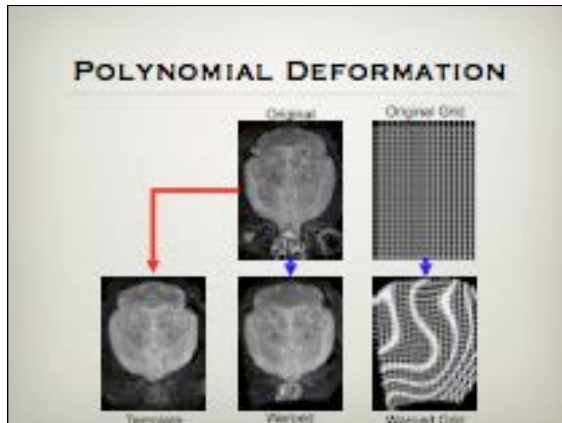
52



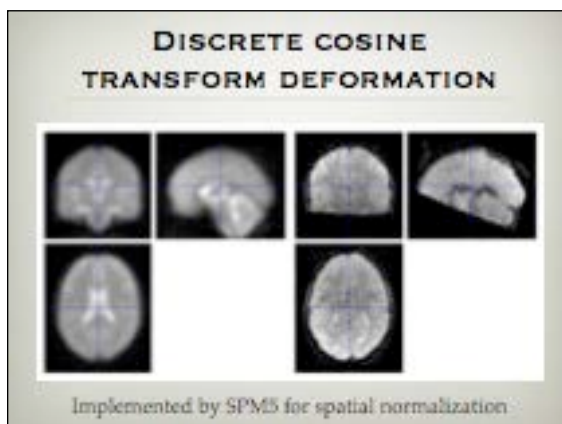
53



54



55



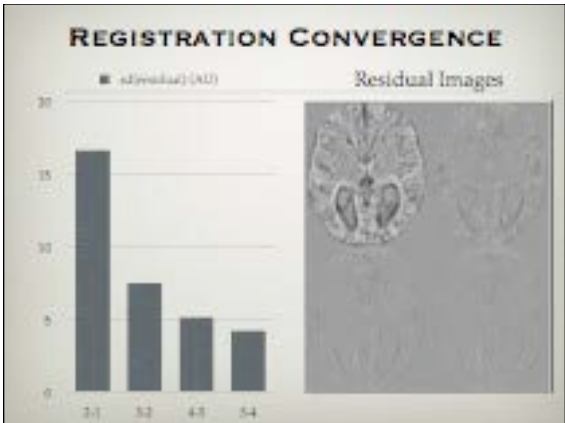
56

MINIMUM DEFORMATION AVERAGE

Pass 1: Initial affine coregistration to MNI template

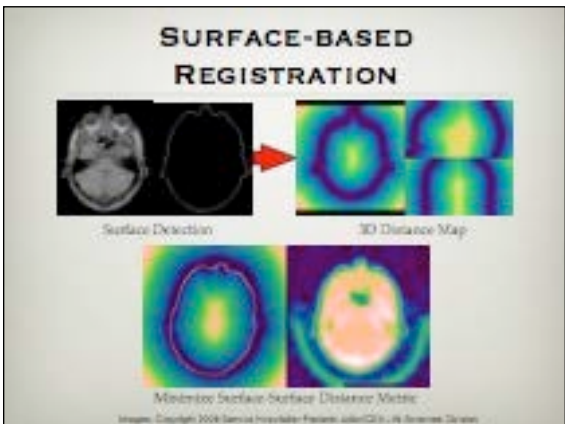
Mean Image Individual subjects at pass 5

The image displays two columns of brain MRI slices. The left column, labeled 'Mean Image', shows three axial slices of a brain, numbered 1, 2, and 3. The right column, labeled 'Individual subjects at pass 5', shows a 3x3 grid of brain MRI slices, representing individual subjects. The slices are arranged in a grid, with the first row containing three slices, the second row containing three slices, and the third row containing three slices. The slices are labeled with numbers 1 through 9, indicating the order of subjects.



GROUP MEDIAN MDT FOR AGCC

The figure displays three brain MRI slices illustrating the group median MDT for AGCC. The top-left slice is an axial view, the top-right is a sagittal view, and the bottom-left is a coronal view. Each slice shows a blue line representing the median MDT across the brain. The bottom-right position in the 2x2 grid is empty.



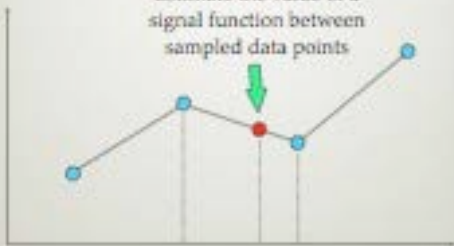


SPATIAL INTERPOLATION

61

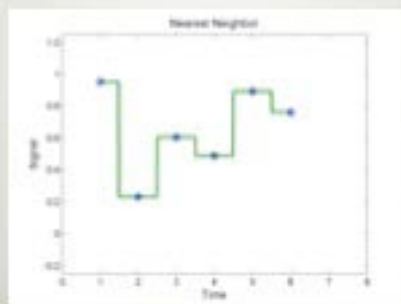
WHAT IS INTERPOLATION?

Estimate the value of a
signal function between
sampled data points



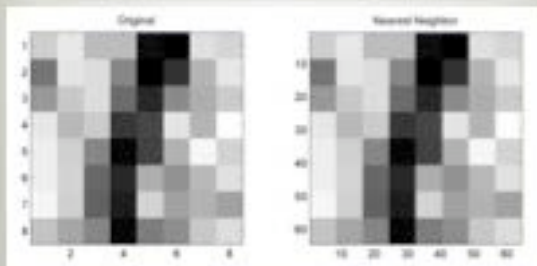
62

NEAREST-NEIGHBOR



63

NEAREST NEIGHBOR



Fastest interpolation. Preserves voxel structure of image.
Used when field is quantized (eg Atlas)

64

LINEAR INTERPOLATION

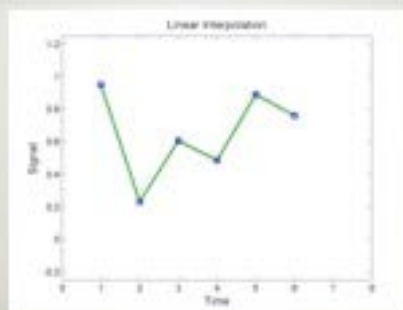
Assume signal varies linearly between samples

$$S(x) = S(x_0) + \frac{S(x_1) - S(x_0)}{x_1 - x_0}(x - x_0)$$

NOT continuous in the first derivative (C1 continuous)

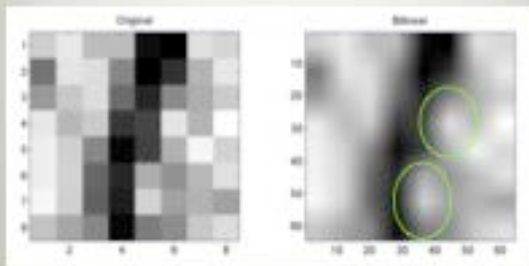
65

LINEAR INTERPOLATION



66

LINEAR INTERPOLATION



Fast interpolation commonly used for image display ("digital zoom"). "Star artifact" can be distracting at high magnification.

67

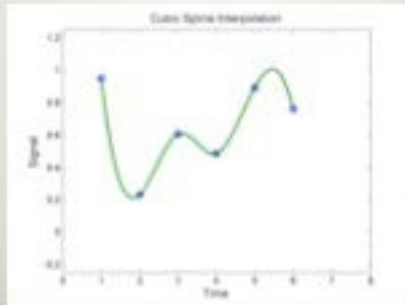
CUBIC SPLINE

Specific example of a piecewise polynomial (PP)



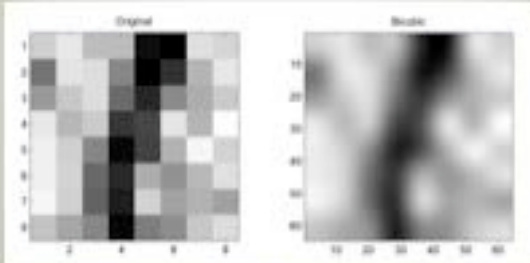
68

CUBIC SPLINE INTERPOLATION



69

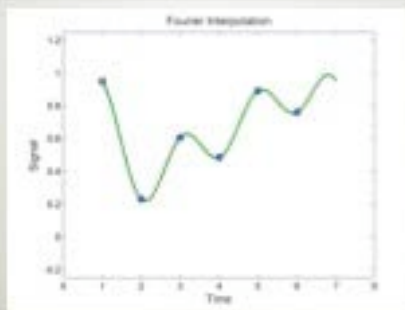
CUBIC INTERPOLATION



Smother representation of intensity without "star artifacts" evident in linear interpolation.

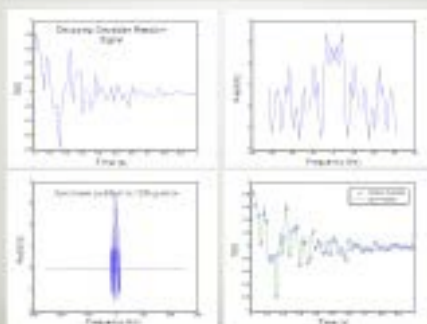
70

FOURIER INTERPOLATION



71

ZERO PADDING



72



IMAGE SEGMENTATION

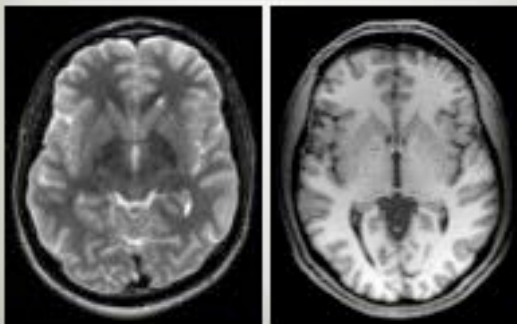
73

IMAGE SEGMENTATION

- Identification of different materials or tissues within an image.
- Approaches:
 - Manual
 - Fully-automated
 - Semi-automated

