Basic Nuclear Magnetic Resonance

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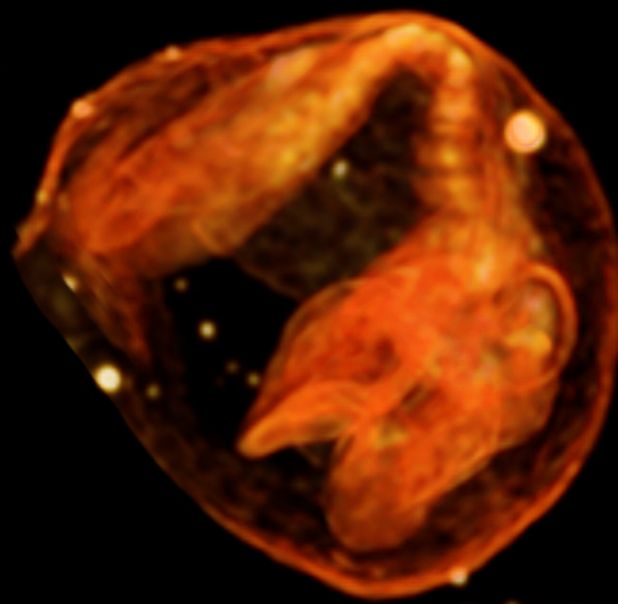
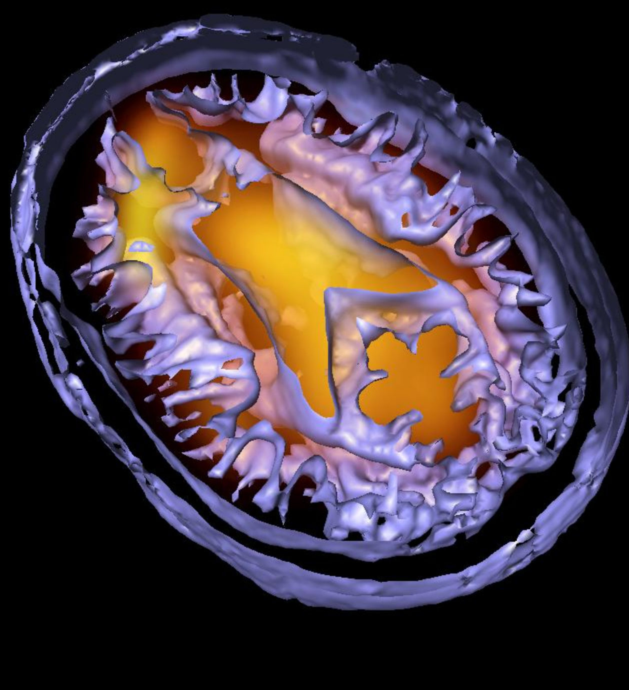
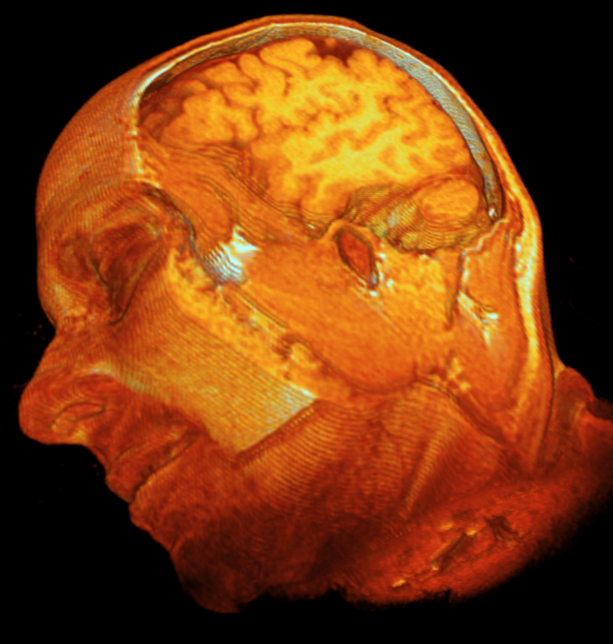
Magnetic Resonance Theory Course 2004

Caltech Brain Imaging Center

California Institute of Technology



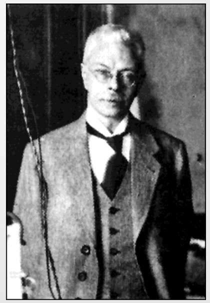




HISTORY OF NMR

Early Years 1890-1937

1896 Pieter Zeeman observes sodium D line splitting in a magnetic field



1921 Stern and Gerlach atomic beam/magnetic field experiments

1933 Stern and Gerlach observe nuclear spin effect in hydrogen beam

1890 | 1900 | 1910 | 1920 | 1930 | 1940

1896 Sir Joseph Larmor publishes magnetic field effect on radiation frequency

1927 Pauli and Darwin include spin in quantum mechanics

1937 Rabi predicts and observes NMR in a LiCl beam

1945/6 First NMR in bulk materials

– EM Purcell/Torrey/Pound at Harvard - Paraffin at 30MHz
– F Bloch/Hansen/Packard at Stanford - Water at 8MHz

From NMR to MRI

1950 Hahn Spin Echoes

1951 Packard and Arnold observe temperature dependence of chemical shift

1952 Nobel Prize in Physics to Bloch and Purcell

1973 First Magnetic Resonance Images by Paul Lauterbur

1975 Fourier Transform MRI suggested by Richard Ernst

1977 Mansfield develops Echo Planar Imaging

1977 Damadian first human MRI

Late 1980s Ogawa observes BOLD response at Bell Labs

1950 1960 1970 1980 1990 2000

Modern MRI

1991 Nobel Prize in Chemistry to Ernst

MAGNETISM

Forms of Magnetism

- Paramagnetism
  - Electronic Paramagnetism
  - NUCLEAR PARAMAGNETISM
  - Superparamagnetism
- Diamagnetism
  - Most organic compounds
- Ferromagnetism
  - Metallic Fe, Ni, Co

Magnetic Susceptibility

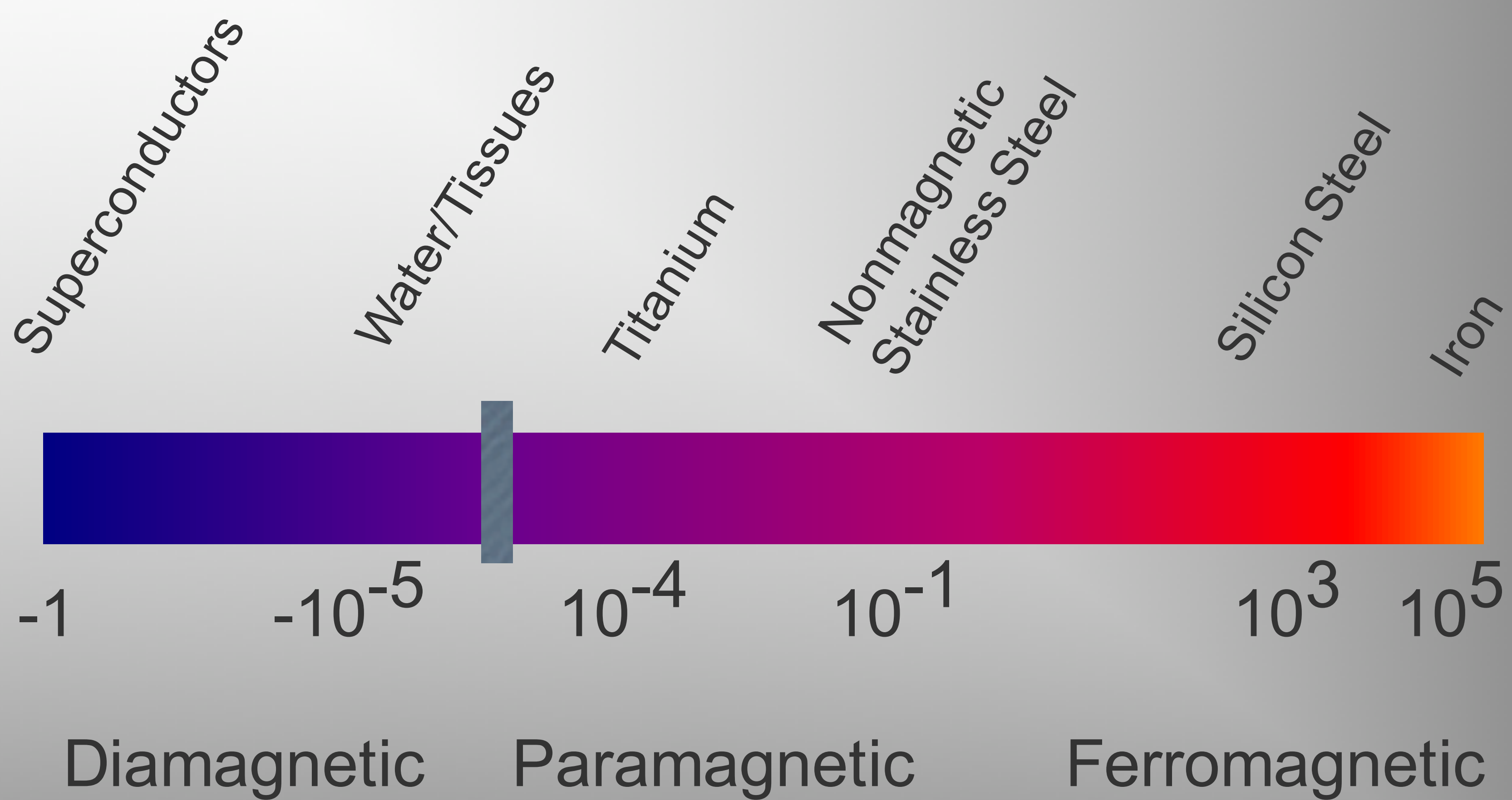
Susceptibility is dimensionless and relates the magnetization of a material to an applied magnetic field.

$$\mathbf{M} = \chi \mathbf{H}$$

$$\mathbf{B} = \mu_0 (\mathbf{H} + \mathbf{M})$$

$$= \mu_0 \mathbf{H} (1 + \chi)$$

Susceptibility Spectrum



Magnetic Units

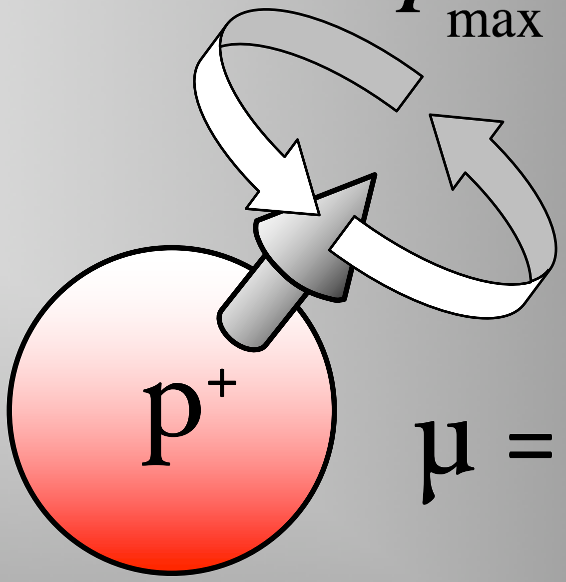
Quantity	Symbol	Unit	Unit Abbr	SI Equiv
Magnetic Flux Density	**B**	Tesla	T	kg s^2 A^{-1}
Magnetic Field Strength	**H**	(Oersted)		A m^{-1}
Magnetic Flux		Weber	Wb	m^2 kg s^2 A^{-1}