74

MRI SIGNAL DEGENERACY

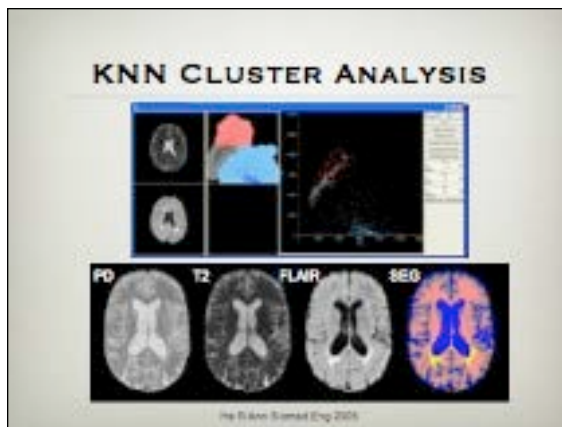


75

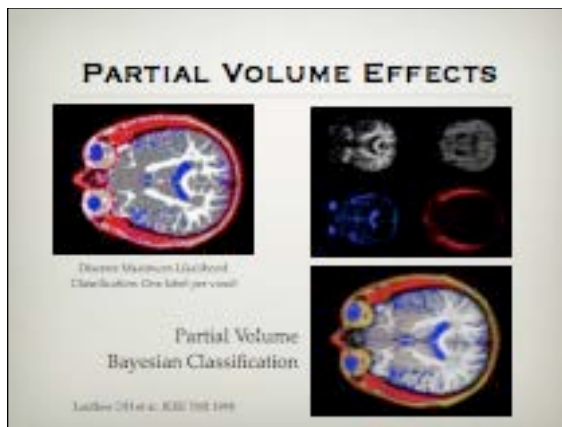
FEATURE SPACES



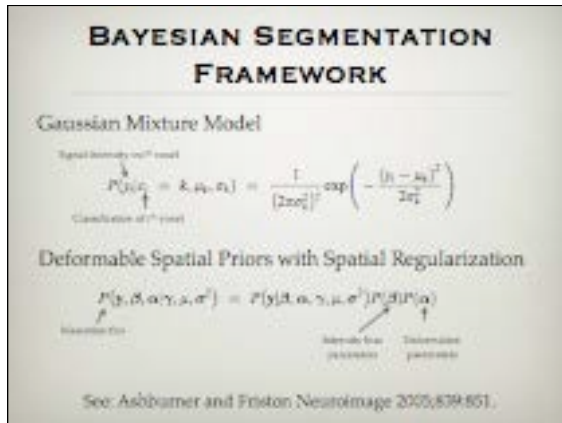
76



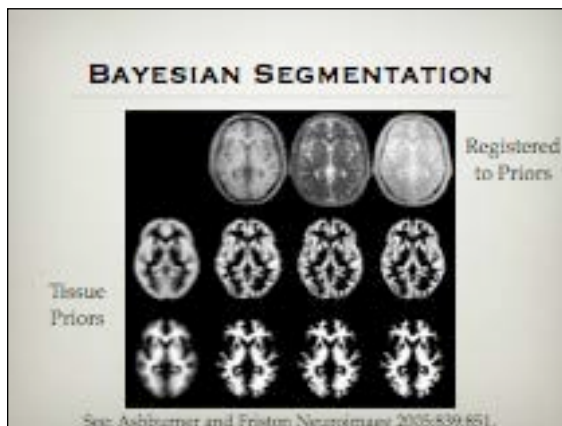
77



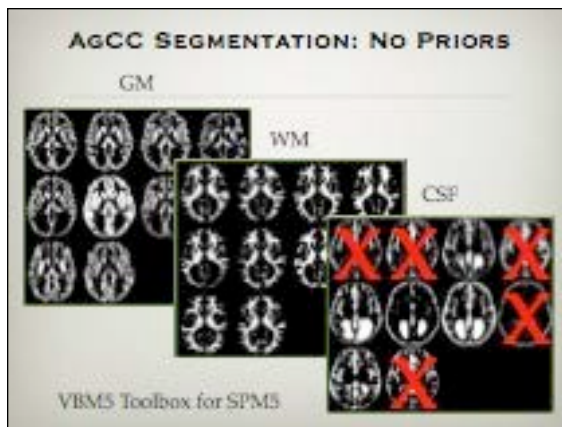
78



79



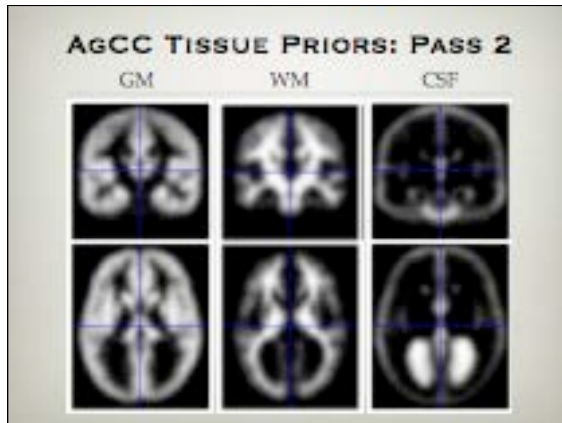
80



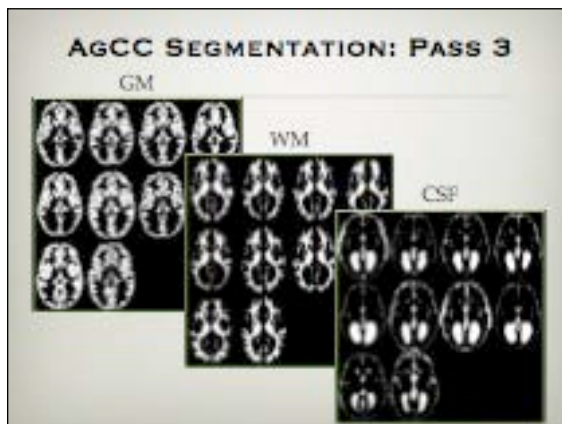
81



82

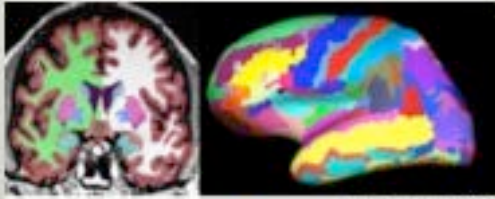


83



84

ATLAS-BASED SEGMENTATION

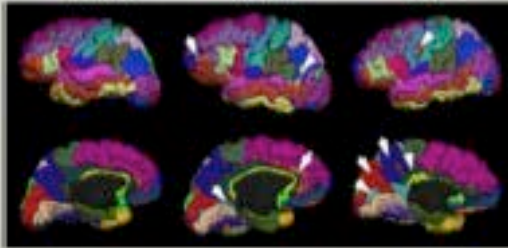


Coregister subject image with template image in standard space (MNI, Talairach).
Allow for variations when segmenting cortex.

85

CORTICAL PARCELLATION

Control Autism AgCC

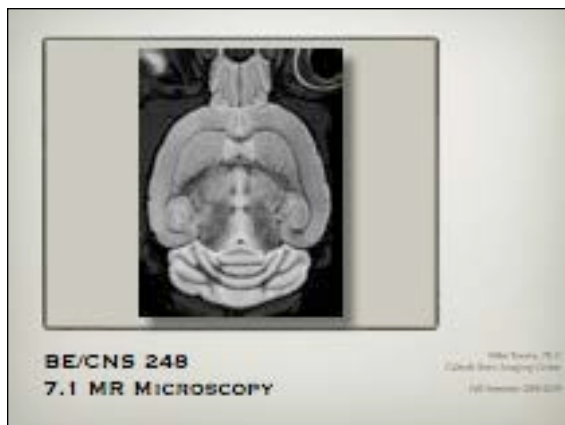


86

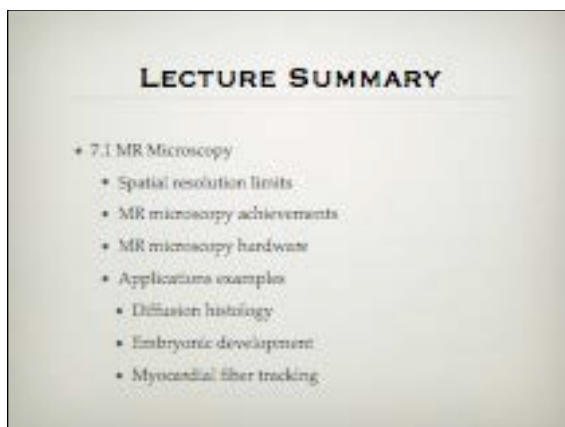
NEXT LECTURE

- 7T MRI Demonstrations
- Bring food samples
- Meet outside CBIC at 1:30pm

87



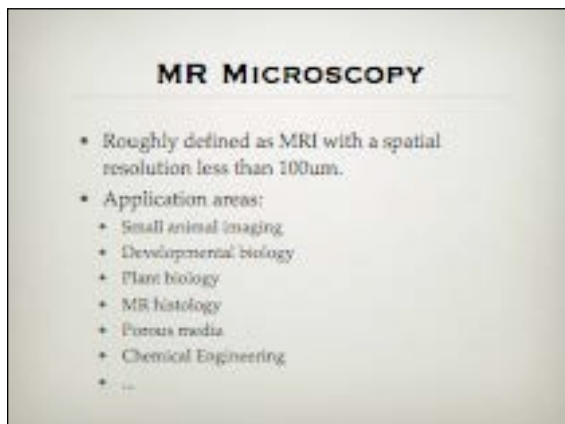
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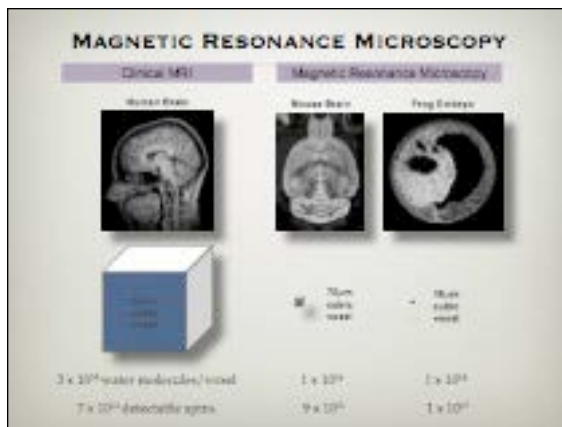
2



3



4



5

LIMITS TO MRM RESOLUTION

Fundamental	Technical
Diffusion	SNR
T2 relaxation	Total Imaging Time
Susceptibility	Hardware

6

DIFFUSION LIMITED RESOLUTION

RMS Displacement

$$\langle x \rangle = \sqrt{2Dt}$$

Diffusion Attenuation

$$S(t) = \exp(-bD)$$

$$= \exp(-\gamma^2 G^2 D t^3)$$

Diffusion Linebroadening

$$\Delta f_{diff} = 0.6 \left(\frac{1}{3} \gamma^2 G^2 D \right)^{\frac{1}{2}}$$

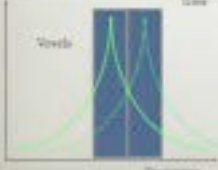
7

T₂-LIMITED RESOLUTION



Signal Window

Nat



Width

Frequency

T₂ Signal Decay

$$S(t) = \exp(-t/T_2)$$

Linewidth

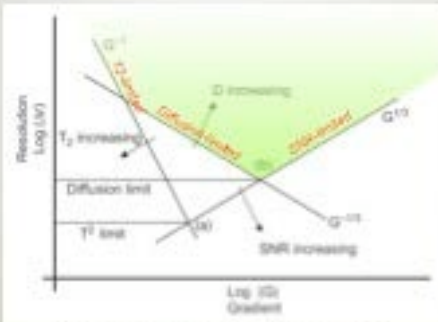
$$\Delta f = \frac{1}{\pi T_2}$$

Optimal Resolution

$$\Delta f^* = \frac{B}{N} = \frac{1}{\pi T_2}$$

8

RESOLUTION LIMITS



Modified from: Cleveland Manifesto Top. Prog. Phys. 2001; 64: 1481

9

INCREASING SIGNAL

- Higher field
- More polarization
- Greater signal induction
- Shorter T_1 (bad)
- Smaller coil-sample distances
- Hyperpolarization

10

REDUCTION OF NOISE IN MRI

- Sample or Subject Noise
 - Important at lower fields / larger subjects
 - Can't cool subject
- RF coil
 - Important at higher fields / smaller samples
 - Can cool coil
- Improve RF preamplifier
- Improve RF receiver electronics
- Minimize external noise sources (EMI)

11

MRM SNR

Signal-to-noise ratio as a function of imaging parameters

$$\frac{S}{\sigma_N} = K (\Delta x \cdot \Delta y \cdot \Delta z) \sqrt{\frac{N_{av}}{\Delta f}}$$

Total imaging time proportional to N_{av}

$$N_{av} \propto \frac{1}{\Delta x^6}$$

Total imaging time has an inverse sixth power relation to voxel dimension at constant SNR.

12

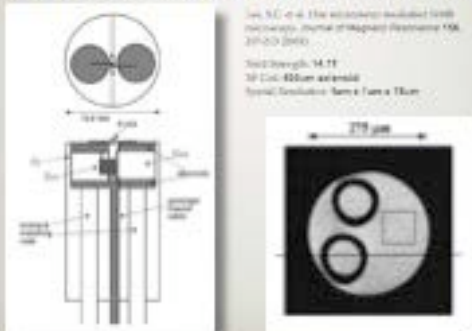
ALL MR MICROSCOPY IS DIFFUSION WEIGHTED

- At high spatial resolutions, the gradient pulses associated with image acquisition have comparable moments to diffusion-weighted pulses.
- All MR microscopy must be considered to be intrinsically diffusion weighted.
- b-matrix calculation must account for cross-terms between imaging and diffusion-weighting gradient pulses.