1 Tesla (SI) = 10,000 Gauss (cgs)

Nuclear Spin and Magnetic Moment

- Spin angular momentum (I) is an intrinsic property of subatomic particles.
- Neutrons and protons (I = 1/2) both contribute to the total nuclear spin.
- A nucleus carries a total positive charge.
- A nucleus has a magnetic moment $\mu = \gamma \hbar I$

$$P_{\max} = \hbar I$$

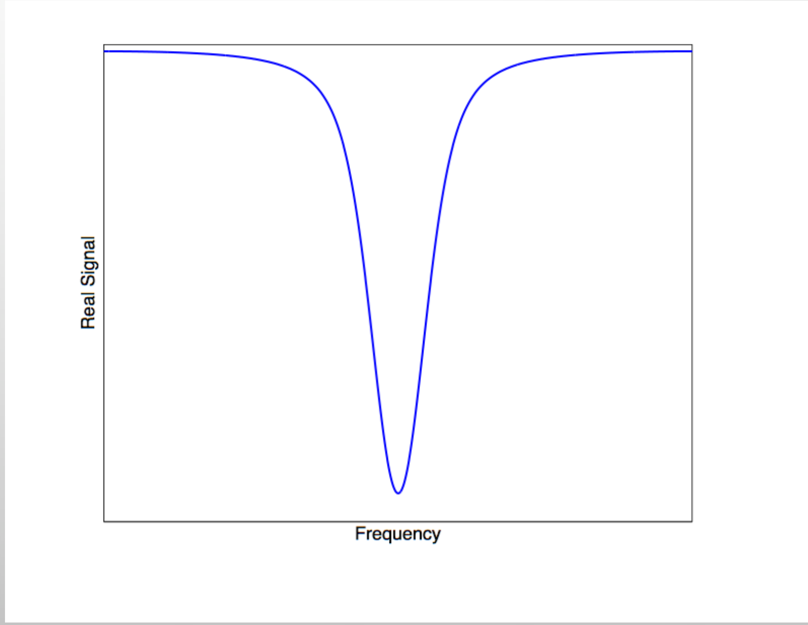


$$\mu = \gamma \mathbf{P}$$

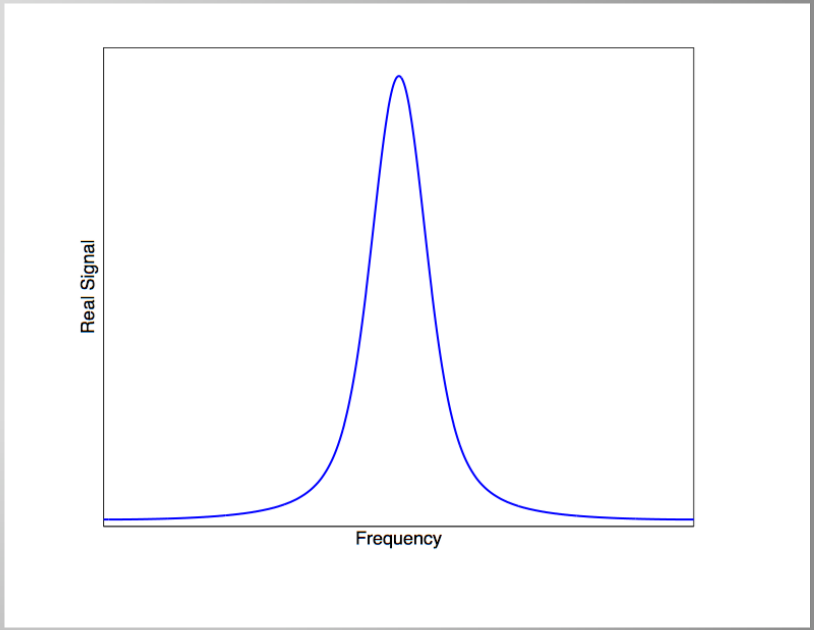
μ approximately 10^{-4} to 10^{-3} Bohr magnetons (10^{-27} to 10^{-26} Am^2)

Nuclear Magnetic Resonance

Continuous Wave Absorption



Pulsed RF Excitation



Increasing RF Frequency
Constant Field Strength

Increasing RF Frequency
Constant Field Strength

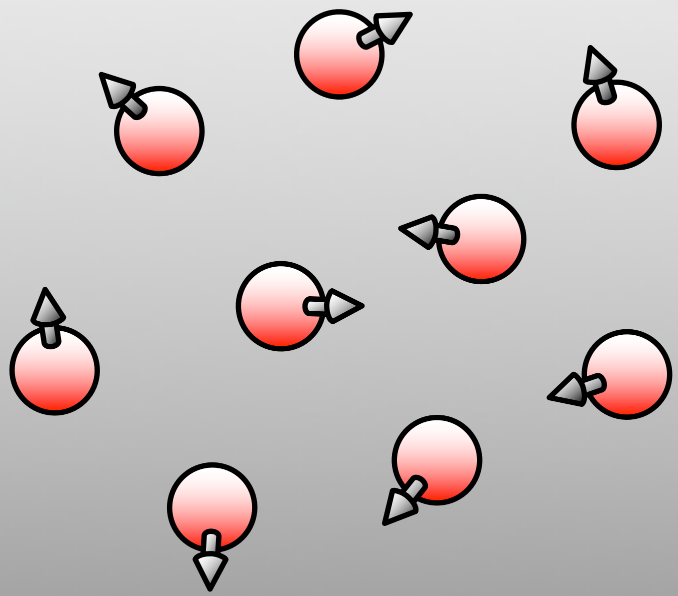
Increasing Field Strength
Constant RF Frequency

Biologically Relevant Nuclei and Spin

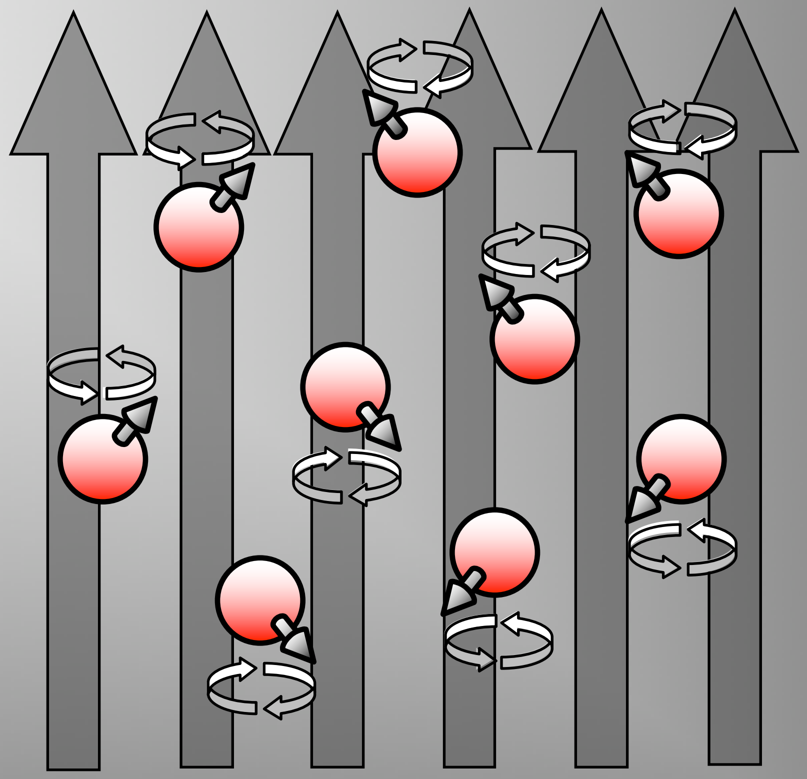
Nucleus	Spin	γ (MHz/T)	Natural Abundance (%)	Sensitivity (%)
^{1}H	1/2	42.58	99.99	100
^{2}H	1	6.54	0.015	0.965
^{12}C	0	-	-	-
^{13}C	1/2	10.71	1.1	1.59
^{14}N	1	3.08	99.63	0.101
^{31}P	1/2	17.24	100	6.63

Nuclear Zeeman Effect

- A magnetic field forces nuclei with a net magnetic moment to align in two or more energy states.

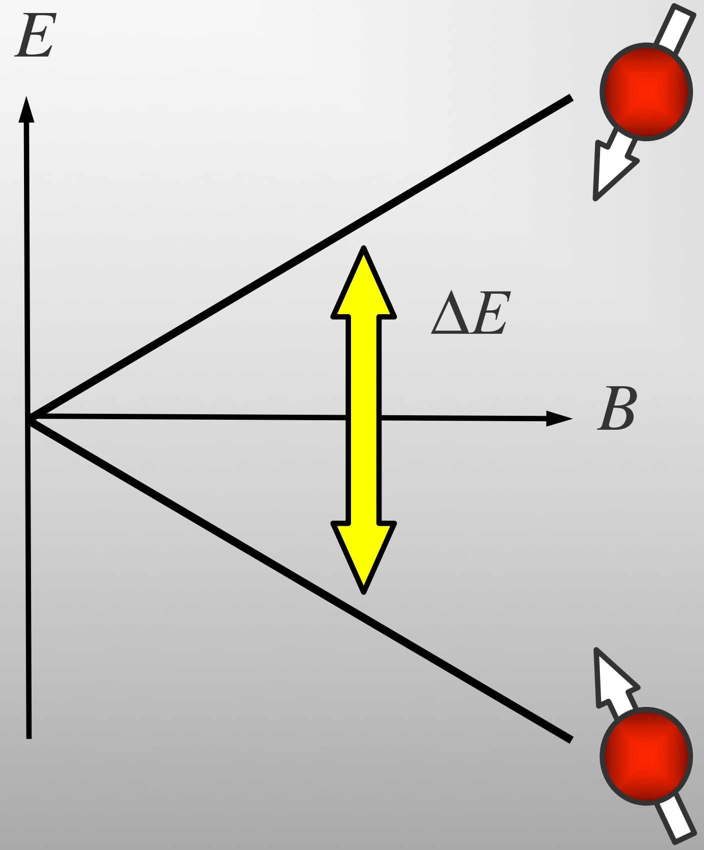


Spins align randomly in absence of a magnetic field



Magnetic Field

Larmor Relation



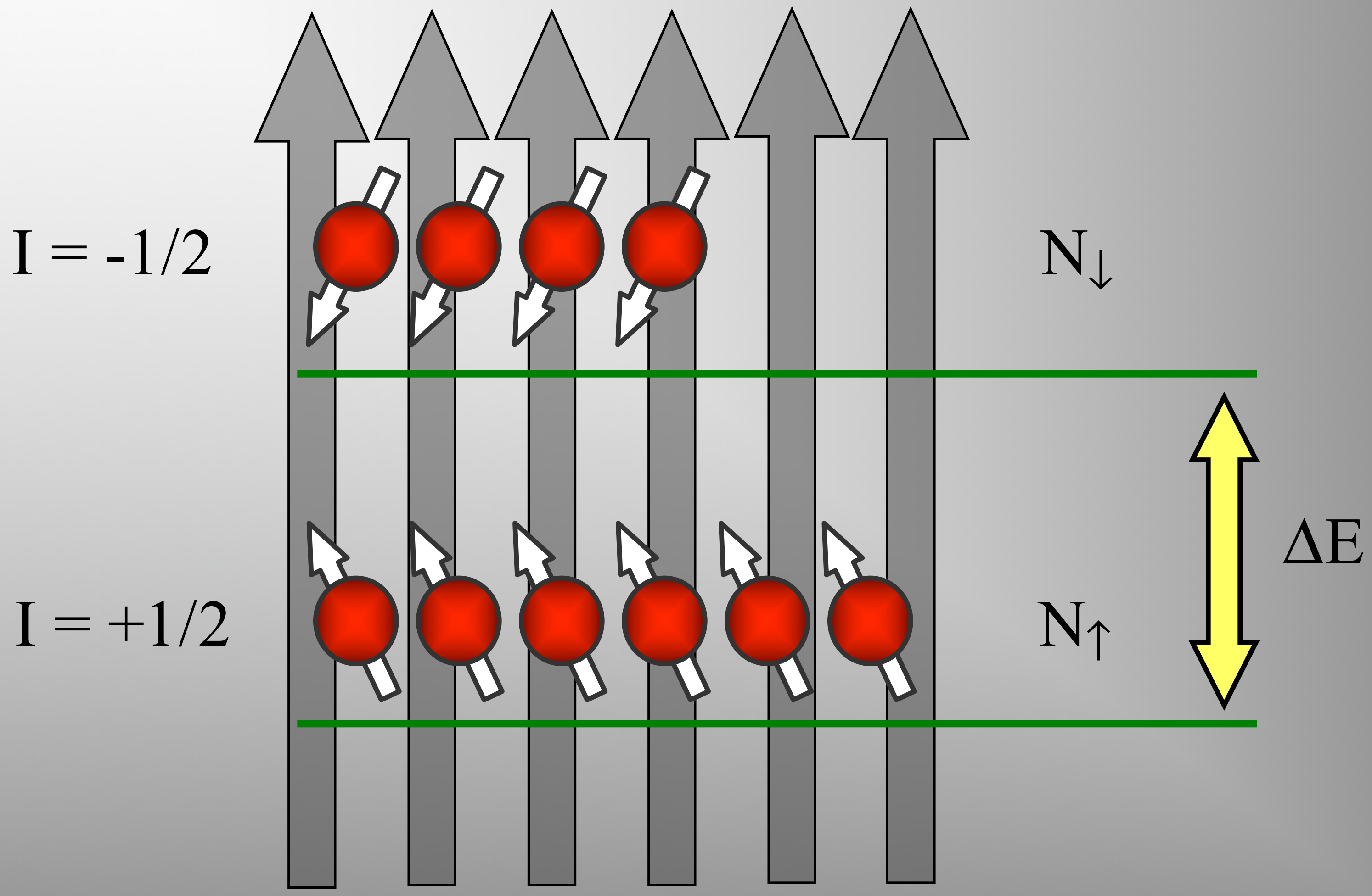
Precessional Frequency

$$\boxed{\Delta E = \hbar\omega = \hbar\gamma B}$$

Photon Energy

Gyromagnetic Ratio

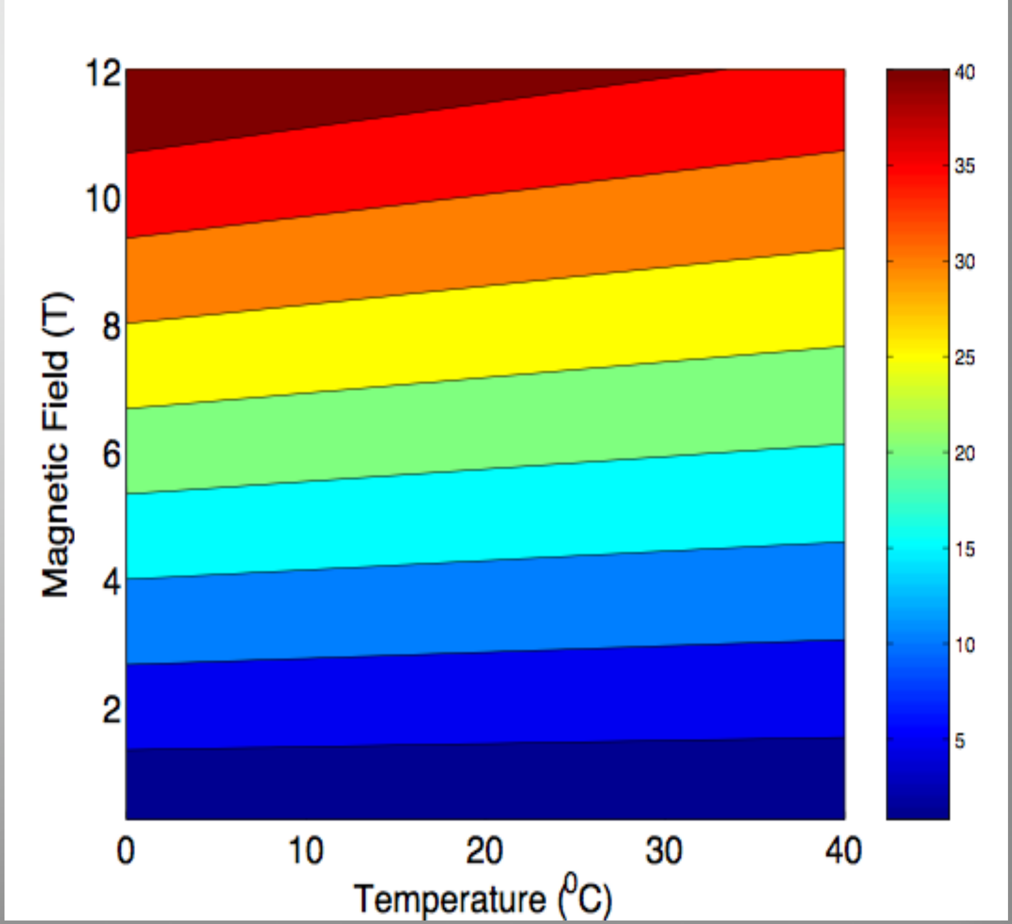
Population of Energy Levels



Population of Energy Levels

Boltzmann distribution of ¹H energy level populations

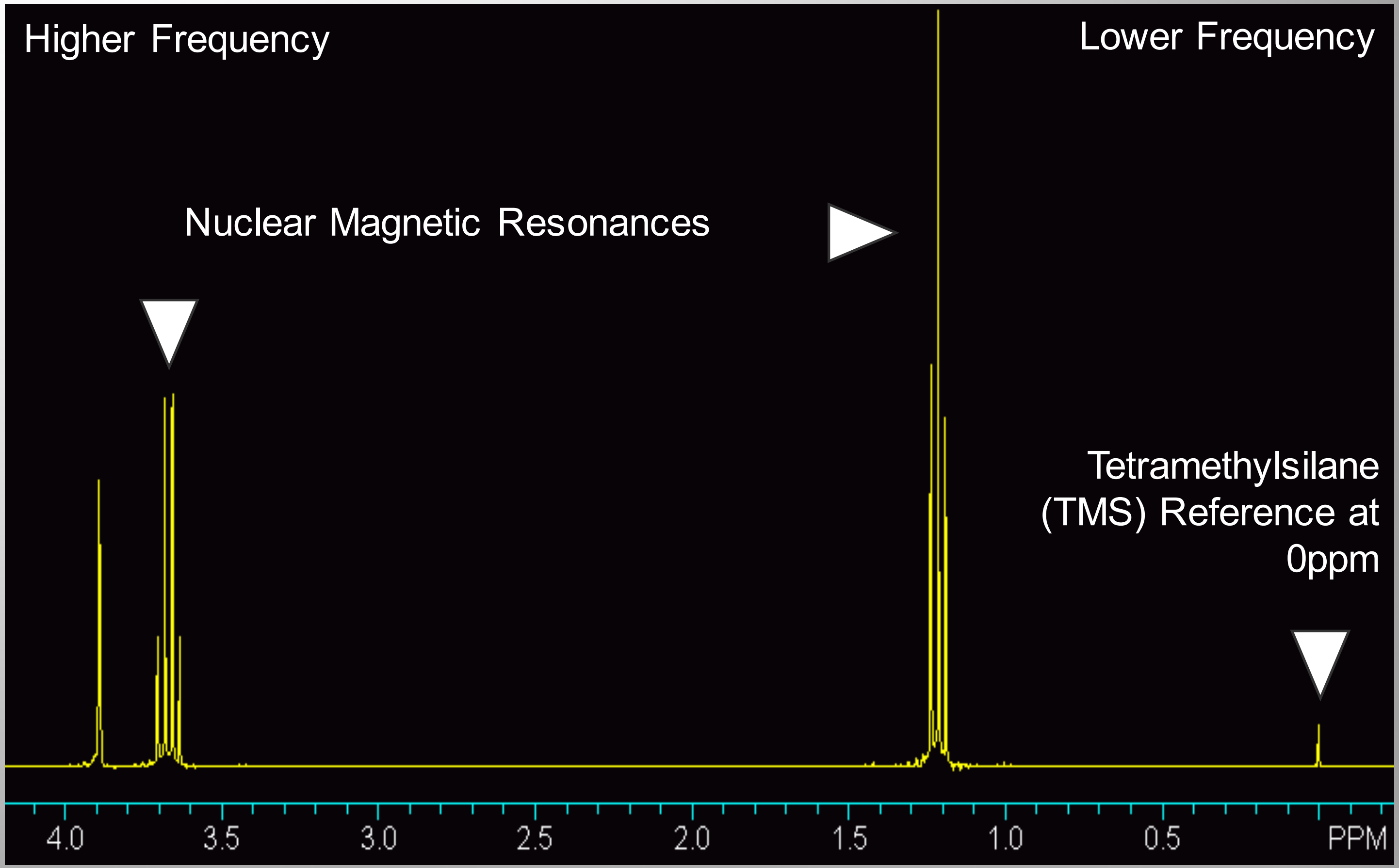
$$N_R = \frac{N_\downarrow}{N_\uparrow} = e^{-\Delta E/kT} = e^{-\hbar\gamma B/kT} < 1$$



Excess fraction in low energy state

$$\Delta N_R = \frac{N_\uparrow - N_\downarrow}{N_\uparrow + N_\downarrow} = \frac{1 - N_R}{1 + N_R}$$