$$b_{ij}(T) = \gamma^2 \int_0^T dt k_i(t) k_j(t)$$

13

MRM RESOLUTION RECORDS



14

MRM RESOLUTION RECORDS



15

MRI OF NEURONS



16



MR MICROSCOPY HARDWARE

17

HIGH FIELD MAGNETS FOR MRM

9.4T 300mm



11.7T 89mm



14.1T 89mm



18

MR MICROSCOPY RF COILS

2.5cm solenoid



2.5cm Alderman-Grant



Iron surface loop



19

MICROSCOPY GRADIENTS

4T/m Cylindrical



12T/m Biplanar



2.5T/m Uniplanar



20



**MR MICROSCOPY IN
DEVELOPMENTAL BIOLOGY**

21

XENOPUS GASTRULATION TO TAILBUD



Williams & Smith / Xenopus

22

XENOPUS DEVELOPMENT IN SEALED TUBE

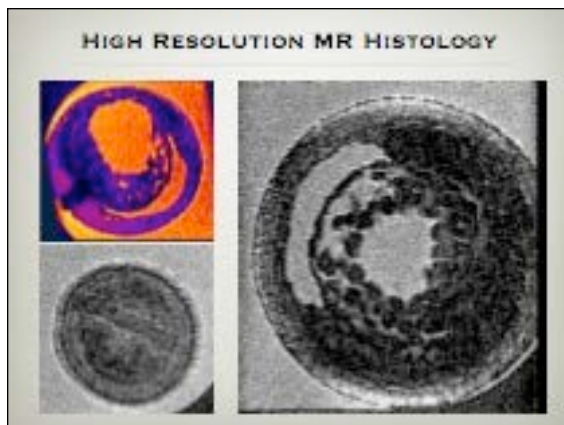


23

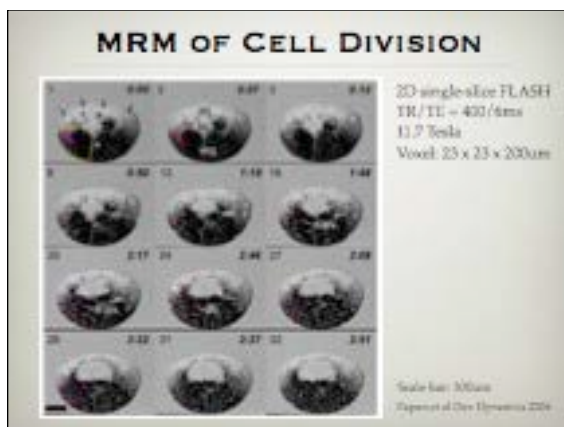
FROM GASTRULA TO TAIL-BUD STAGE



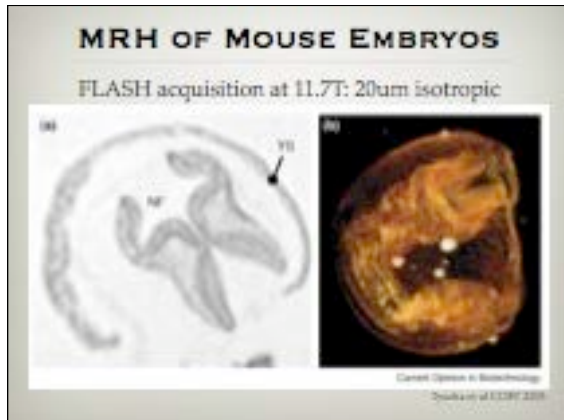
24



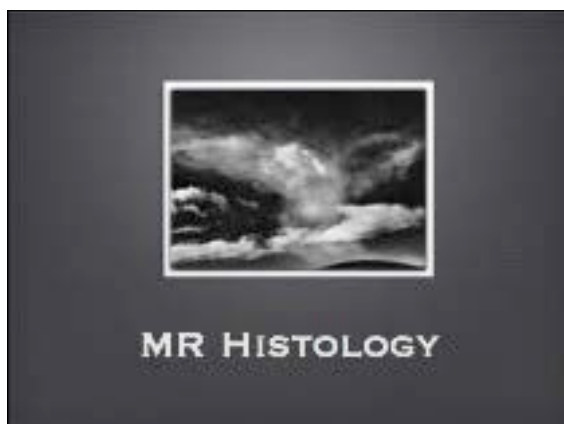
25



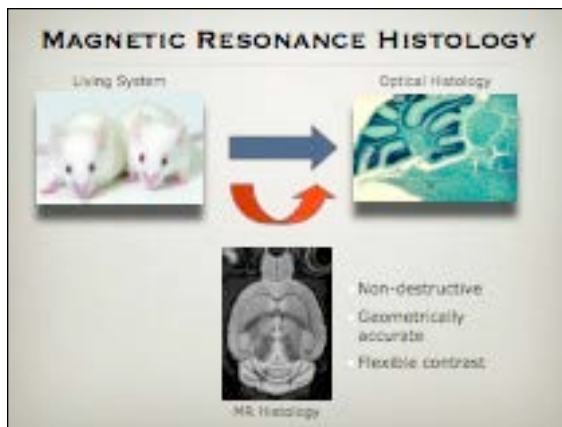
26



27



28

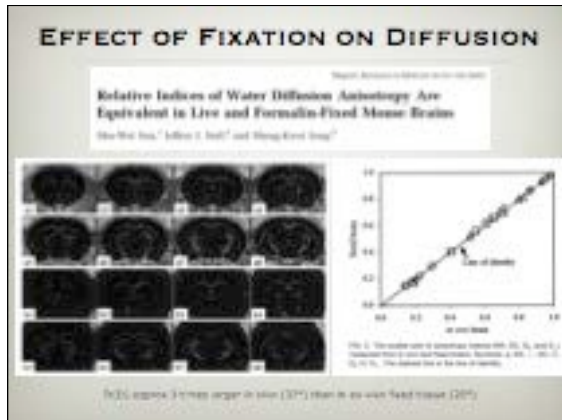


29

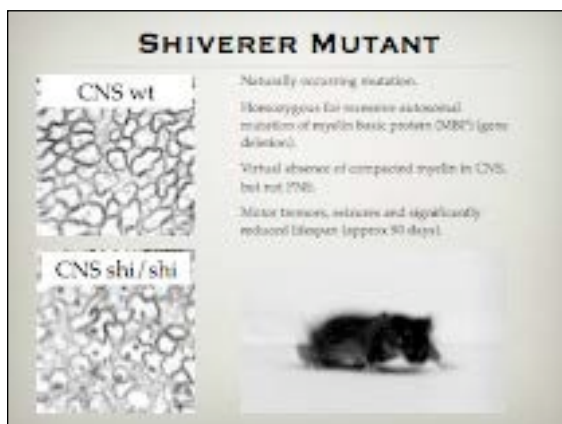
IN VIVO DWI vs DWH

	In Vivo DWI	DWH
Tissue Properties	Living systems	Fixation effects
Sensitivity	Medium-SNR	High SNR
Spatial Resolution	~ Approx 2mm in human brain	1/128-1/256 of sample dimensions
Geometric Accuracy	Partially correctable (DW-EPI)	High geometric accuracy (DWH-GS, DWH-SARSI)
Total Imaging Time	Limited by subject tolerance	Limited by handling

30



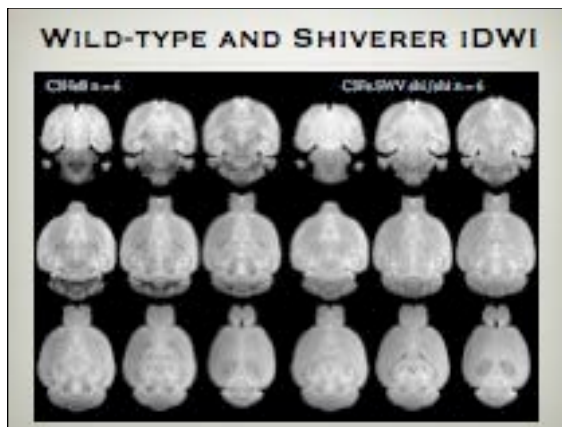
31



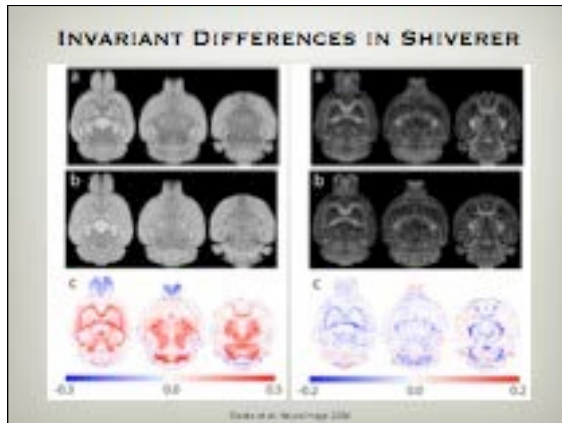
32



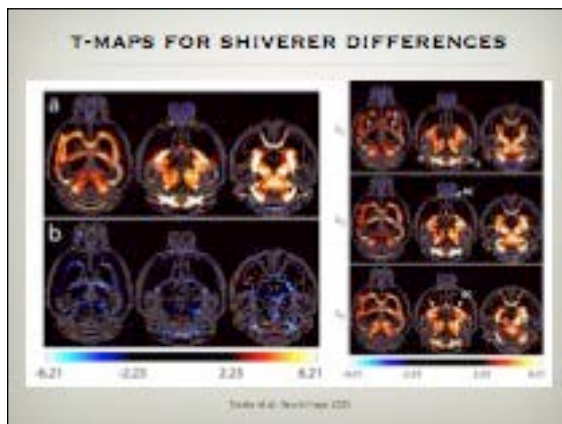
33



34



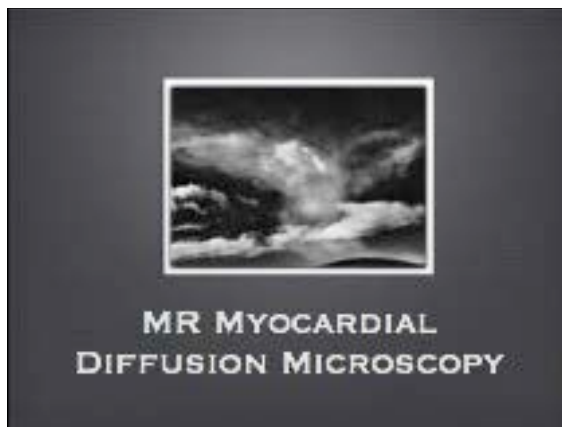
35



36



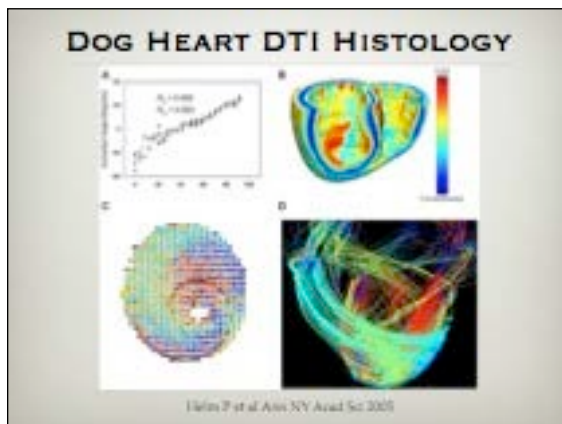
37



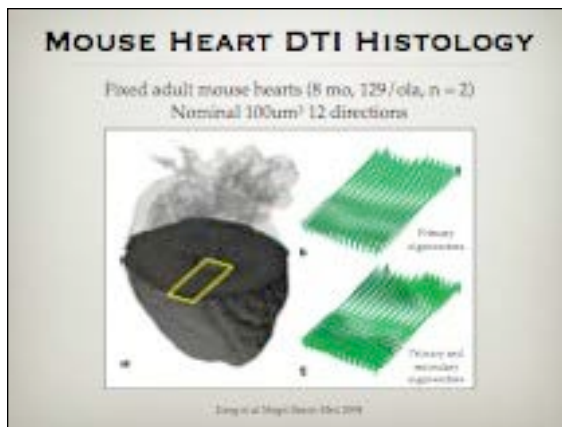
38



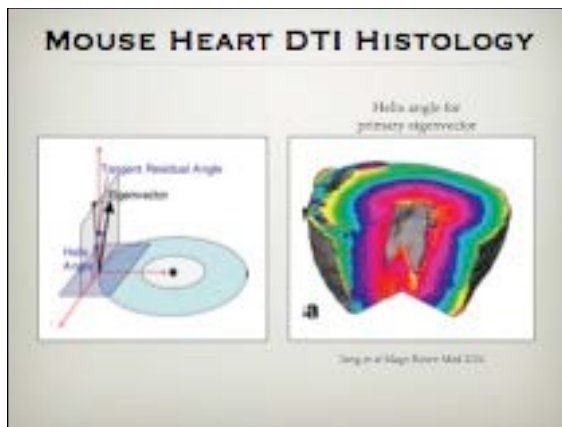
39



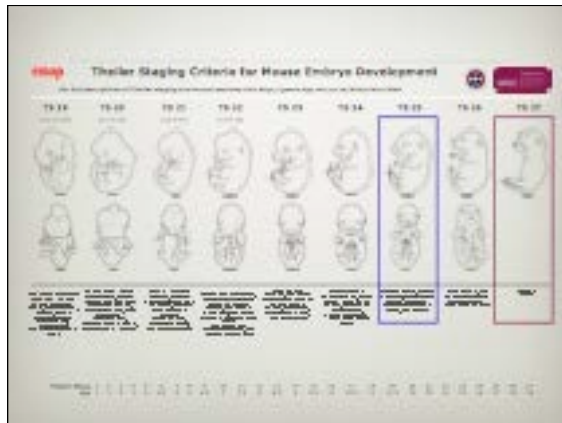
40



41



42



SAMPLE PREPARATION

- Samples equilibrated in 5mM gadoteridol in PBS and 0.01% sodium azide overnight prior to MRI.
- T1 drops to 100-200ms
- Two hearts mounted in Teflon holder immersed in Fomblin.



45

MR MICROSCOPY

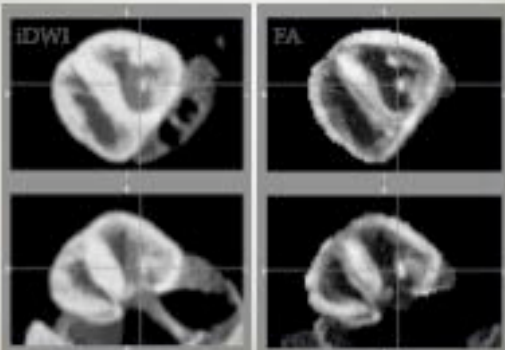
- 11.7 Tesla Vertical Bore Bruker Avance 100X 500
- Micro2.3 T1/T2, R2/T2* gradient set
- 10mm linear Shims
- PGSE imaging sequence
 - TR/TE = 100/12ms
 - 25 DWL 5 ms
 - b = 1200 s/cm²
 - Approx 22 hours total



Nonkali sampled resolution: 300µm³
0.5 voxel Gaussian Filter

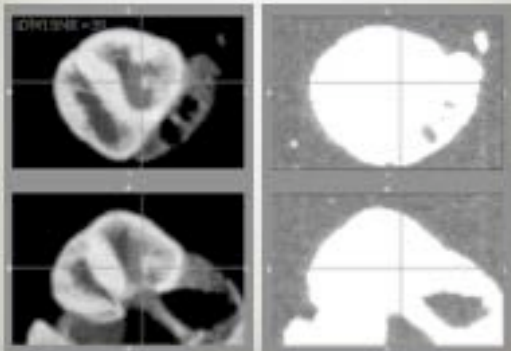
46

MOUSE EMBRYO HEART DTI

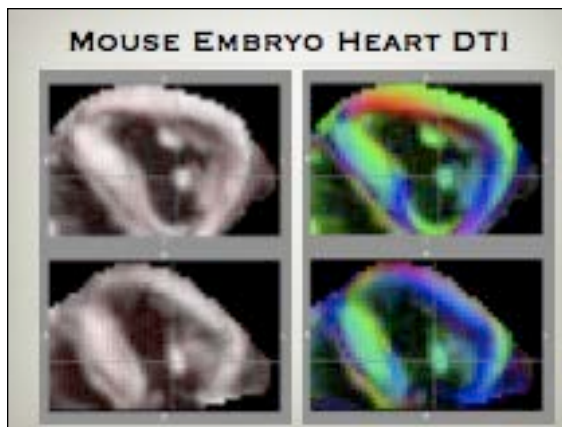


47

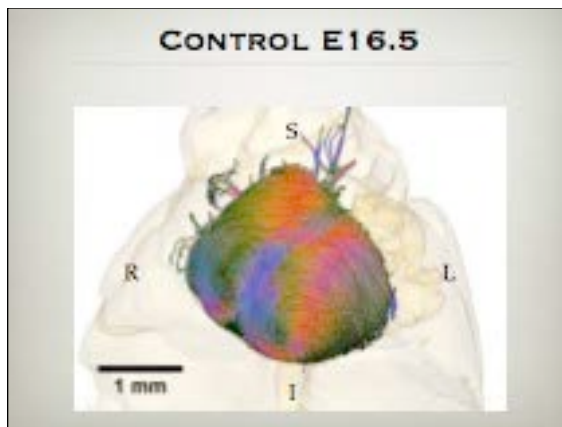
MOUSE EMBRYO HEART DTI



48



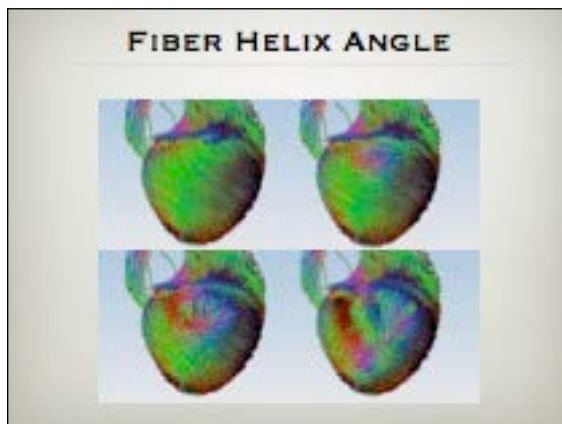
49



50

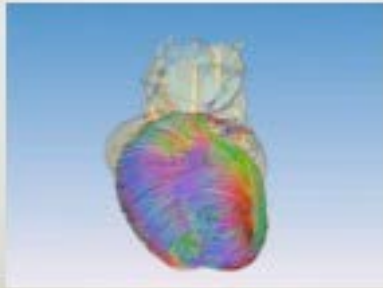


51



52

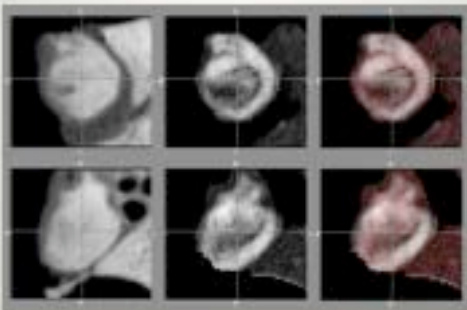
NEWBORN (E19) CONTROL



Caution: Viz LR Flipped

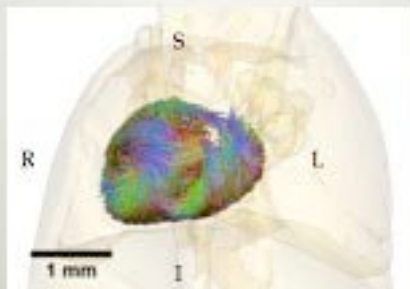
53

HETEROTAXY MUTANT E16.5



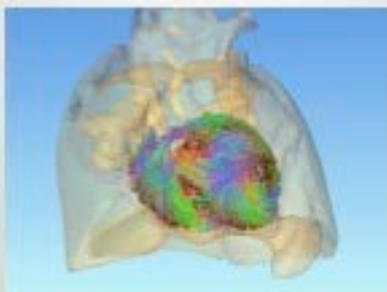
54

HETEROTAXY MUTANT E16.5



55

HETEROTAXY MUTANT



Caution: Viz LR Flipped

56



VEGETABLES

57

MRM OF PLANTS

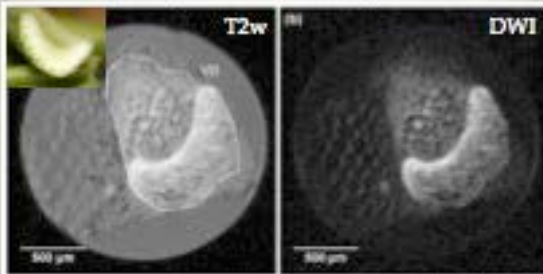


Cerebellum leaf stem at 14.1T (3x3x30um) Lee et al (MR 2009) 150:327

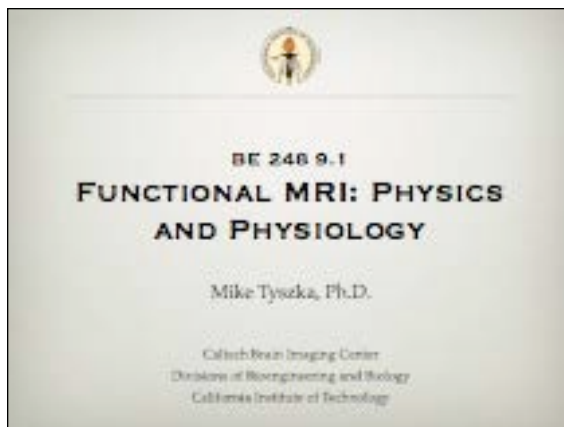
58

DIFFUSION IN PLANT CELLS

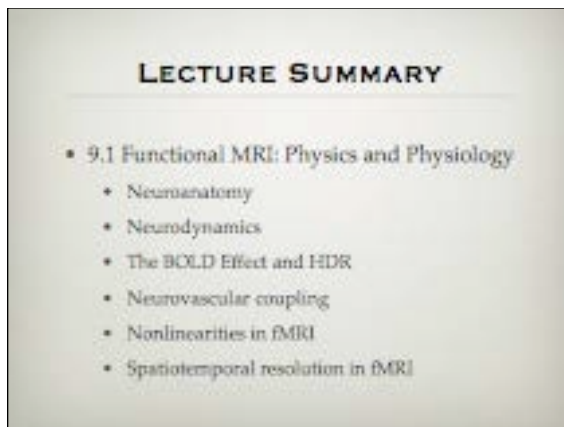
Celery vascular bundle explant



59



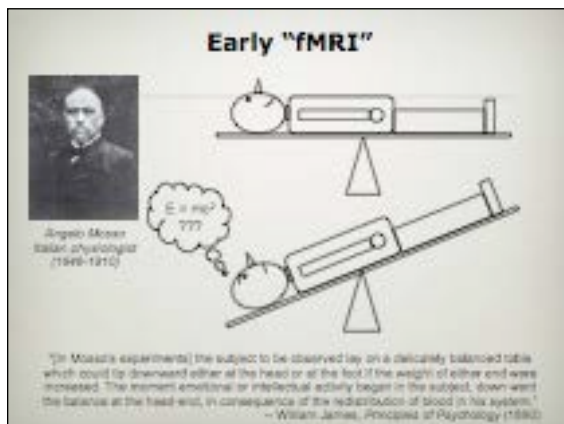
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2



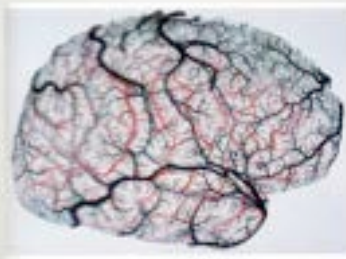
3



4

NEUROVASCULAR ANATOMY

5



from Duvernoy et al., Brain Res Bull 1981

6

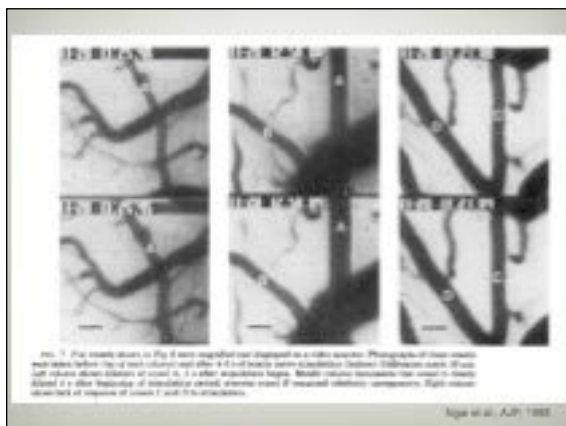
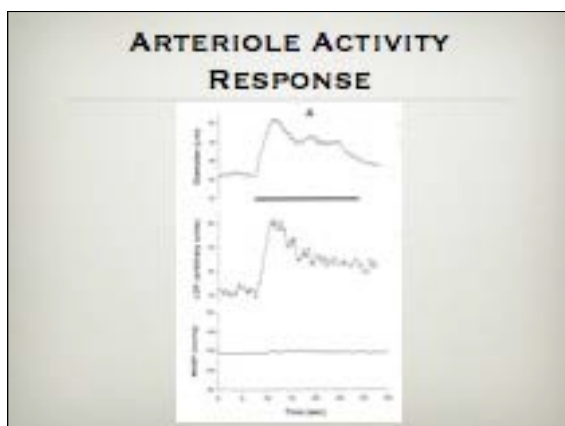
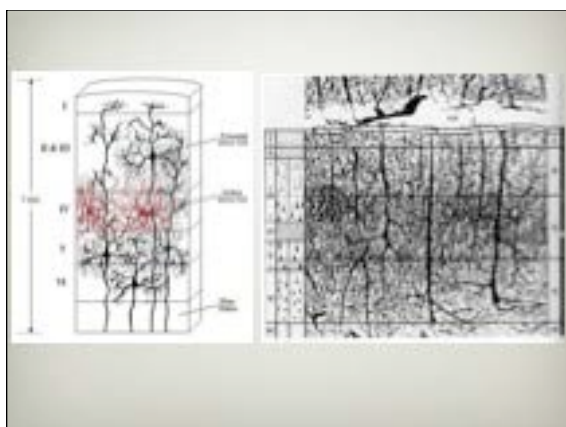
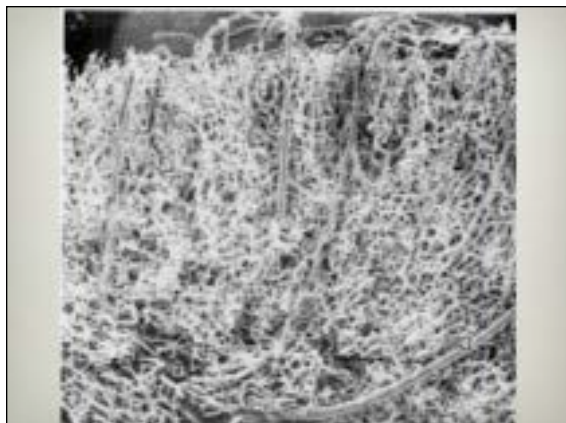


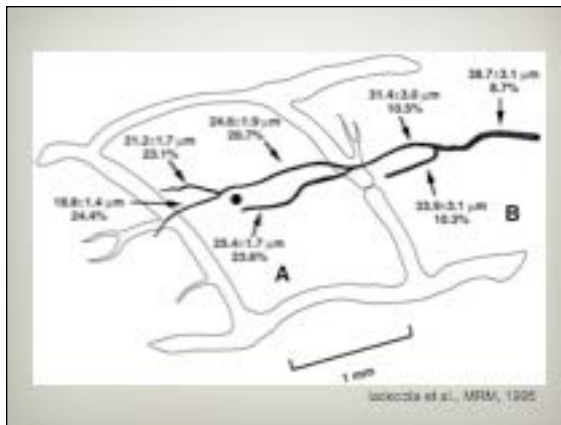
Pial Veins

7



8





13



14

OGAWA ET AL., 1990A

- Subjects: 1) Mice and Rats, 2) Test tubes
- Equipment: High-field MR (7+ T)
- Results 1:
 - Contrast on gradient-echo images influenced by proportion of oxygen in breathing gas
 - Increasing oxygen content → reduced contrast
 - No vascular contrast seen on spin-echo images
- Results 2:
 - Oxygenated blood in tube leads to little signal change, either on spin- or gradient-echo images
 - Deoxygenated blood leads to large susceptibility effects on gradient-echo images

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OGAWA ET AL. 1990

100% O₂

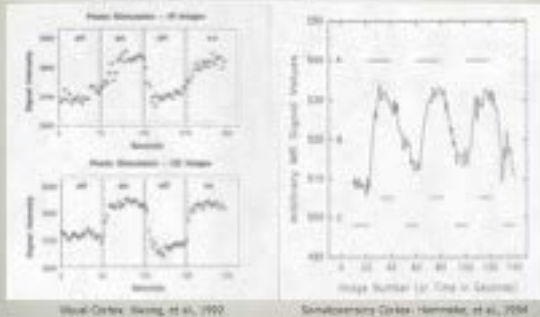
HYPEROXIA
Under anesthesia, rats breathing pure oxygen have some BOLD contrast (black lines).

90% O₂, 10% CO₂

HYPERCAPNIA
Breathing a mix including CO₂ results in increased blood flow, in turn increasing blood oxygenation. There is no increased metabolic load (no task). Therefore, BOLD contrast is reduced.

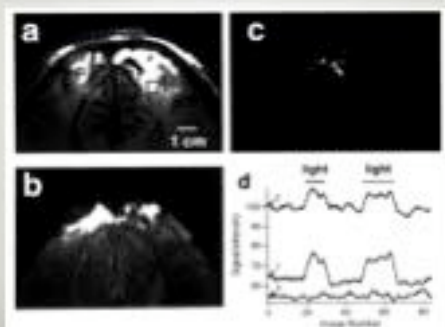
16

USING MRI TO STUDY BRAIN FUNCTION



17

OGAWA ET AL., 1992

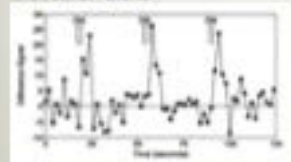


18

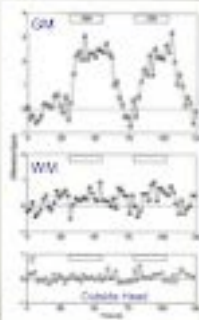
BLAMIRE ET AL., 1992

This was the first event-related fMRI study. It used both blocks and pulses of visual stimulation.

Short Stimuli - Events



Long stimuli - Blocks



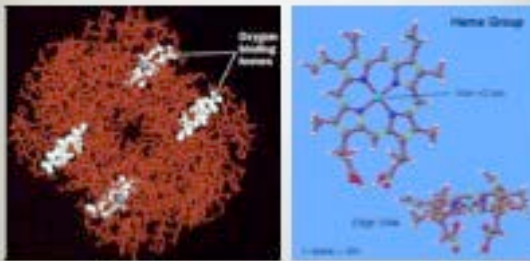
19

BOLD ENDOGENOUS CONTRAST

- + Blood Oxygenation Level Dependent Contrast
- + Deoxyhemoglobin is paramagnetic, oxyhemoglobin is less so.
- + Magnetic susceptibility of blood increases linearly with increasing oxygenation
- + Oxygen is extracted during passage through capillary bed
- + Arteries are fully oxygenated
- + Venous (and capillary) blood has increased proportion of deoxyhemoglobin
- + Difference between oxy- and deoxy- states is greater for veins
- + BOLD sensitive to venous changes

20

HEMOGLOBIN MOLECULE



280 million hb molecules per red blood cell

21

FMRI AND CALTECH

508 CHEMISTRY: PAULING AND CORRELL Phys. N. A. S.

THE MAGNETIC PROPERTIES AND STRUCTURE OF HEMOGLOBIN, OXYHEMOGLOBIN AND CARBONMONOXHEMOGLOBIN

By LINUS PAULING AND CHARLES D. CORRELL

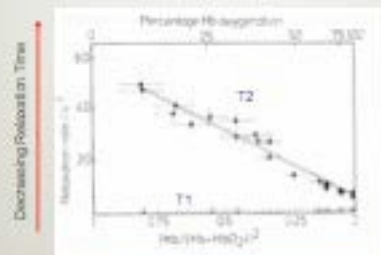
State Chemical Laboratory, California Institute of Technology

Communicated March 10, 1936

Over sixty years ago, on November 9, 1945, Michael Faraday investigated the magnetic properties of dried blood and made a note "Must try moist fluid blood." If he had determined the magnetic susceptibilities of arterial and venous blood, he would have found them to differ by a large amount (as much as twenty per cent for completely oxygenated and completely deoxygenated blood); this discovery without doubt would have excited much interest and would have influenced appreciably the course of research on blood and hemoglobin.¹