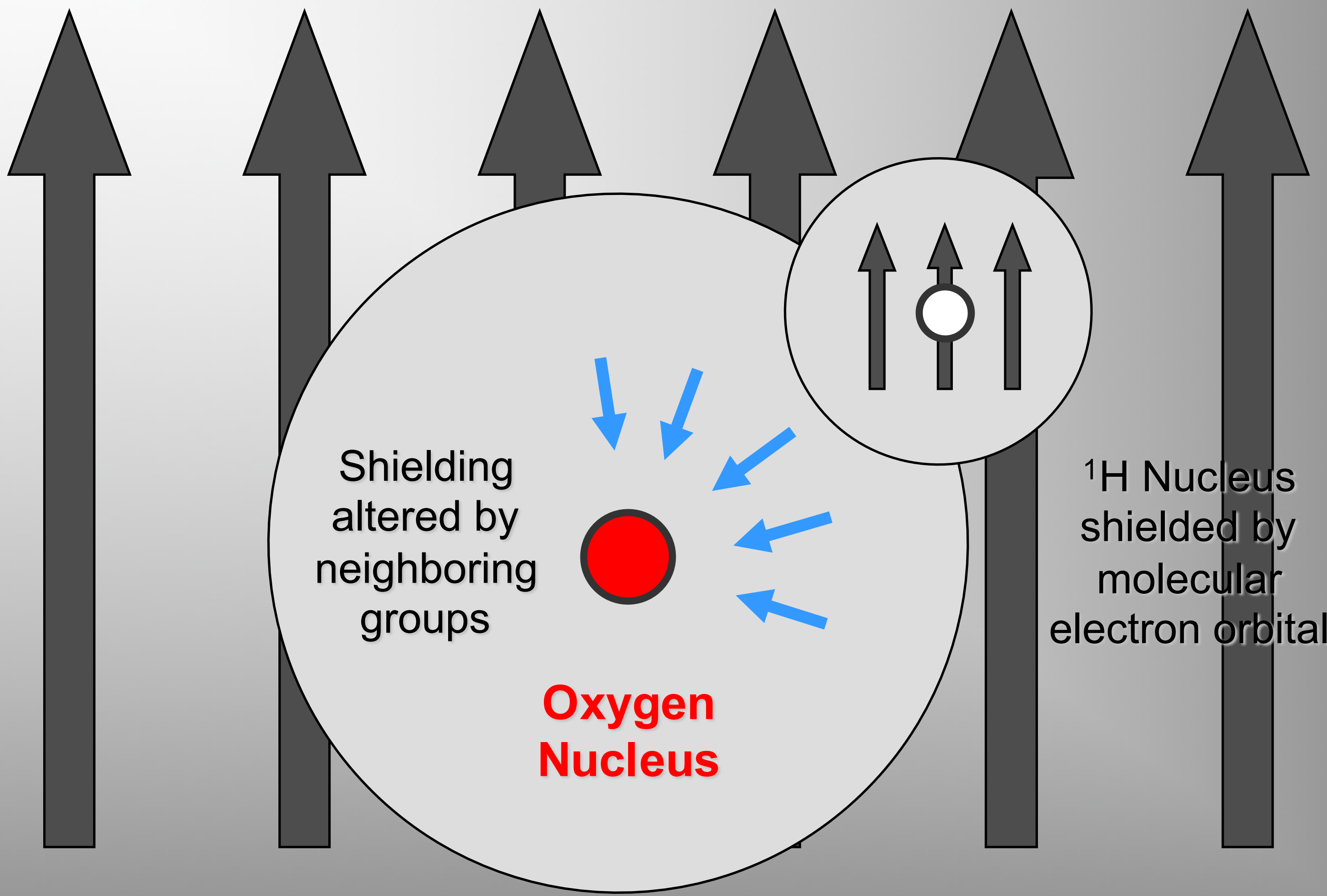
NMR Spectra : Chemical Fingerprints



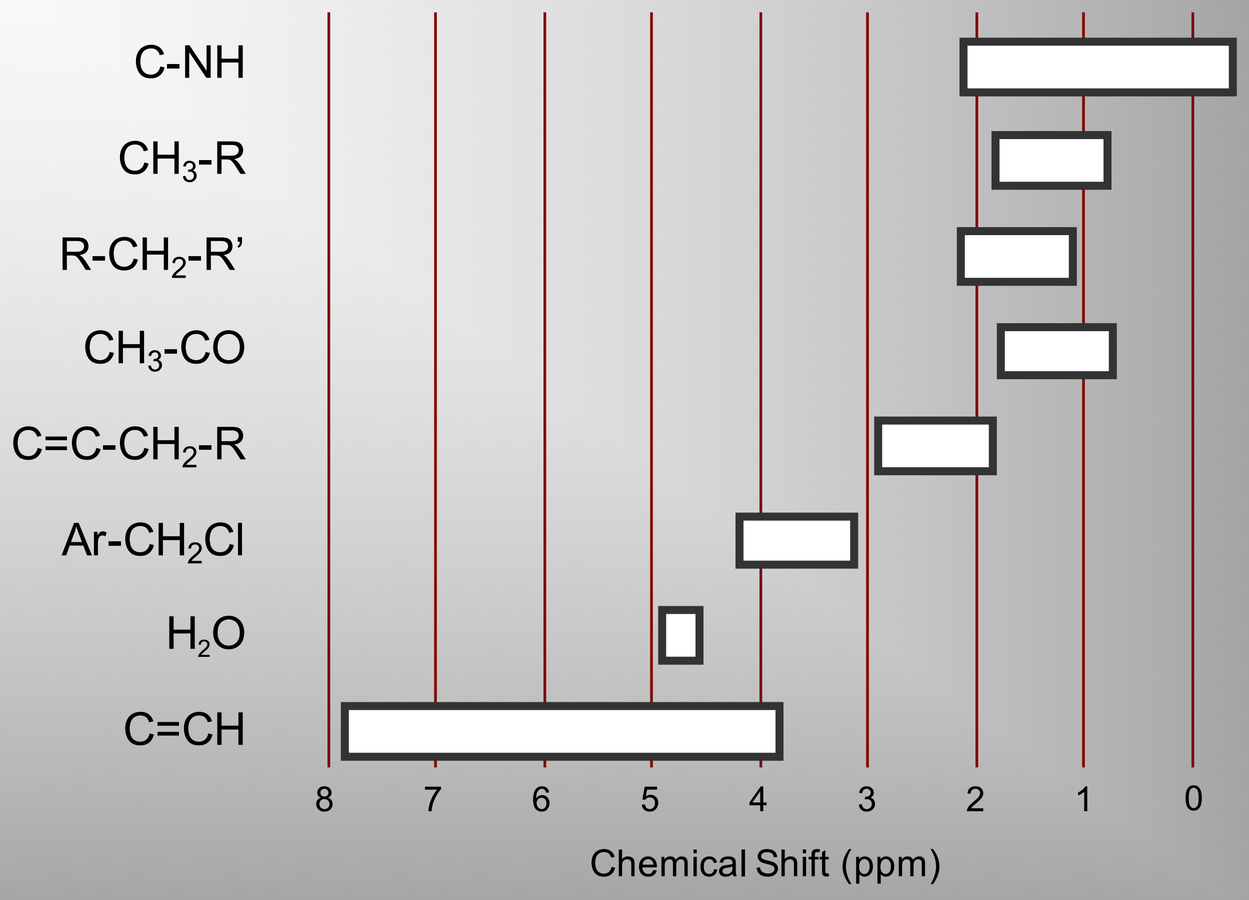
300MHz High Resolution 1H NMR Spectrum of Ethanol

The frequency scale is in parts per million (ppm) and runs from right to left.

The Origin of Chemical Shift

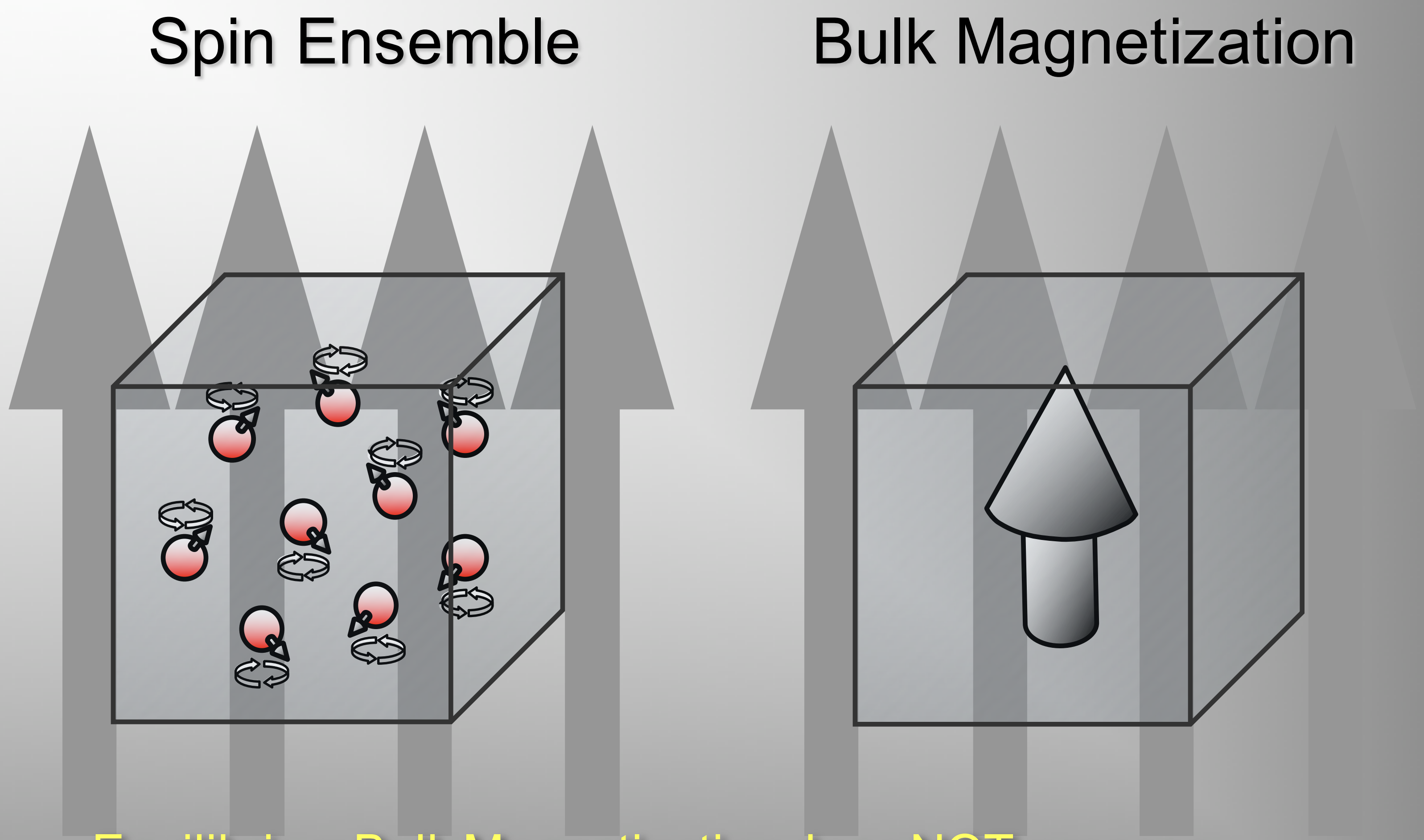


Chemical Shift Ranges



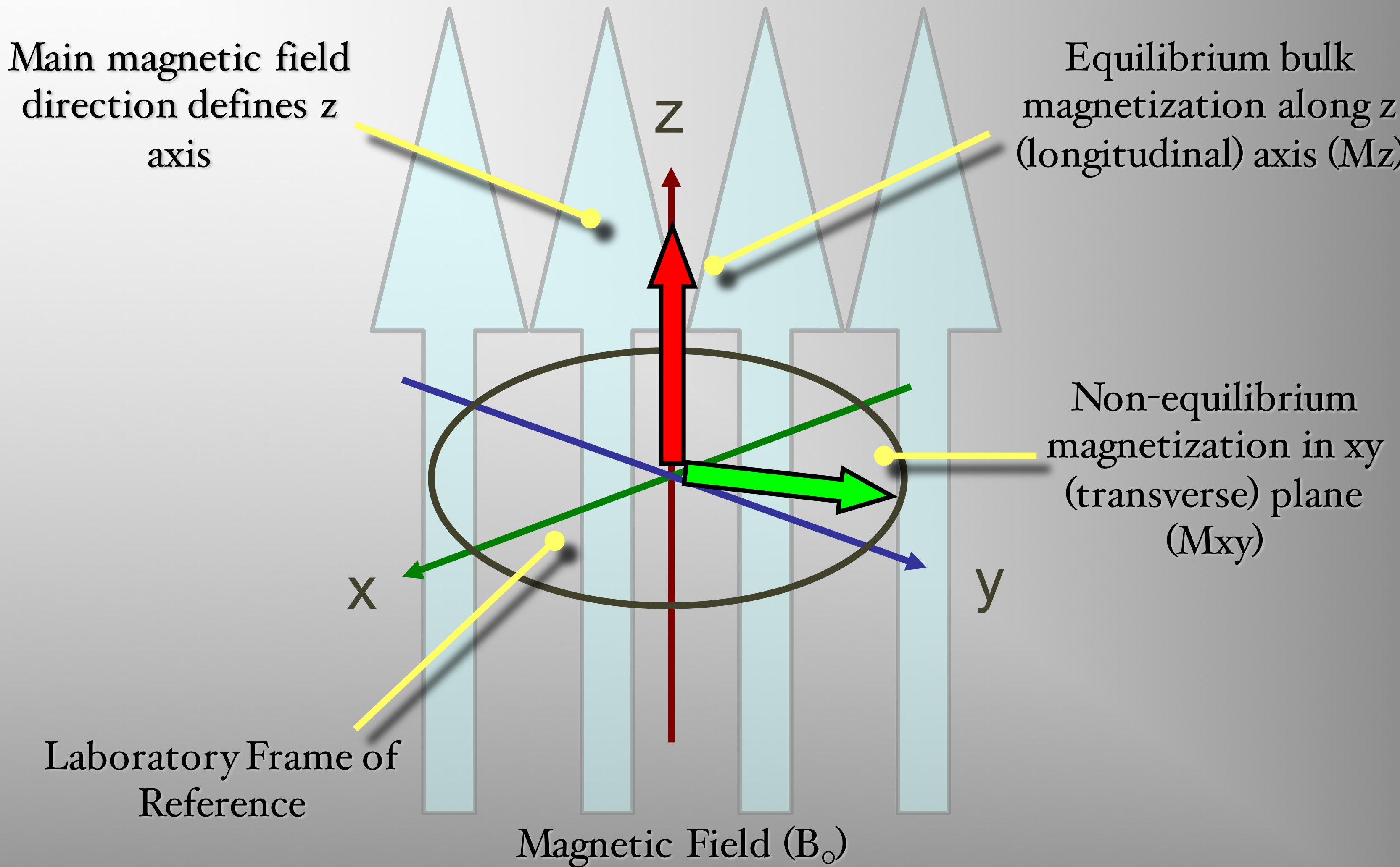
Adapted from Sorrel TN Interpreting Spectra of Organic Molecules. University Science Books, Mill Valley CA. 1988

Bulk Magnetization



- Equilibrium Bulk Magnetization does NOT precess
- A typical MRI voxel (1µl) contains roughly 10^{19} spins

The Semi-classical Vector Model



The Bloch Equations

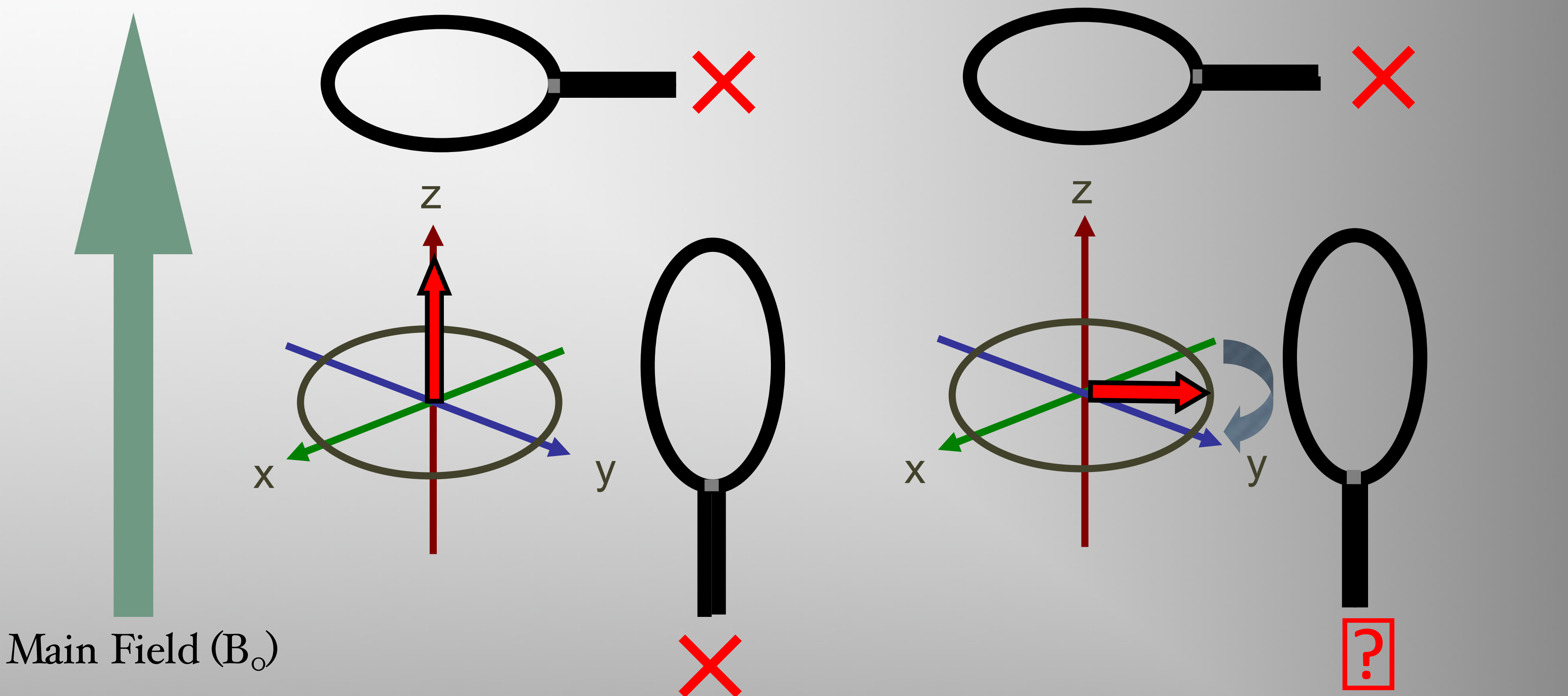
Provide a classical description for the evolution of the magnetization with time.