22

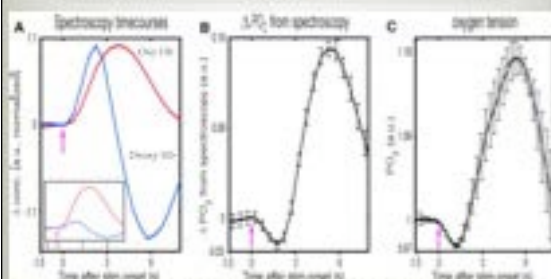
BLOOD DEOXYGENATION AND RELAXATION RATES



Thulborn et al., 1992

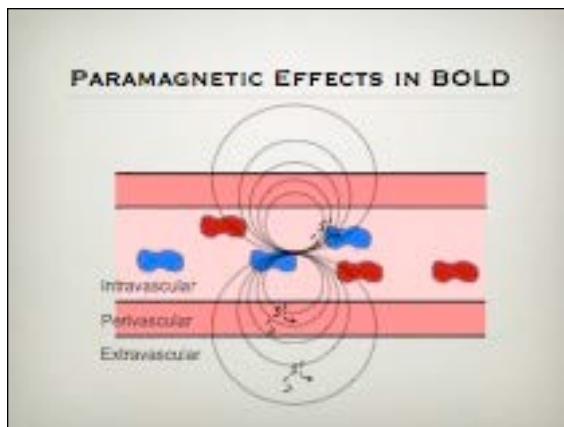
23

VENOUS O2 AND STIMULATION

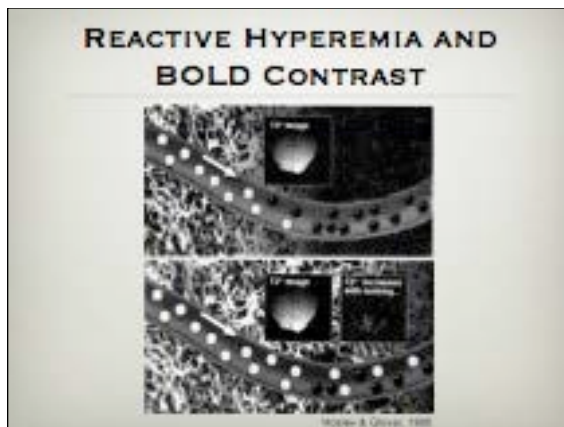


Vanzetta and Grinvald, Science, 286: 1555-1558, 1999

24



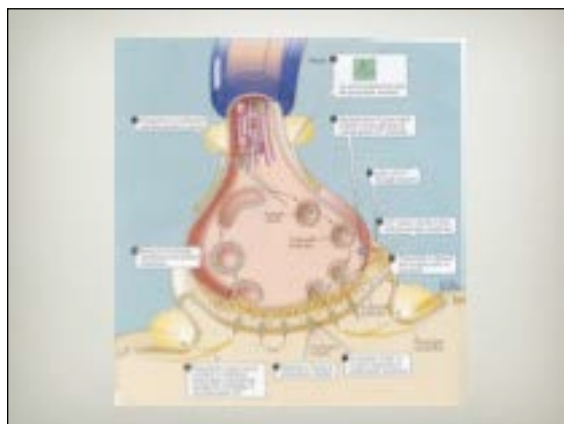
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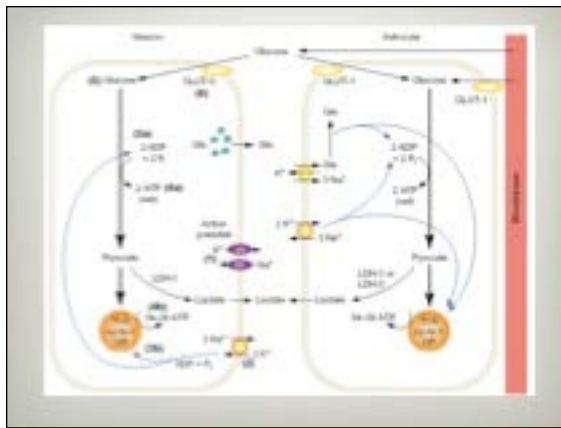
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27



28



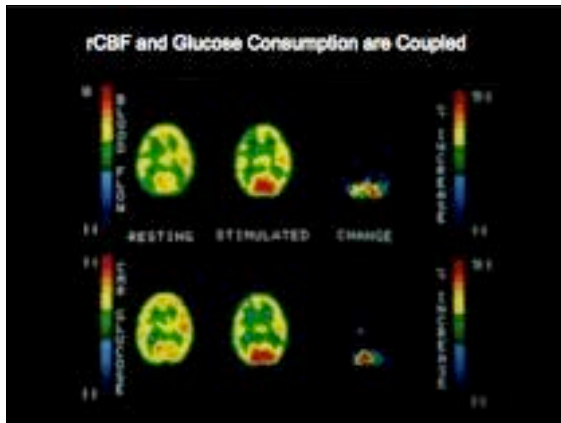
29

PET STUDIES OF BRAIN FUNCTION

Image from M. Raichle

- Injection of radioactive tracer
- Measure presence of tracer during performance of task
- Quantification of activity during single conditions
- Advantages
 - Conceptually simple
 - Allows functional measurement
- Disadvantages
 - Invasive (injected mechanism)
 - Limited repeatability
 - Very low temporal resolution

30

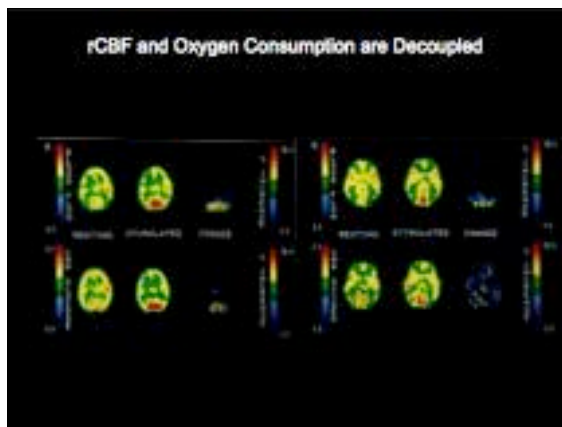


31

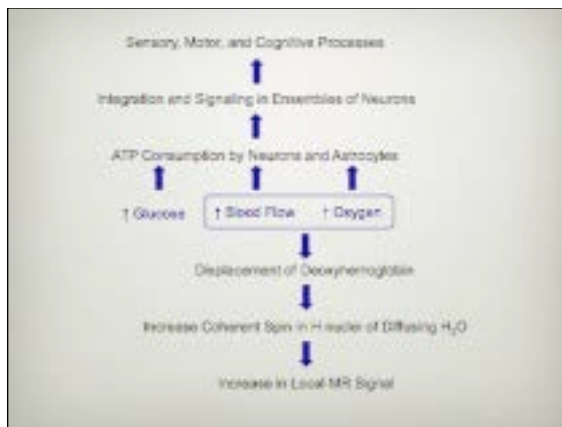
RCBF AND CMRO₂

- Cerebral Blood Flow (CBF) and Cerebral Metabolic Rate of Oxygen (CMRO₂) are coupled under baseline conditions
- PET measures CBF well, CMRO₂ poorly
- fMRI measures CMRO₂ well, CBF poorly
- CBF about 5 ml/g/min under baseline conditions
 - Increases to max of about 7-8 ml/g/min under activation conditions
- CMRO₂ only increases slightly with activation
 - Note: A large CBF change may be needed to support a small change in CMRO₂

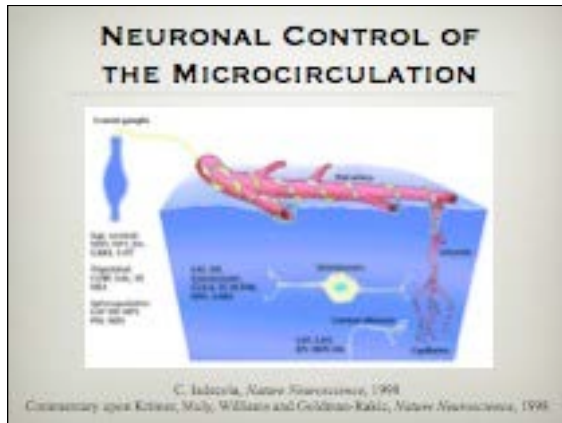
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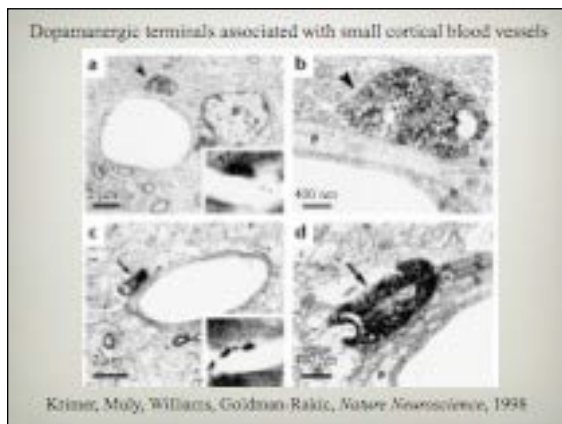
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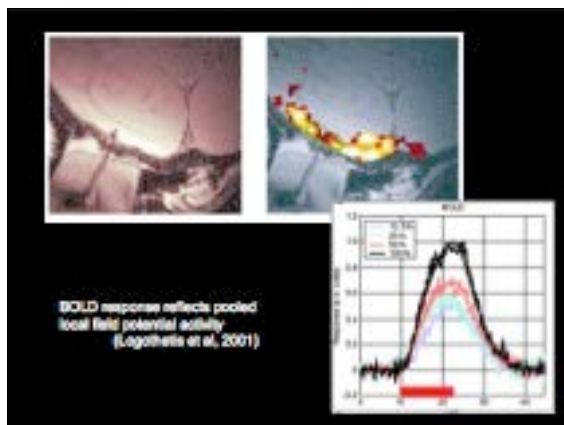
34



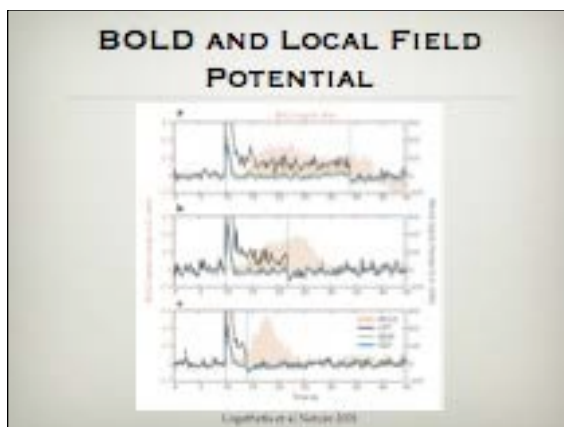
35



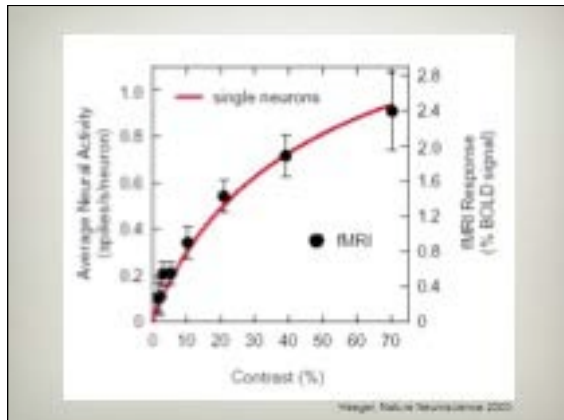
36



37



38



39

THE HEMODYNAMIC RESPONSE

40

In this period of intense research in the neurosciences, nothing is more promising than functional magnetic resonance imaging (fMRI) and positron emission tomography (PET) methods, which localize brain activities. These functional imaging methodologies map neurophysiological responses to cognitive, emotional, or sensory stimulations. The rapid experimental progress made by using these methods has encouraged widespread optimism about our ability to understand the activities of the mind on a biological basis. However, the relationship between the signal and neurobiological processes related to function is poorly understood, because the functional imaging signal is not a direct measure of neuronal processes related to information transfer, such as action potentials and neurotransmitter release. Rather, the intensity of the imaging signal is related to neurophysiological parameters of energy consumption and blood flow. To relate the imaging signal to specific neuronal processes, two relationships must be established.

The first relationship is between the intensity of the imaging signal and the rate of neurophysiological energy processes, such as the cerebral metabolic rates of glucose (CMRglc) and of oxygen (CMRO₂).

The second and previously unresolvable relationship is between the neurophysiological processes and the activity of neuronal processes. It is necessary to understand these relationships to directly relate functional imaging studies to neurobiological research that tests the relationship between the regional activity of specific neuronal processes and mental processes.

41

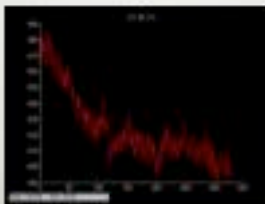
BASIC FORM OF THE HEMODYNAMIC RESPONSE



42

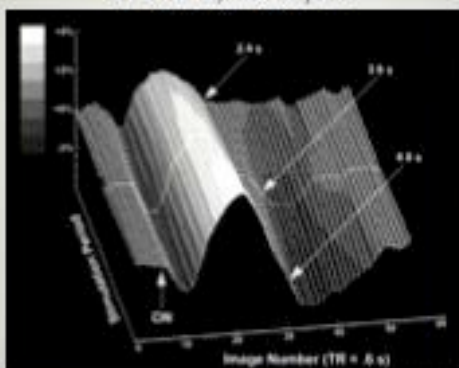
BASELINE PERIOD

- Why include a baseline period in epoch?
- Corrects for scanner drift across time



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BOLD Hemodynamic Response



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SPATIAL BOLD RESPONSE TO VISUAL STIMULUS



Hu, Lu, Ungerleider, 1997

45

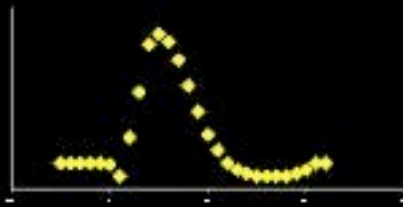
INITIAL DIP (HYPO-OXIC PHASE)

- Transient increase in oxygen consumption, before change in blood flow
- Menon et al., 1996; Hu, et al., 1997
- Shown by optical imaging studies
- Malinen & Grinvald, 1996
- Smaller amplitude than main BOLD signal
- 10% of peak amplitude (e.g., 0.1% signal change)
- Potentially more spatially specific
- Oxygen utilization may be more closely associated with neuronal activity than perfusion response

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Rise (Hyperoxic Phase)

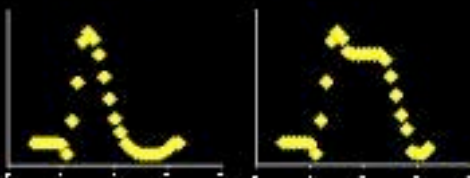
- Results from vasodilation of arterioles, resulting in a large increase in cerebral blood flow
- Inflection point can be used to index onset of processing



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Peak – Overshoot

- Over-compensatory response
 - More pronounced in BOLD signal measures than flow measures
- Overshoot found in blocked designs with extended intervals
 - Signal saturates after ~10s of stimulation



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SUSTAINED RESPONSE

- Blocked design analyses rest upon presence of sustained response
 - Comparison of sustained activity vs. baseline
 - Statistically simple, powerful
- Problems
 - Difficulty in identifying magnitude of activation
 - Little ability to describe form of hemodynamic response
 - May require detrending of raw time course

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UNDERSHOOT

- CBF more locked to stimuli than CBV
 - Increased blood volume with baseline flow leads to decrease in MR signal
- More frequently observed for longer-duration stimuli (>10s)
 - Short duration stimuli may not evidence
 - May remain for 10s of seconds

50

ISSUES IN HRF ANALYSIS

- Delay in the HRF
 - Hemodynamic activity lags neuronal activity
- Amplitude of the HRF
- Variability in the HRF
- HRF as a relative measure

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VARIABILITY IN THE HEMODYNAMIC RESPONSE

- Across Subjects
- Across Sessions in a Single Subject
- Across Brain Regions
- Across Stimuli

52

SPATIAL RESOLUTION IN FMRI

53

WHAT SPATIAL RESOLUTION DO WE WANT?

- Hemispheric
 - Lateralization studies
 - Selective attention studies
- Systems / lobes
 - Relation to lesion data
- Centimeter
 - Identification of active regions
- Millimeter
 - Topographic mapping (e.g., motor, vision)
- Sub-millimeter
 - Oscillatory Dominance Concerns
 - Cortical Layers

54

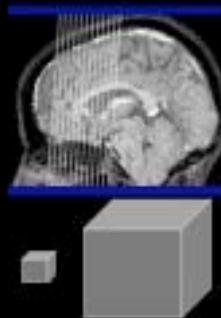
WHAT DETERMINES SPATIAL RESOLUTION?

- Voxel Size
 - In-plane Resolution
 - Slice thickness
- Spatial noise
 - Head motion
 - Artifacts
- Spatial blurring
 - Smoothing (within subject)
 - Coregistration (within subject)
 - Normalization (within subject)
 - Averaging (across subjects)

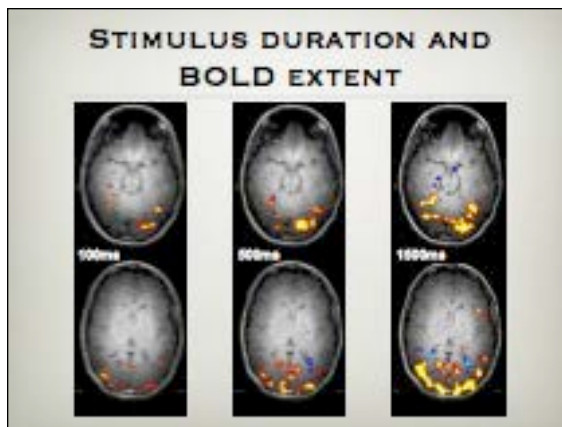
55

Costs of Increased Spatial Resolution

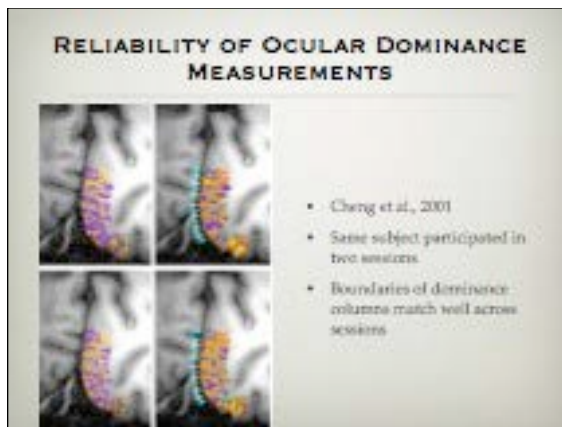
- Acquisition Time
 - In-plane
 - Higher resolution takes more time to fill k-space (resolution $\propto$ size of k-space)
 - #Slices/second
 - Sample rates for 64x64 images
 - Early Data-MV: 2-4 s/s
 - GE EPI: 12 s/s
 - Duke Spiral: 14 s/s
 - Duke Inverse Spiral: 20+ s/s
- Reduced signal per voxel
 - What is our dependent measure?



56



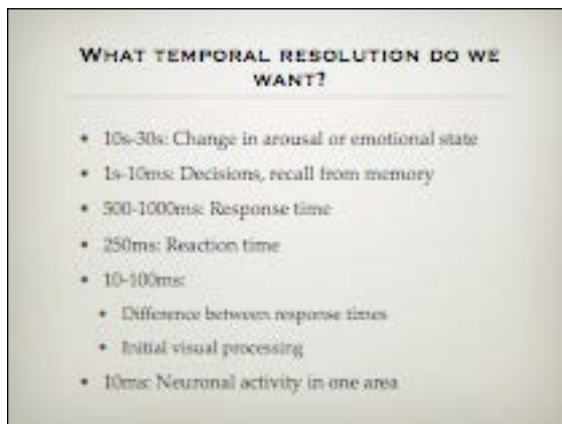
57



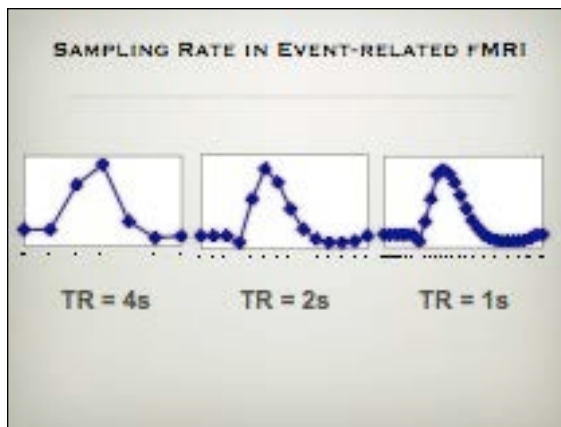
58

TEMPORAL RESOLUTION

59



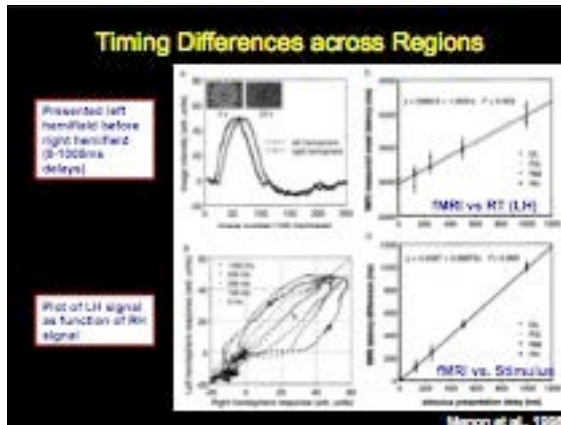
60



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- ### COSTS OF INCREASED TEMPORAL RESOLUTION
- Reduced signal amplitude
 - Shorter flip angles must be used (to allow reaching of steady state), leading to reduced signal
 - Fewer slices acquired
 - Usually, throughput expressible as slices per unit time

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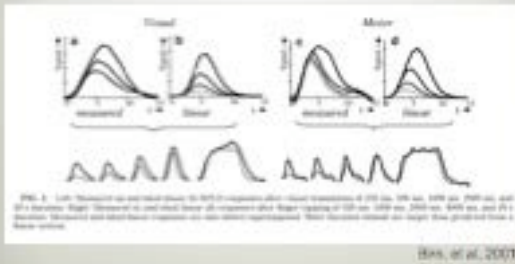


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- ### EFFECTS OF STIMULUS DURATION
- Short stimulus durations evoke BOLD responses
 - Amplitude of BOLD response often depends on duration
 - Stimuli > 100ms evoke reproducible BOLD responses
 - Form of response changes with duration
 - Latency to peak increases with increasing duration
 - Onset of rise does not change with duration
 - Rate of rise increases with duration
 - Key issue: decoupling duration of stimulus with duration of neuronal activity

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DIFFERENCES IN NONLINEARITY ACROSS BRAIN REGIONS

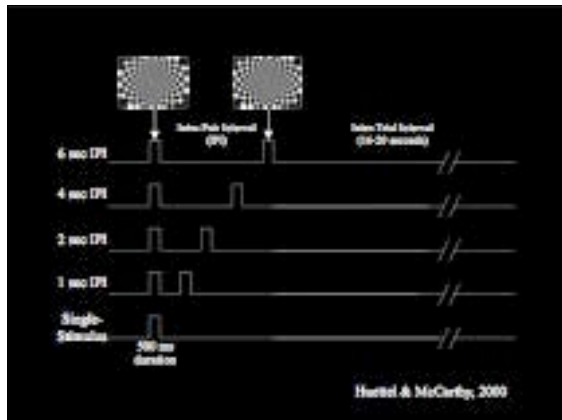


65

REFRACTORY PERIODS

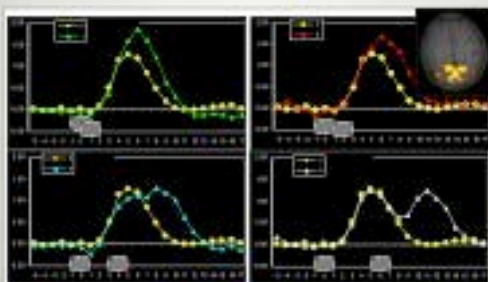
- Definition: a change in the responsiveness to an event based upon the presence or absence of a similar preceding event
- Neuronal refractory period
- Vascular refractory period

66



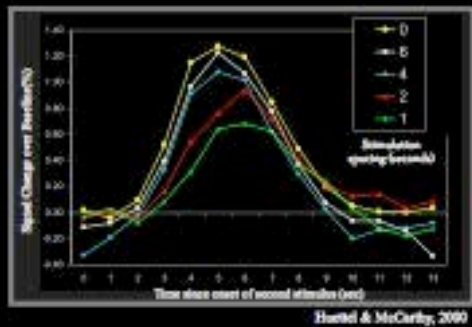
67

HEMODYNAMIC RESPONSES TO CLOSELY SPACED STIMULI



68

Refractory Effects in the fMRI Hemodynamic Response

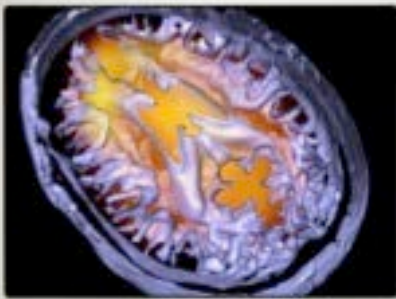


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NEXT LECTURE

- Data analysis for fMRI
 - Preprocessing
 - General Linear Model
 - Statistical Parametric Mapping

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BE/CNS 248
9.2 BASIC FMRI ANALYSIS

Nick Topley PhD
UCL Centre for Brain Imaging
10/11 November 2006-2007

1

LECTURE SUMMARY

- 9.2 Functional MRI Processing
 - BOLD data acquisition
 - Preprocessing
 - General Linear Model
 - Statistical Inference

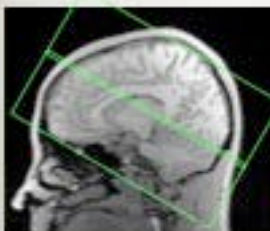
2

RESOURCES

- SPM for Dummies Course 2006
- Functional Imaging Laboratory,
University College London, UK
- fMRI Minicourse 2005
- Rik Henson, MRC Cognition and Brain
Sciences Unit, Cambridge University,
UK

3

TYPICAL BOLD IMAGING



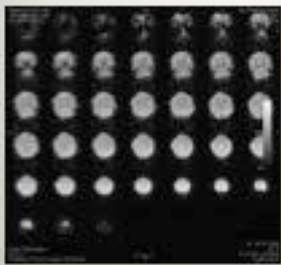
Multislice image volume
angled approximately -30°
relative to gradient XY
plane.

Between 32 and 40 3mm
slices covering whole brain.

In-plane resolution
approximately 3mm (FOV:
64 x 64cm, 64 x 64 matrix)

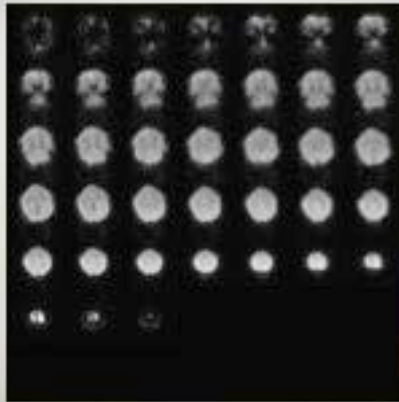
4

TYPICAL BOLD IMAGING



FID-EPI (T2*-weighted)
Single shot
TR/TE: 2000/30ms
Flip angle: 70°
Fat suppression
GRAPPA 2
1 Average

5



6

FMRI DESIGNS

7

FMRI DESIGN TYPES

- Blocked Designs
- Event-Related Designs
 - Periodic Single Trial
 - Jittered Single Trial
 - Staggered Single Trial
- Mixed Designs
 - Combination blocked/event-related
 - Variable stimulus probability

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WHAT ARE BLOCKED DESIGNS?