$$\frac{dM_x}{dt} = \gamma(\mathbf{M} \times \mathbf{B})_x - \frac{M_x}{T_2}$$

$$\frac{dM_y}{dt} = \gamma(\mathbf{M} \times \mathbf{B})_y - \frac{M_y}{T_2}$$

$$\frac{dM_z}{dt} = \gamma(\mathbf{M} \times \mathbf{B})_z + \frac{M_0 - M_z}{T_1}$$

Detecting Magnetization

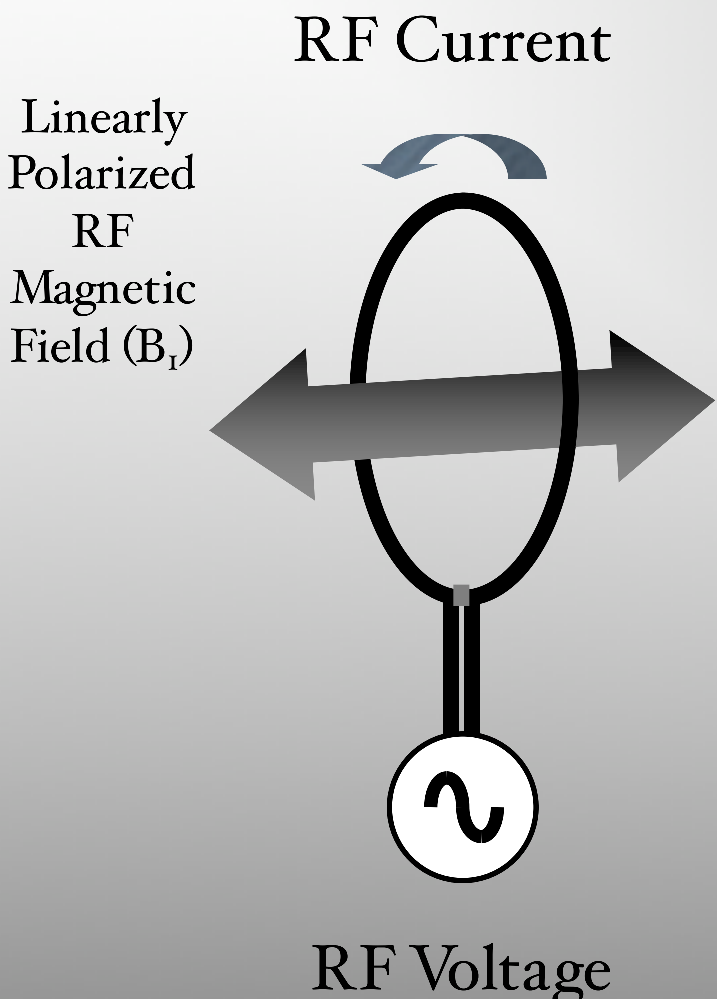


Equilibrium bulk magnetization does not precess => no induction in any coil

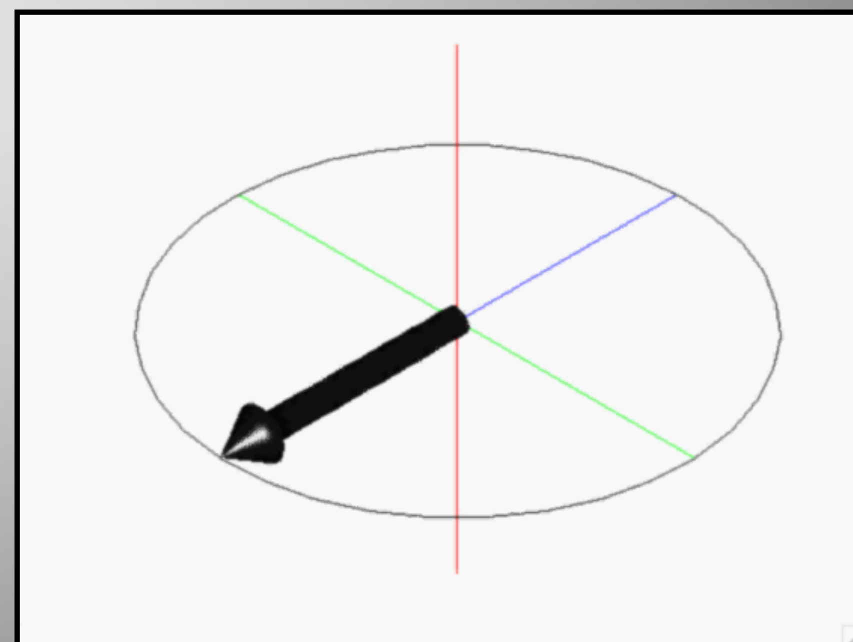
Bulk transverse magnetization precesses about the main field => detected by a loop perpendicular to main field

So How Do We Generate Transverse Magnetization?

Linear and Circular Polarization

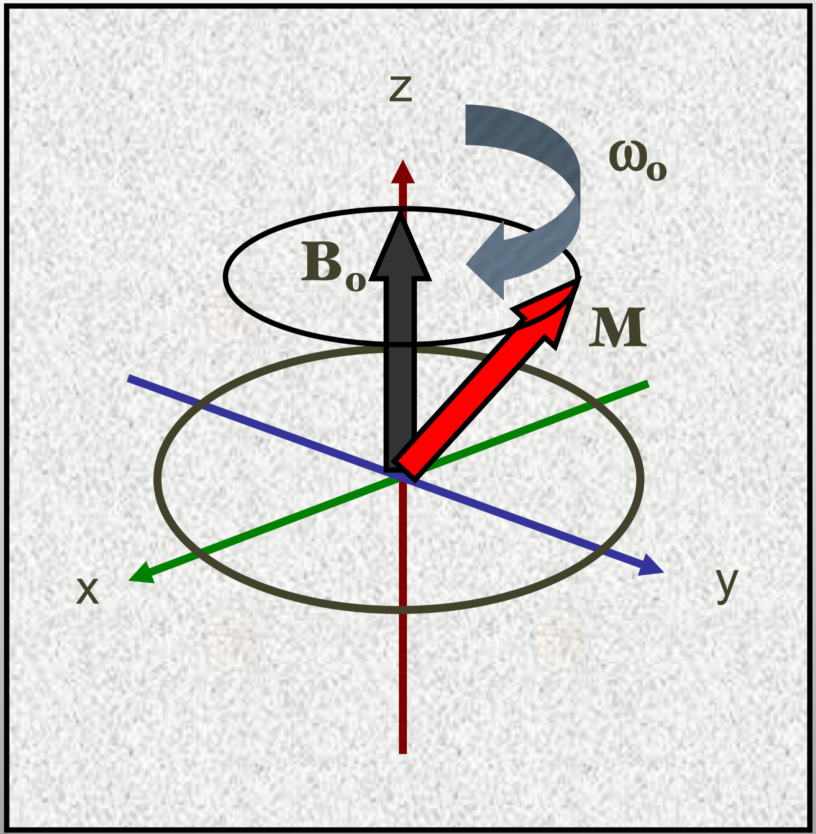


Decomposition of a linearly polarized field into two counter-rotating circularly polarized fields

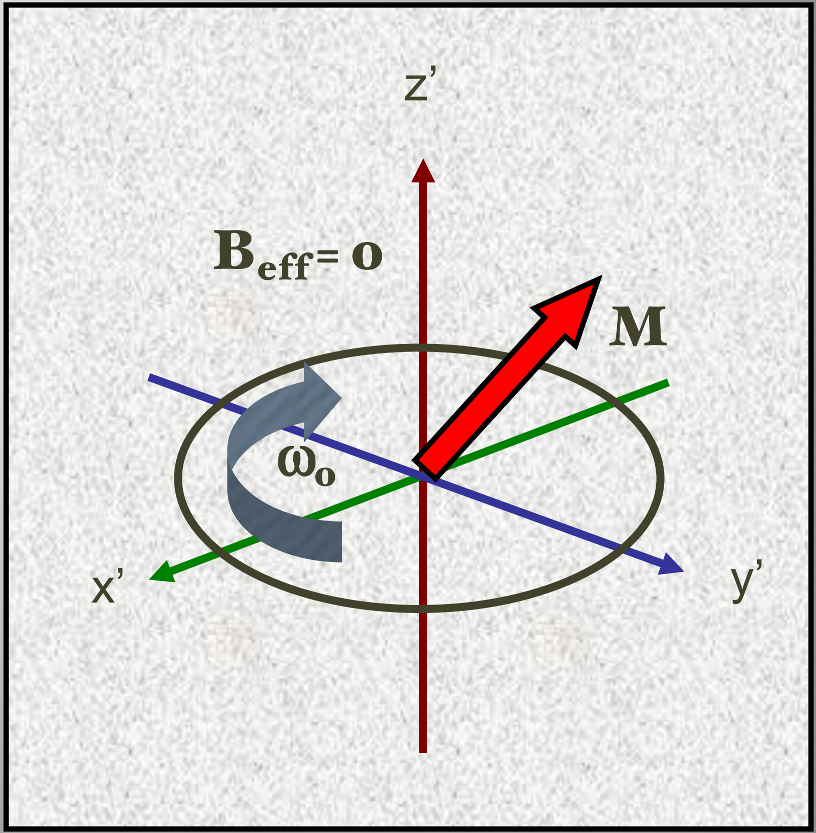


The Rotating Frame

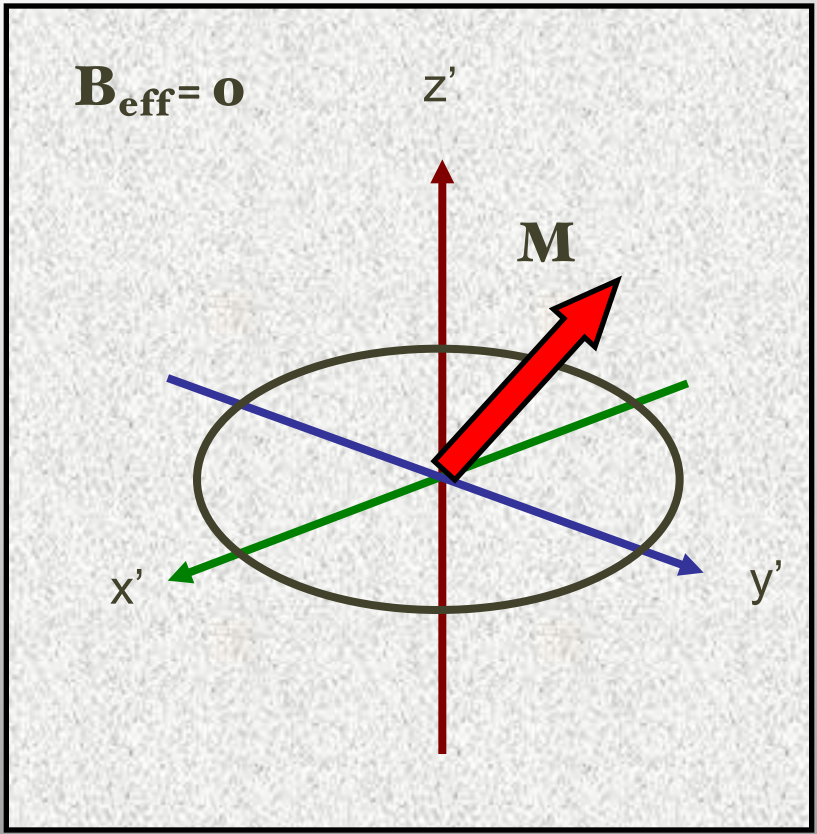
Laboratory Frame

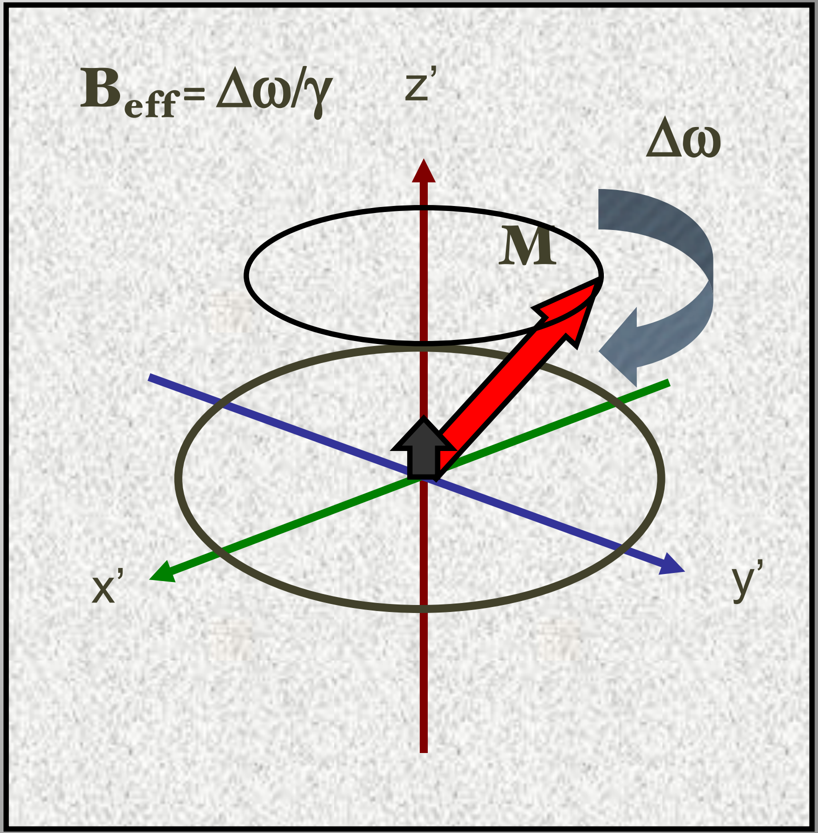


Rotating Frame



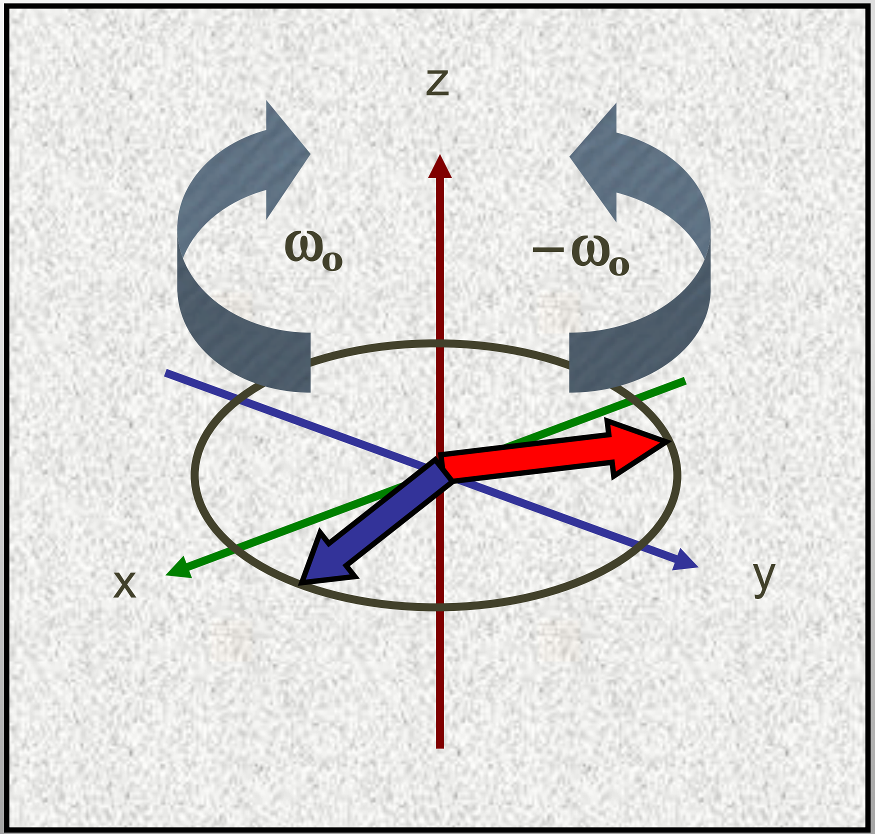
Off-resonance in the Rotating Frame



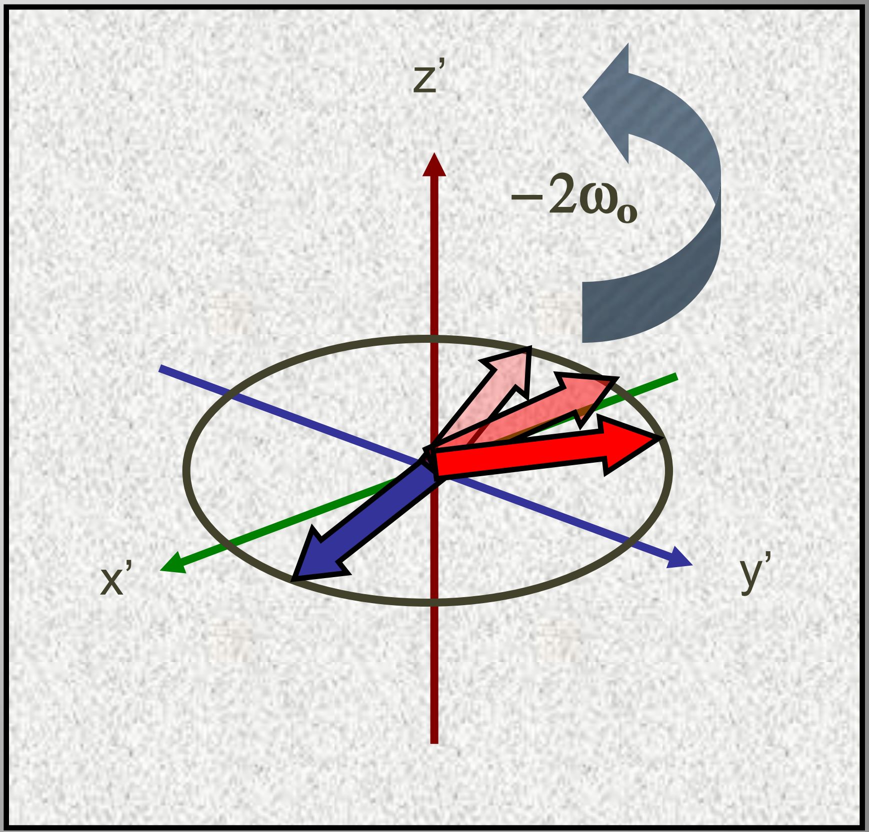


Circularly polarized RF: Laboratory and Rotating Frames

Laboratory Frame

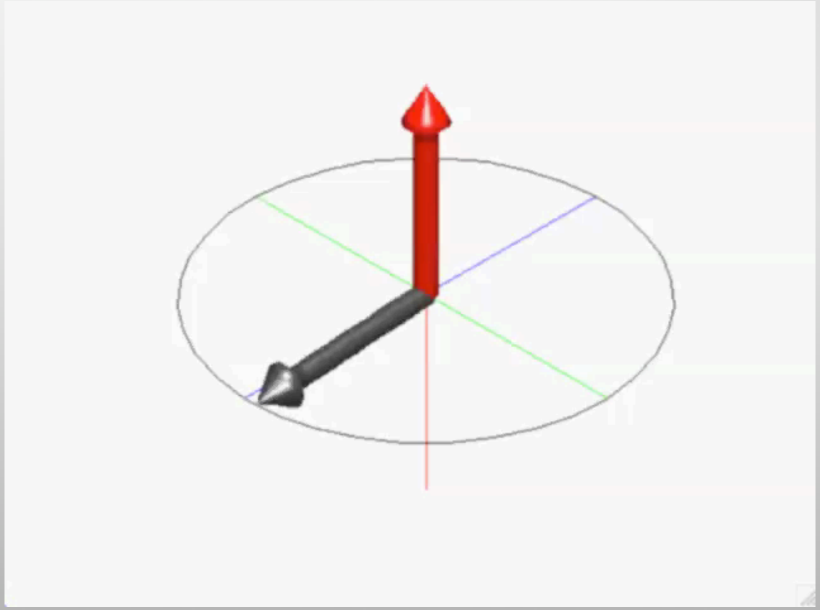


Rotating Frame

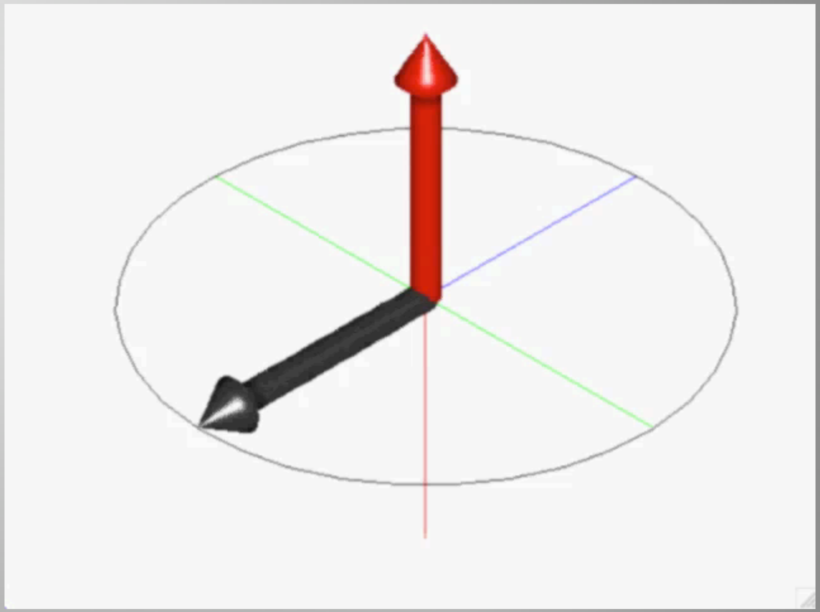


Circularly polarized RF: Laboratory and Rotating Frames

Laboratory Frame



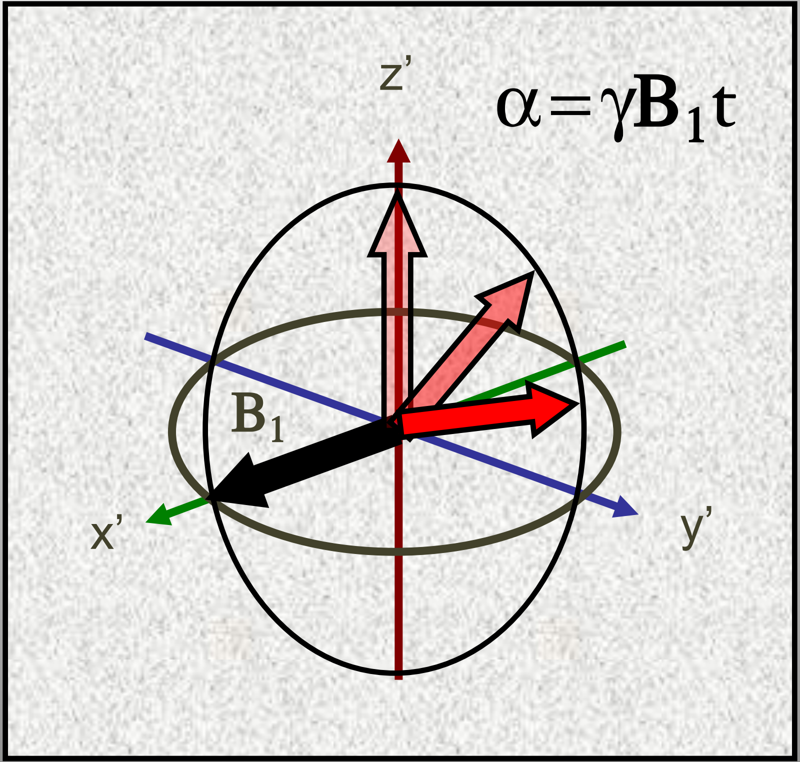
Rotating Frame



And that's how we generate transverse magnetization!