- Blocked designs segregate different cognitive processes into distinct time periods



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WHAT BASELINE SHOULD YOU CHOOSE?

- Task A vs. Task B
 - Example: Squeezing Right Hand vs. Left Hand
 - Allows you to distinguish differential activation between conditions
 - Does not allow identification of activity common to both tasks
 - Can control for uninteresting activity
- Task A vs. No-task
 - Example: Squeezing Right Hand vs. Rest
 - Shows you activity associated with task
 - Resting state uncontrolled

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LIMITATIONS OF BLOCKED DESIGNS

- Very sensitive to signal drift
 - Sensitive to head motion, especially when only a few blocks are used.
- Poor choice of baseline may preclude meaningful conclusions
- Many tasks cannot be conducted repeatedly
- Difficult to estimate the HDR

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WHAT ARE EVENT-RELATED DESIGNS?

- Event-related designs associate brain processes with discrete events, which may occur at any point in the scanning session.



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WHY USE EVENT-RELATED DESIGNS?

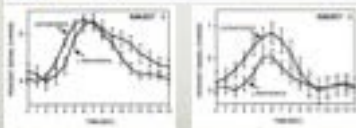
- Some experimental tasks are naturally event-related
- Allows studying of trial effects
- Simple analyses
 - Selective averaging
 - No assumptions of linearity required
- Can isolate HRFs associate with different task components

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EVENT-RELATED VS BLOCKED DESIGNS



Blocked



ER

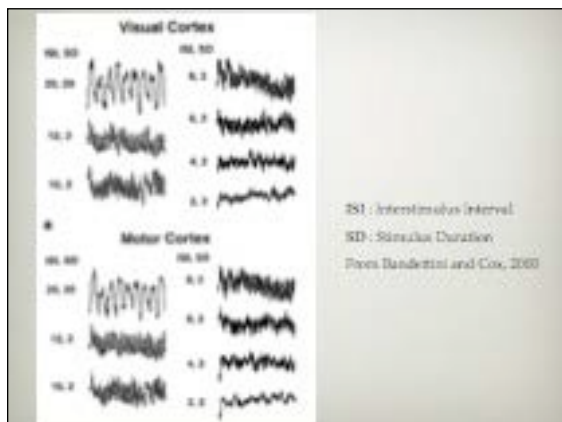
14

PERIODIC SINGLE-TRIAL DESIGN

- Stimulus events presented infrequently with a long inter-stimulus interval (ISI)



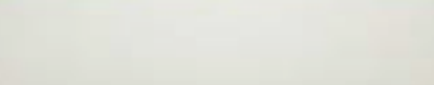
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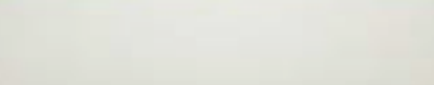


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JITTERED SINGLE-TRIAL DESIGNS

- Varying the timing of trials within a run



- ## JITTERED SINGLE-TRIAL DESIGNS
- Varying the timing of trials within a run
- 



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EXTRACTING DIFFERENT TASK COMPONENTS

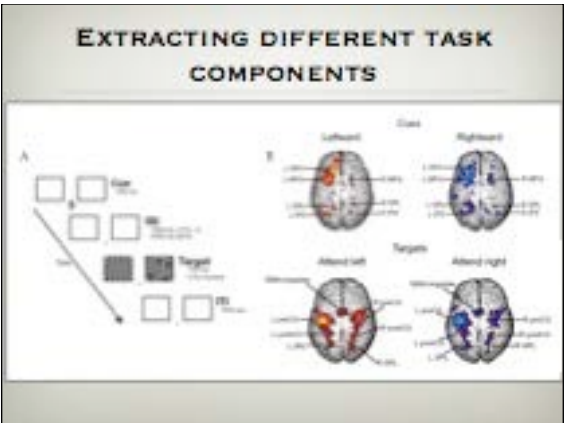
A

Easy (100% correct)
Hard (75% correct)
Target (50% correct)
No (25% correct)

B

Leftward
Rightward
Anterior left
Anterior right

LIP
LIPp
LIPm
LIPd
LIPs
LIPv
LIPh
LIPl
LIPr
LIPt
LIPb
LIPc
LIPf
LIPg
LIPj
LIPk
LIPn
LIPo
LIPp
LIPq
LIPr
LIPs
LIPt
LIPu
LIPv
LIPw
LIPx
LIPy
LIPz



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LIMITATIONS OF EVENT-RELATED DESIGNS

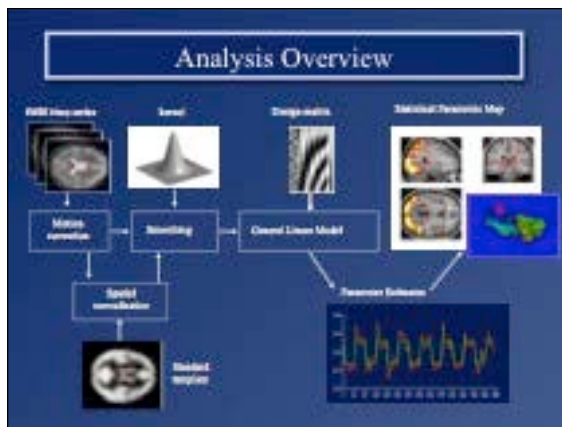
- Differential effects of ISI
 - Long intervals do not optimally increase stimulus variance
 - Short intervals may result in refractory effects
- Detection ability dependent on form of HDR
- Length of "event" may not be known

- ## LIMITATIONS OF EVENT-RELATED DESIGNS
- Differential effects of ISI
 - Long intervals do not optimally increase stimulus variance
 - Short intervals may result in refractory effects
 - Detection ability dependent on form of HDR
 - Length of "event" may not be known

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FMRI ANALYSIS

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PREPROCESSING

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THE NEED FOR PREPROCESSING

- Attempt to condition data prior to statistical inference.
- Reduce non-task related variability.
- Frequent use of prior knowledge (localization, extent, HRF, ...)

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ELEMENTS OF PREPROCESSING

- Slice Timing Correction
- Motion Correction
- Coregistration
- Normalization
- Spatial Smoothing

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SLICE TIMING CORRECTION

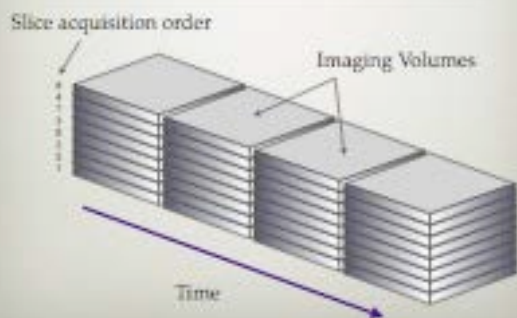
25

MOTIVATION FOR SLICE TIMING CORRECTION

- Most fMRI acquired using 2D multislice EPI
- Individual slices acquired throughout the TR
- TR typically 1-10s
- Slice timing correction attempts to interpolate signal time course at standardized timepoints

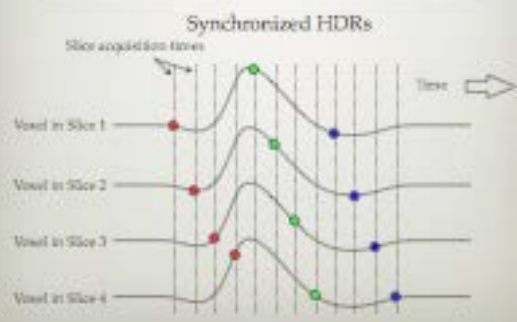
26

VOLUME TIMESERIES



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SLICE TIMING DELAY



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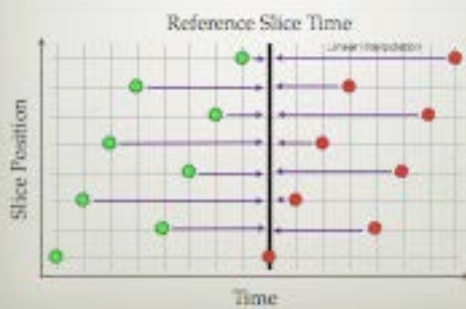
SLICE TIMING ERRORS

If simultaneous slice acquisition is assumed:



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SLICE TIMING CORRECTION



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MOTION CORRECTION

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CORRECTING HEAD MOTION

- Typically achieved using rigid body coregistration of all subsequent volumes to the first volume in the series.
- Assumes little to no geometric distortion of the head.
- SPM can combine motion correction with geometric distortion correction.

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1. Registration

- Iterative procedure (Gauss-Newton search)
- Additional scaling parameter
- New matrix of alignment parameters written to file (N is number of scans)
- Distortion matrices on * may file updated for each volume (do not have to be updated)
- Slice-timing correction can be performed before or after registration (depending on acquisition)

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2. Transformation (reslicing)

- Application of registration parameters involves **re-sampling** the image to create new voxels by interpolation from existing voxels
- Interpolation can be nearest neighbour (0-order), bi-linear (1st-order), (windowed) Fourier sinc, or in SPM2, variable **"b-splines"**

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Residual Errors after Realignment

- Interpolation errors, especially with tri-linear interpolation and small-window sinc
- fMRI (EPI):
 - Ghosts (and other artifacts) in the image (which do not move in a rigid body)
 - Rapid movements within a scan (which cause non-rigid image deformation)
 - Spin excitation history effects (residual magnetization effects of previous scans)
 - Interaction between movement and local field inhomogeneity, giving non-rigid distortion

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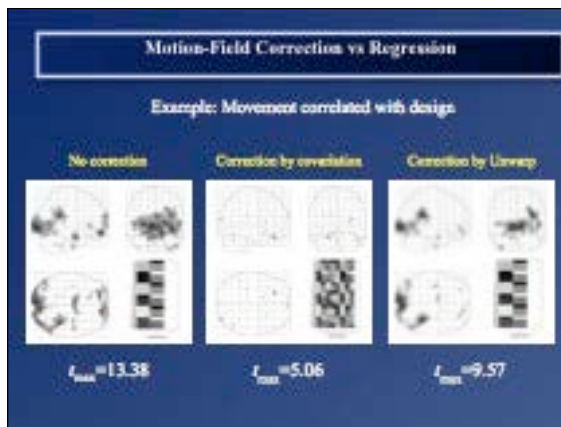
Motion-Field Interaction Model

- One could include the movement parameters as confounds in the statistical model of activations
- However, this may remove activations of interest if they are correlated with the movement
- Better to incorporate physical knowledge, eg to model how field changes as function of pitch and roll in moving phase-encoding is de-/accelerated...
- ... using Taylor expansion (about mean realigned image)

$$\Delta B_z = \partial B_z / \partial \phi \Delta \phi + \partial B_z / \partial \theta \Delta \theta$$

- Requires 15 estimate movement parameters ($\Delta \phi, \Delta \theta, 2$ estimate deformation fields, 11 to estimate movement ...)
- Fields expressed by spatial basis functions (3D discrete cosine and sine)

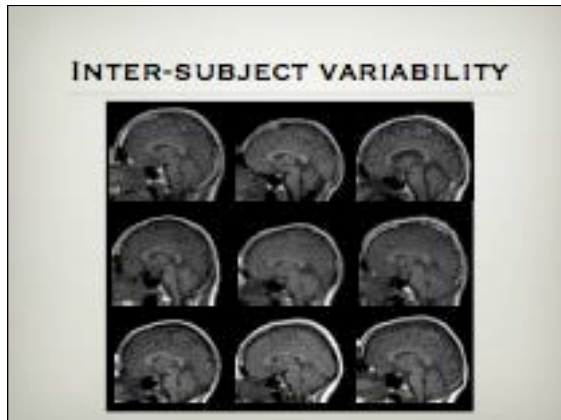
36



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SPATIAL NORMALIZATION

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- Reasons for Normalisation**
- Inter-subject averaging
 - extrapolate findings to the population as a whole
 - increase statistical power above that obtained from single subject
 - Reporting of activations in co-ordinates within a standard stereotaxic space
 - e.g. the space described by **Talairach & Tournoux**
 - **Label-based** approaches: Warp the images such that defined landmarks (points/lines/surfaces) are aligned
 - but few readily identifiable landmarks (and normally defined!)
 - **Intensity-based** approaches: Warp to images to maximise some voxel-wise similarity measure
 - eg. squared error, assessing intensity correspondence (within stability)
 - Normalization constrained to correct for only gross *differences*; residual variability accommodated by subsequent spatial smoothing

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Summary

- Describes transformation that minimizes the sum of squared difference between an image and a (combination of) template image(s)
- Two stages:
 1. **affine registration** to match size and position of the images
 2. **non-linear warping** to match the overall brain shape
- Uses a **B-spline framework** to combine affine and warps

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Stage 1. Full Affine Transformation

- The first part of normalization is a 12 parameter affine transformation
 - 3 translations
 - 3 rotations
 - 3 shears
 - 3 scales
- Beware if template image is anisotropy (eg because of image distortions in EPI functional T1)

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Stage 2. Nonlinear Warps

- Deformations consist of a linear combination of smooth basis images
- These are the lowest frequency basis images of a 3-D discrete cosine transform
- Brain masks can be applied (eg for lesions)

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STANDARDIZED SPACES

- Talairach Space (proportional grid system)
 - From atlas of Talairach and Tournoux (1988)
 - Based on single subject (60y, Female, Cadaver)
 - Single hemisphere
 - Related to Brodmann coordinates
- Montreal Neurological Institute Space (MNI) and ICBM
 - Combinations of many MRI scans on normal controls
 - All right-handed subjects
 - Approximated to Talairach space
 - Slightly larger
 - Taller from AC to top by 3mm, deeper from AC, to bottom by 3mm
- Used by SPM, National BOLD Database, International Consortium for Brain Mapping

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SHOULD YOU NORMALIZE?