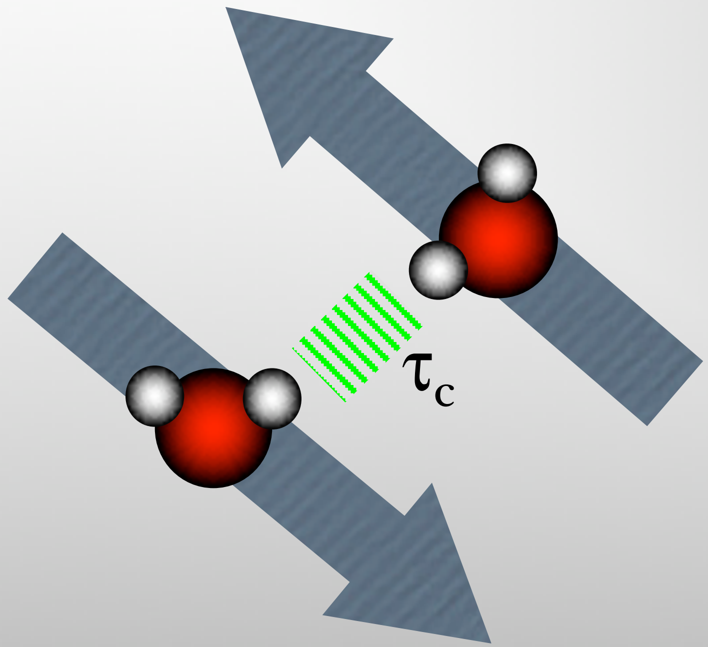
Precession about B1: The Flip Angle

- Magnetization precesses about B_1 (+ B_{eff}) for as long as the RF pulse is on.
- Total "flip angle", α, is proportional to B_1 (+ B_{eff}) and pulse duration.

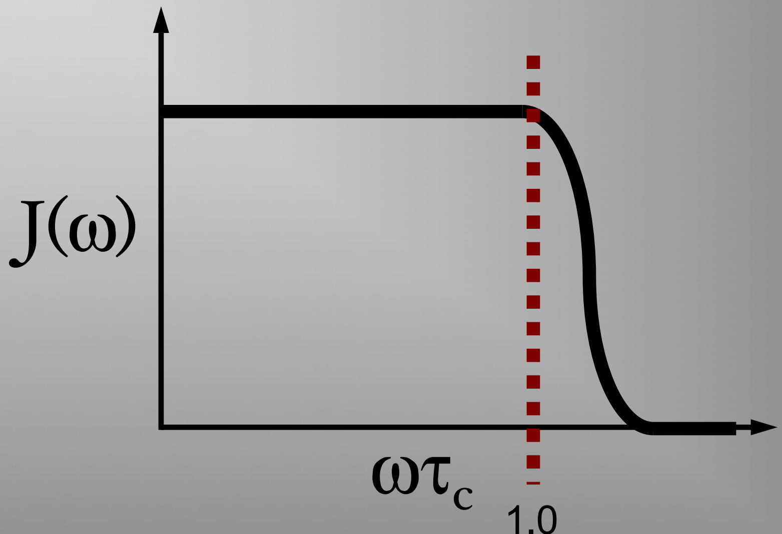


Relaxation

Correlation Time and the Spectral Density Function



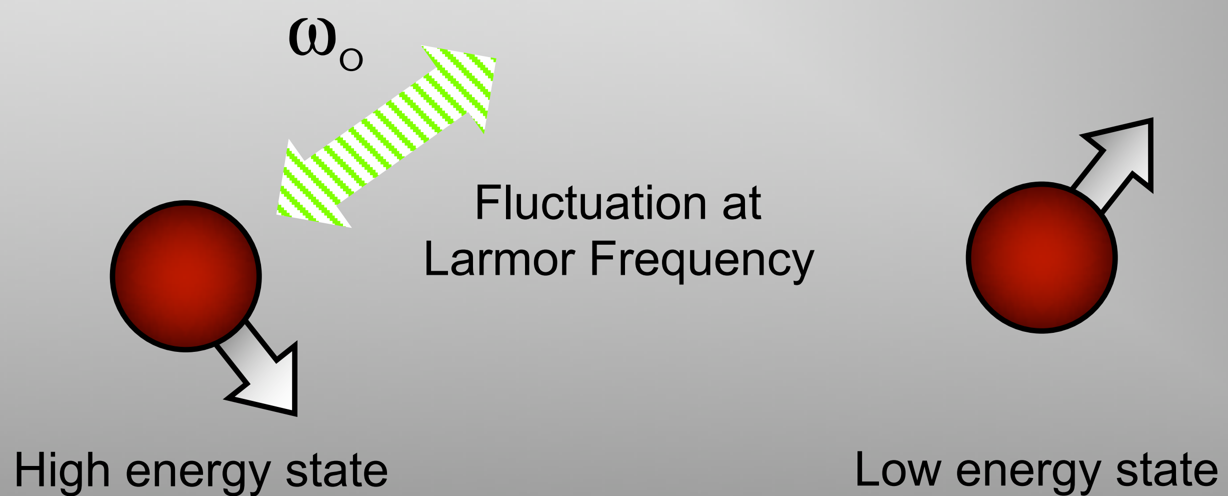
Nuclear spins experience short duration (τ_c) dipole-dipole interactions giving rise to a spectrum of magnetic field fluctuation frequencies (spectral density function)



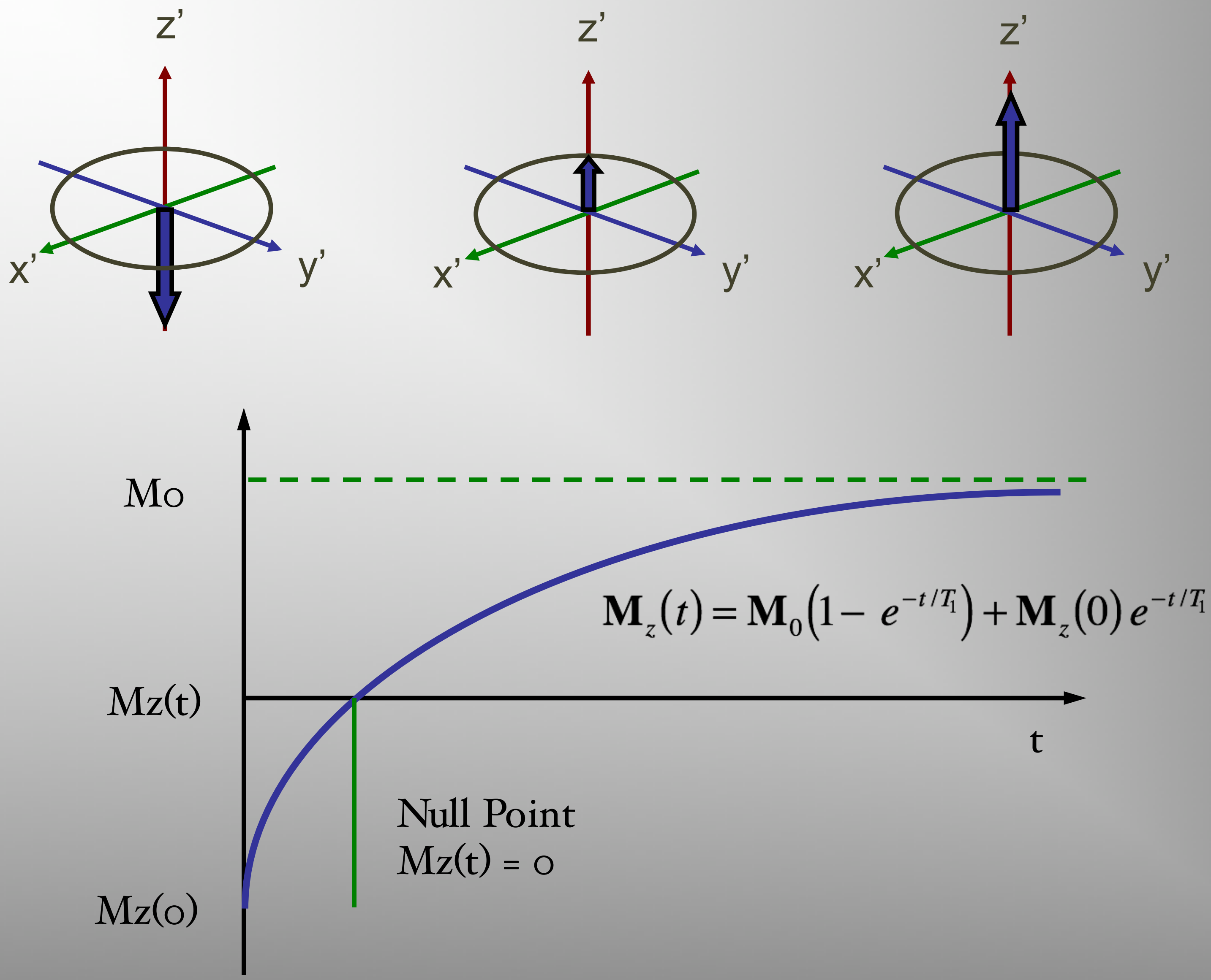
Basis of Bloebergen-Percell-Pound (BPP) relaxation theory

T1 (Spin-Lattice) Relaxation

- Spins do not spontaneously decay from higher to lower energy states because the resonant frequency is very low (MHz).
- T1 relaxation is driven ONLY by field fluctuations at the Larmor frequency.
- The longitudinal magnetization decays exponentially to equilibrium
- Typical T1 for liquid water in tissues ranges from ms to seconds.



T1 (Spin-Lattice or Longitudinal) Relaxation



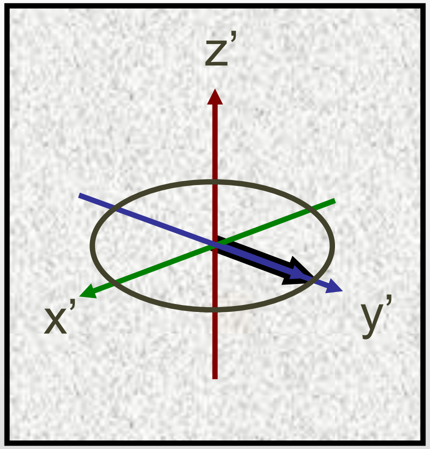
T2 (Spin-Spin or Transverse) Relaxation

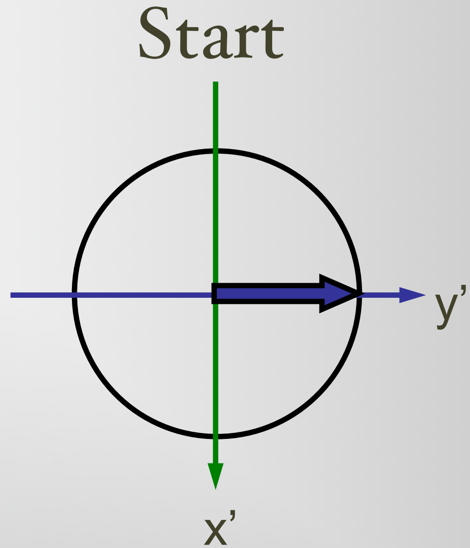
- T2 relaxation is driven by field fluctuations at both zero frequency AND the Larmor frequency.
- Spins are in phase following an RF pulse.
- The transverse magnetization decays exponentially to zero due to cumulative dephasing.
- Typical T2 for liquid water in tissues ranges from ms to seconds.

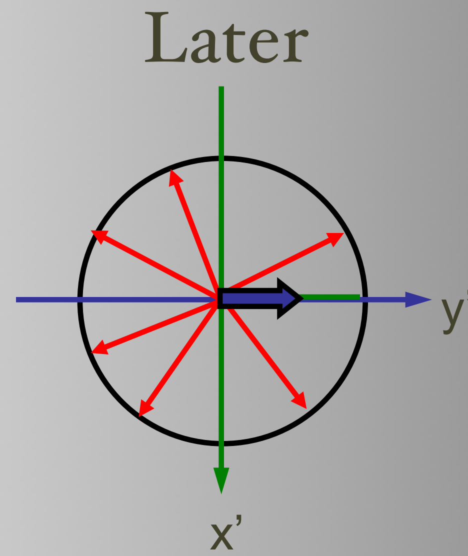
Fluctuation at either zero frequency or the Larmor Frequency causes spin-spin energy exchange.

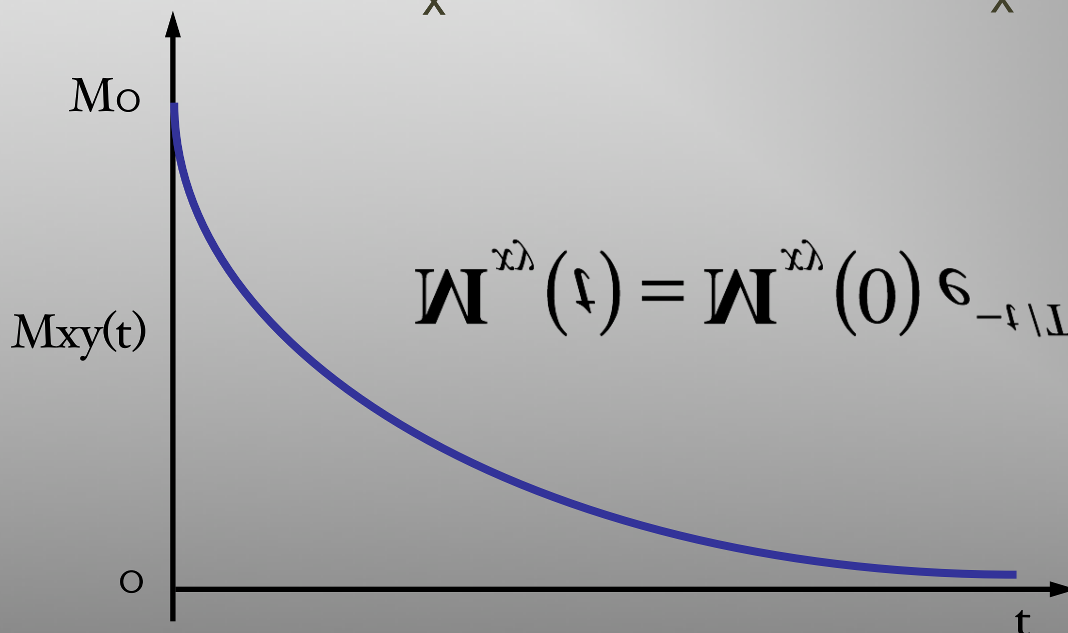
Total energy preserved.
Phase randomized.

T2 (Spin-Spin) Relaxation





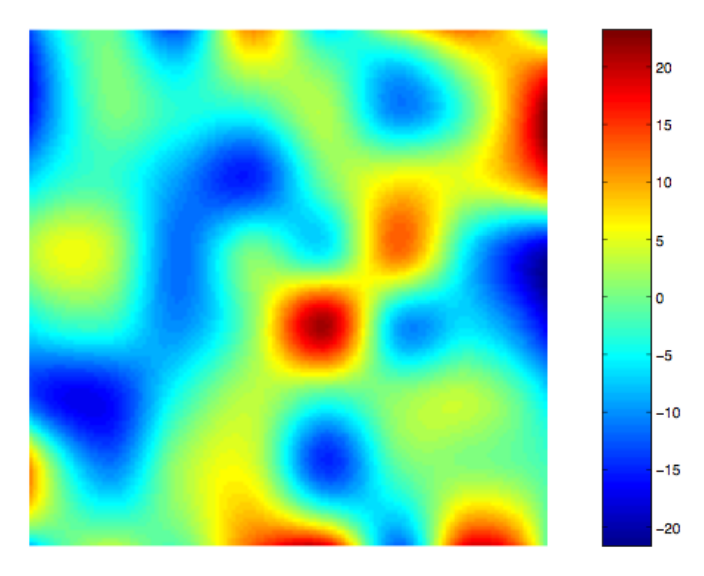


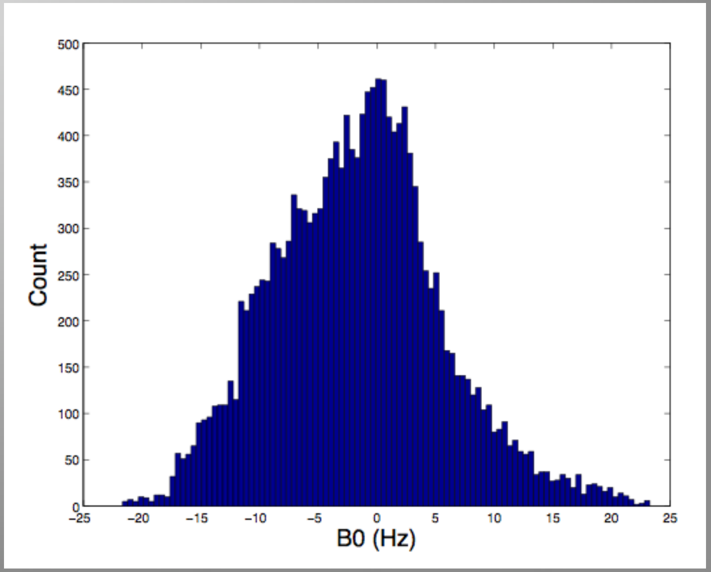


$$M_{xy}(t) = M_{xy}(0)\, e^{-t/T2}$$

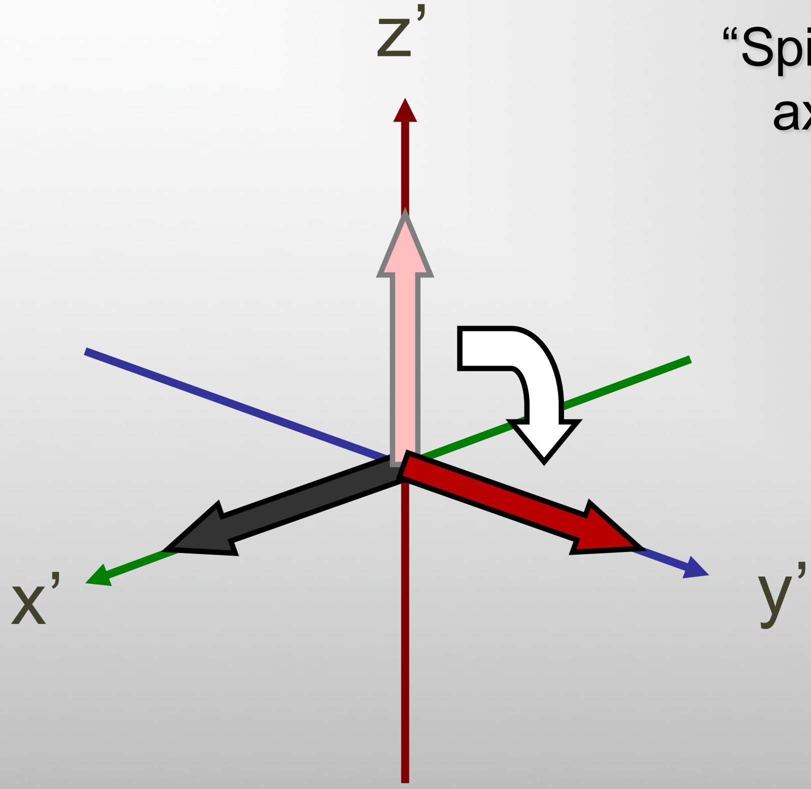
T2* Relaxation

- B_o inhomogeneities within a volume (right) cause additional dephasing of total Mxy.
- Decreases apparent T2 by providing an additional dephasing mechanism.
- T2* is NOT an exponential process - B_o distribution (below right) would have to be Lorentzian).
- Basis of BOLD and susceptibility sensitive imaging.
- Addressed in part by magnetic field shimming (homogeneity optimization.



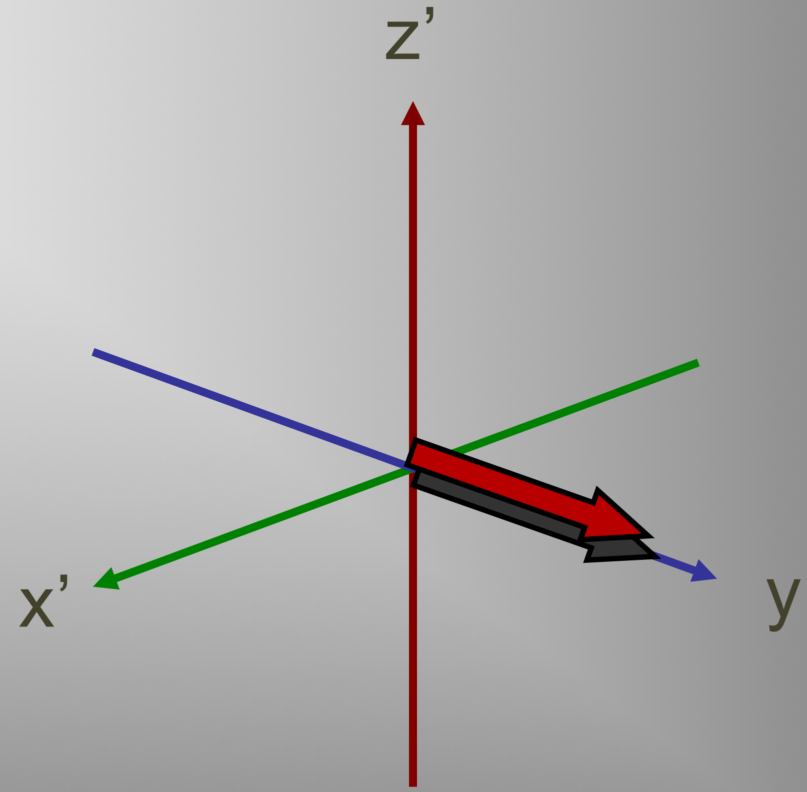


T1ρ Relaxation

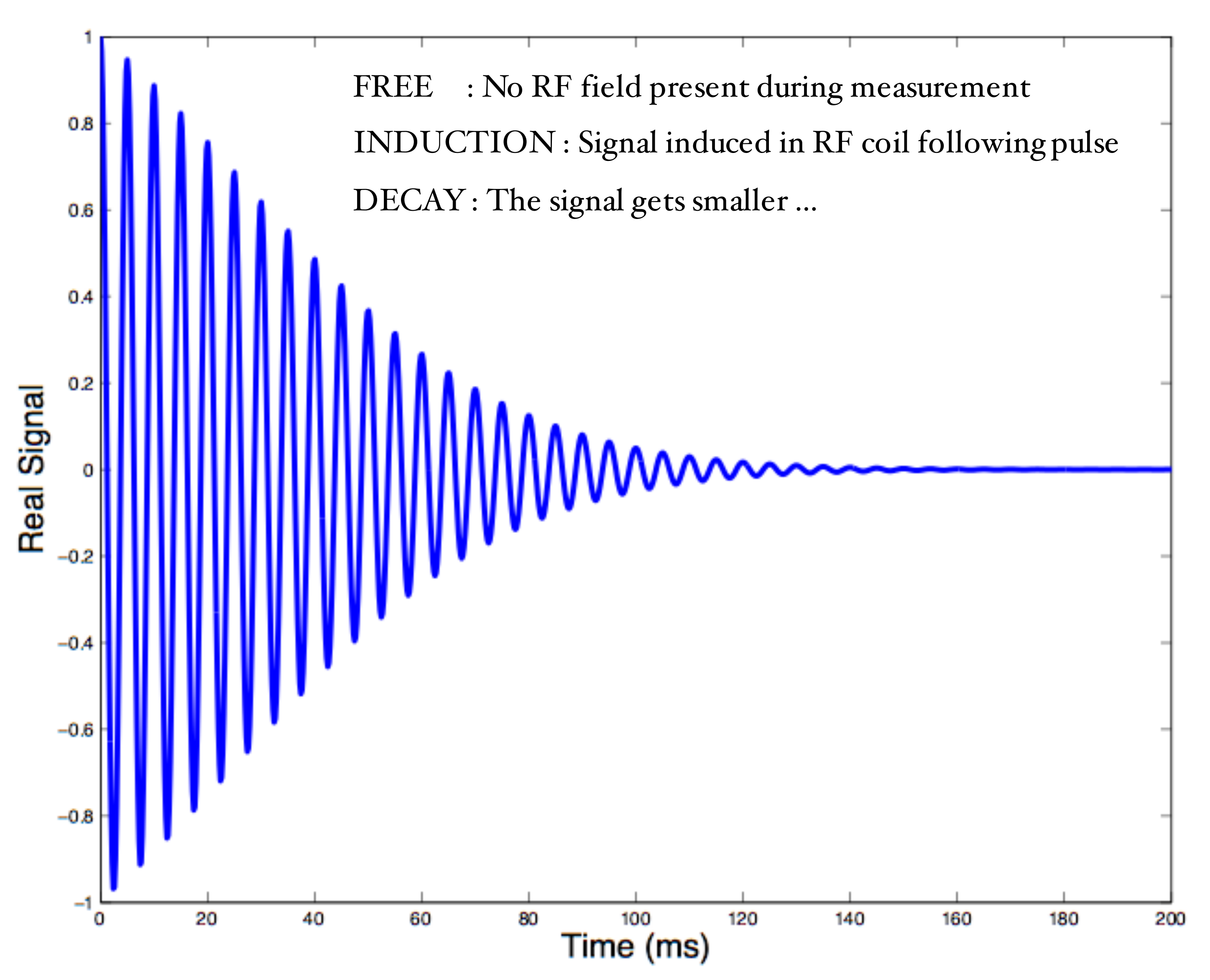


Excite with a 90 degree pulse along the x’ axis

“Spin Lock” using an RF pulse along the y’ axis. Spin locked relaxation rate is $T_1\rho$



The Free Induction Decay



Next Lecture : Basic MRI

Turning the NMR Signal into an Image