• Advantages

- Allows generalization of results to larger population
- Improves comparison with other studies
- Provides coordinate space for reporting results
- Enables averaging across subjects

• Disadvantages

- Introduces spatial blurring
- May reduce activation strength by subject averaging
- Time consuming, potentially problematic
- Doing bad normalization is much worse than not normalizing

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SPATIAL SMOOTHING

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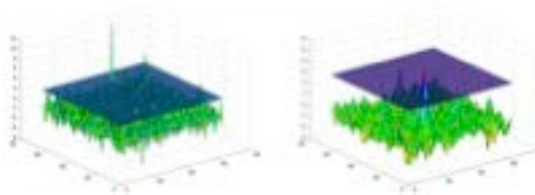
Reasons for Smoothing

- Potentially increase signal to noise (reduced filter frequency)
- Inter-subject averaging (allowing for residual differences after normalization)
- Kernel defined in terms of FWHM (full width at half maximum) of filter - usually ~16-20mm (PET) or ~4-8mm (fMRI) of a Gaussian
- Ultimate smoothness is function of applied smoothing and intrinsic image smoothness (smoothness expressed as "noise" - BLSmooth Elements)



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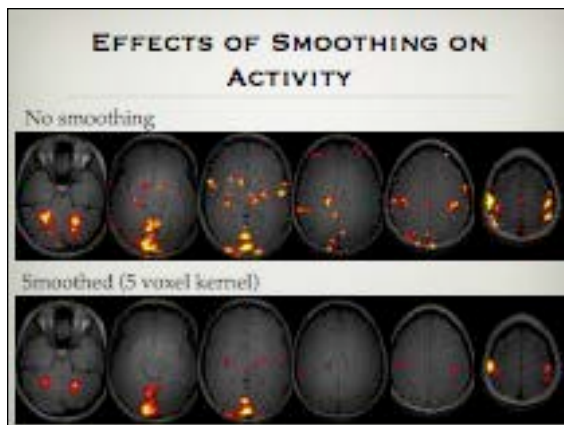
FALSE DISCOVERY AND NOISE



Unsmoothed

Gaussian Smoothed

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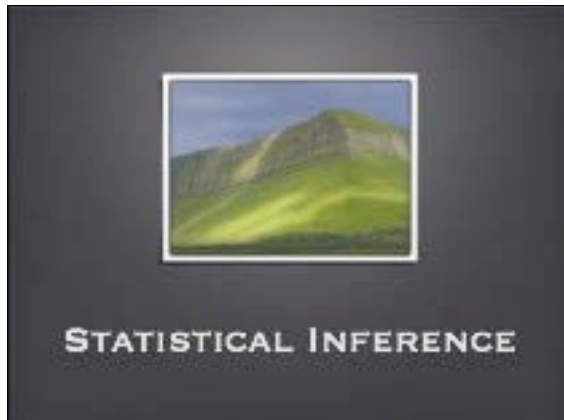


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SHOULD YOU SPATIALLY SMOOTH?

- **Advantages**
 - Increases Signal to Noise Ratio (SNR)
 - Matched Filter Theorem: Maximum increase in SNR by filter with same shape/size as signal
 - Reduces number of comparisons
 - Allows application of Gaussian Field Theory
 - May improve comparisons across subjects
 - Signal may be spread widely across cortex, due to intersubject variability
- **Disadvantages**
 - Reduces spatial resolution
 - Challenging to smooth accurately if size/shape of signal is not known

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General Linear Model

- **Parametric statistics**
 - one sample *t*-test
 - two sample *t*-test
 - paired *t*-test
 - Anova
 - AnCova
 - correlation
 - linear regression
 - multiple regression
 - *F*-tests
 - etc...

all cases of the
General Linear Model

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General Linear Model

- Equation for single (and all) voxels:

$$y_j = x_{j1} \beta_1 + \dots + x_{jp} \beta_p + \epsilon_j \quad \epsilon_j \sim N(0, \sigma^2)$$

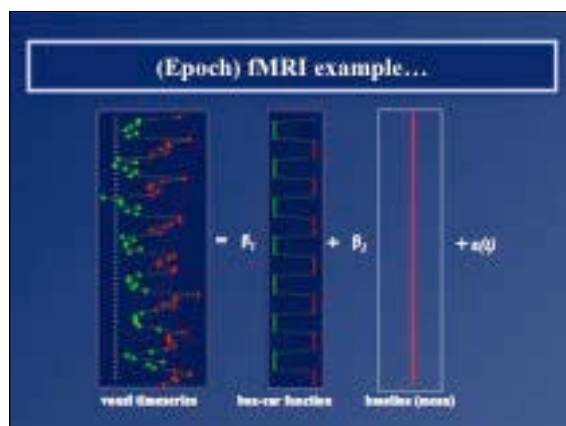
y_j : data for voxel, $j = 1 \dots N$
 x_{jp} : explanatory variables / covariates / regressors, $p = 1 \dots P$
 β_p : parameters / regression slopes / weights / fixed effects
 ϵ_j : residual errors, independent & identically (normally) distributed
- Equivalent matrix form:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\epsilon}$$

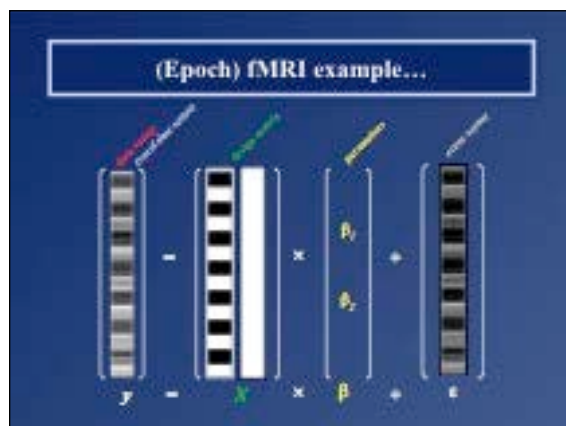
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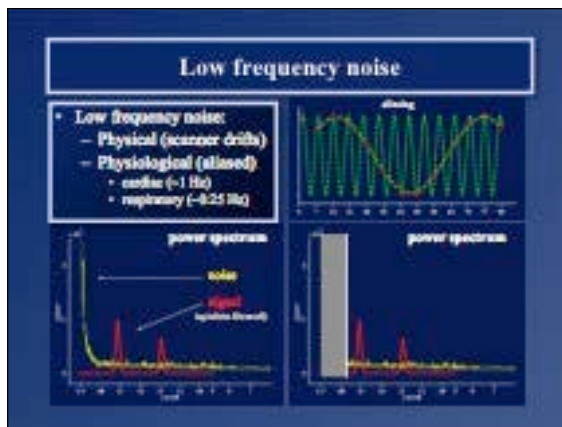
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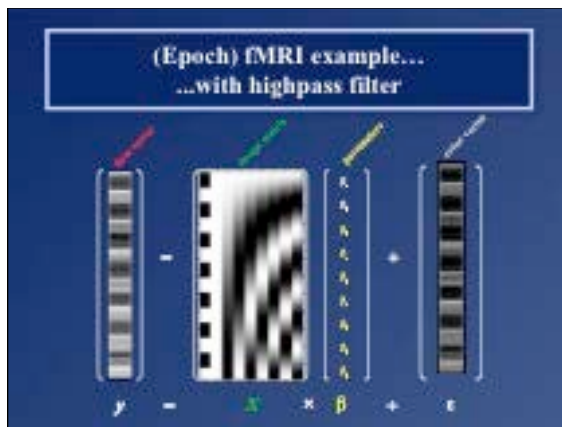
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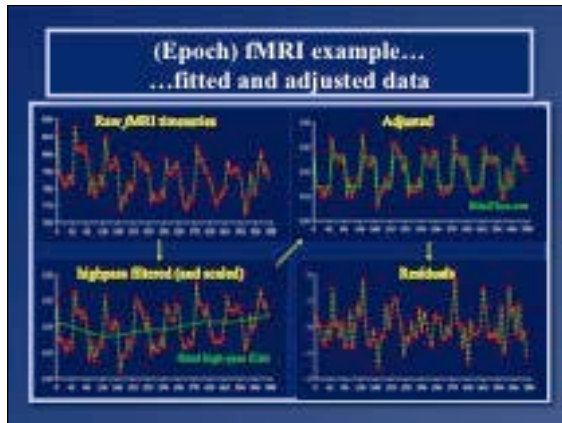
56



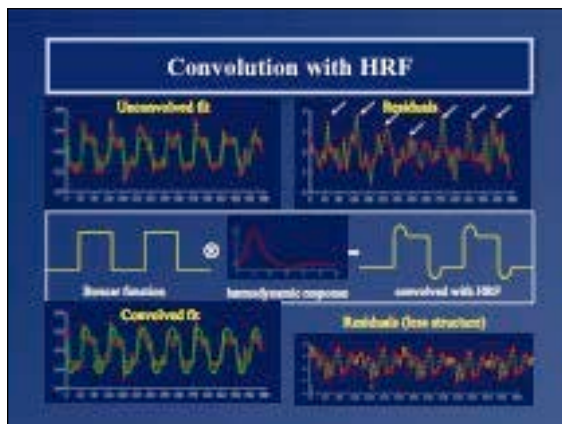
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GLM Estimation

- Estimate parameters from least squares fit to data, y :

$$\hat{\beta} = (X^T X)^{-1} X^T y \quad (\text{OLS estimate})$$
- Fitted response is:

$$\hat{Y} = X \hat{\beta}$$
- Residual errors and estimated error variance are:

$$r = y - \hat{Y} \quad \hat{\sigma}^2 = r^T r / df$$

where df are the degrees of freedom (assuming full rank):

$$df = N - \text{rank}(X) \quad (= N - P \text{ if } X \text{ full rank})$$

$$(K = I - X X^T) r = 0 \quad \hat{\sigma}^2 = \text{trace}(K) / df$$

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GLM Statistical Inference

- Specify contrast (hypothesis), c , a linear combination of parameter estimates, $c^T \hat{\beta}$

- Calculate Test statistic for this contrast:

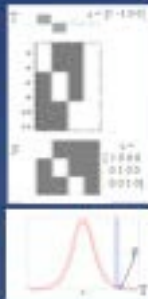
$$T_{\text{contrast}} = c^T \hat{\beta} / \sqrt{\text{var}(c^T \hat{\beta})} = c^T \hat{\beta} / \sqrt{c^T (X^T X)^{-1} c}$$

(c is a vector), or an F-statistic:

$$F_{\text{contrast}} = [(c^T \hat{\beta} - c^T \beta_0)^2 / (c^T (X^T X)^{-1} c)] / [(r^T r) / (N - P)]$$

where c and β_0 reflect the reduced model specified by c (which is a matrix)

- Probability of falsely rejecting Null hypothesis, $\beta_0: c^T \beta = 0$ ("p-value")



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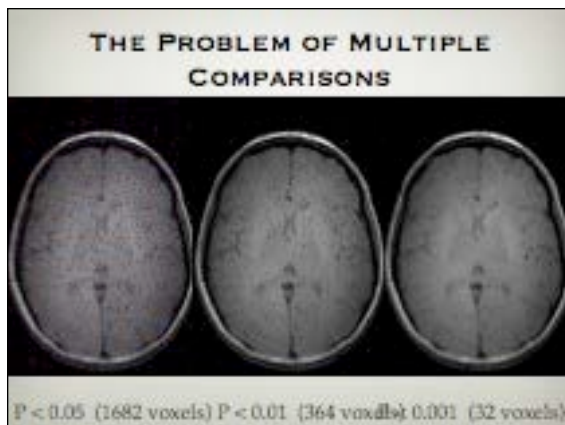
MULTIPLE COMPARISON CORRECTION

63

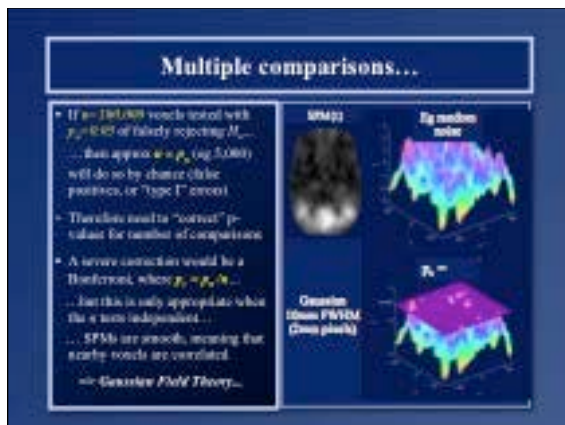
TYPES OF ERRORS

		Reality	
		H ₀ (Null) True	H ₀ (Null) False
Output of Statistical Test	Accept H ₀ (Null) True	TRUE POSITIVE	FALSE POSITIVE Type I Error
	Reject H ₀ (Null) False	FALSE NEGATIVE Type II Error	TRUE NEGATIVE

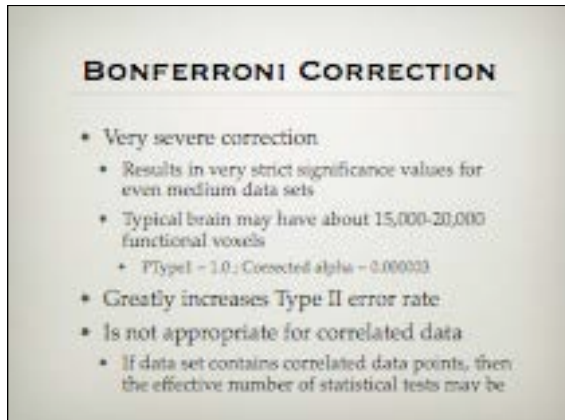
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66



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GAUSSIAN FIELD THEORY

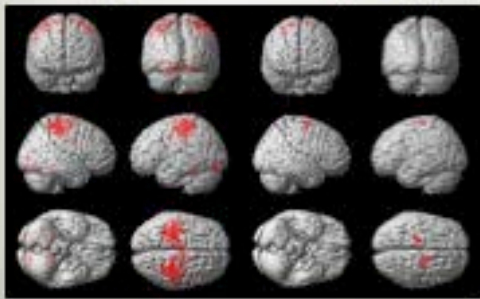
- Approach developed by Worsley and colleagues to account for multiple comparisons
- Forms basis for much of SPM
- Provides false positive rate for fMRI data based upon the smoothness of the data
- If data are very smooth, then the chance of noise points passing threshold is reduced

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FINAL PRODUCT

Tapping > 0

Both > (Left + Right)



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COURSE PAPER ASSIGNMENT

- Journal article review
 - Choose one high impact peer-reviewed journal article from the MRI literature.
 - Write a 3 page critical review of this paper with appropriate comparison to related papers.
- Due Tuesday December 16th 2008

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